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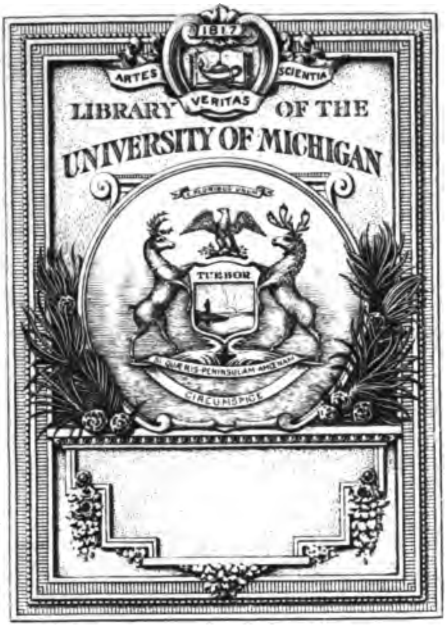
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THE
ELEMENTS
OF
OPTICS.

Emerson, William
Cyclomathesis

THE ELEMENTS OF OPTICS.

IN FOUR BOOKS.

Book I. Simple Optics, or Direct Viſion.
Book II. Catoptrics, or Reflected Viſion.
Book III. Dioptrics, or Refracted Viſion.
Book IV. The Conſtruction of Optical Instruments.

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PREFACE.

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THE *Ancients, whilſt they made a pretence of ſtudying Philoſophy, and ſearching out the cauſes of natural things, had no other method to proceed by, than forming hypothetical conjectures out of their own heads according to their various fancies, without any foundation in the nature of things. And then they endeavoured to defend their hypotheſes, by plauſible arguments of wit, and the figures of Logic; by which they flattered themſelves, they had fully eſtabliſhed their hypotheſes, and given a true account of natural cauſes and effects, and that beyond the poſſibility of any oppoſition. And all this while they had not the leaſt thought of making ſo much as one experiment, to confirm the truth of their theories; which is the only poſſible way, that any theory in nature can be eſtabliſhed. By this means it came to paſs, that their philoſophy was no more than a philoſophy of words and empty ſounds, conſiſting of endleſs terms of art, without any meaning or ideas annext to them. And whilſt they proceeded in this abſurd method, true philoſophy was quite at a ſtand, and made no manner of progreſs; and the nature of things remained perfectly unknown. Let any operation in nature be propoſed, and they preſently had a name for it; and a few terms of art were to explain its nature, though they could not give us the leaſt notion, how or by what* *cauſes ſuch an operation or effect was brought about. And thus they imagined they could ſolve every difficulty in nature, by the mere magic of ſounds. Examples of this way of proceeding are endleſs; but there is nothing in which they have blundered more egregiouſly, than in explaining the nature of light, and the manner of viſion.*

But the moderns finding themſelves no wiſer by this philoſophical gibberiſh, found it neceſſary to purſue another method. They ſaw plainly, that the nature of things could never be known without careful obſervation and experience; therefore they ſet about to make all the experiments upon any ſubject, that could poſſibly be made. By this means they could find out what cauſes could produce ſuch and ſuch effects, what cauſes and effects were linked together, and always accompanied one another; and what proportion they bore to one another; and after they had found theſe things in one ſubject, by applying the ſame method to other ſubjects of the ſame kind; they could find how far theſe laws were general, and agreeable to all of that ſort. In proſecuting this method, it is found, that there are ſome properties that belong but to a few kinds of bodies, and ſome that belong to a great many kinds; and ſome properties that belong univerſally to all bodies: this teaches us ſomething; and the nature and properties of bodies, thus found out by experiments, is called Experimental Philoſophy. *By this method our great* Newton, *the Prince of Philoſophers, found out the nature of light, he analized it, and diſſolved it into its conſtituent particles. He has ſhewn us, that light conſiſts of particles of different colours, and different degrees of refrangibility; and that the colour of each ray of it is permanent and laſting; and its degree of refrangibility invariable: that the particular colour of it can never be changed by any refraction or reflection. But before that, Philoſophers were all of opinion, that light had no difference or variety of its con-*

ſtituent parts, but was in its own nature, ſimple and uniform; and that colours were nothing but certain modifications of the light, or dilatations of the rays, cauſed by reflexions and refractions; whereas they depend upon a real ſeparation of the heterogeneous rays. And all optic writers were generally of opinion, that the reflexion of light was cauſed by the particles thereof ſtriking againſt the ſolid parts of bodies, which they could not enter into, and thence repelled back again, like other reflected bodies. But Sir J. Newton *has ſhewn, that they are driven back and reflected before they touch the body; and that by ſome inherent power in the reflecting body, which acts at a ſmall diſtance from its ſurface, in lines perpendicular thereto.*

Who could ever have thought, that the opacity of bodies depended on their poroſity; and yet the ſame great Philoſopher has ſhewn, that their opacity ariſes from the multiplicity of reflections within the pores of bodies, when the particles are of a determinate ſize. And the ſame bodies become tranſparent, when their pores are filled with a medium of the ſame denſity as the particles of the body.

It is now put paſt diſpute, that viſion is cauſed by the rays of light entering the eye; and painting the image of the object, upon the retina at the bottom of the eye. If theſe rays paſs thro' the ſame medium continued between the object and the eye, then they come from every part of the object to the eye in right lines; then this is called Direct Viſion. *When the rays come not directly to the eye from the object, but fall upon ſome poliſhed body, and are reflected from thence to the eye, in which they form the image of that object; then this ſort of viſion is called* Reflex Viſion. *Here the eye properly ſees the image cauſed by the reflexion, and not the object itſelf. A body muſt be truly poliſhed to give a perfect image by reflection; for a rough uneven ſurface ſcatters the rays, that they make no image, or a very deformed one. If* *the rays paſſing from the object to the eye, go thro' different mediums, as air, water, glaſs, &c. then the rays will be refracted at entering a different medium, and come to the eye in other right lines, of different direction from the former; and this is called* Refracted Viſion. *So that there are three different caſes of viſion, whereby objects may be ſeen; and theſe make three diſtinct parts of the ſcience of* Optics; *of which the firſt is the moſt ſimple, and the laſt the moſt complex.*

Since we find the properties of light to agree with the known properties of body, we cannot ſuppoſe light to be any thing elſe but a body, conſiſting of exceeding ſmall particles; which are continually ſhaken off by the vibrating motion of the luminous body, and expelled by the repellent power thereof, with incredible velocity from it in all directions. Some of which, by their exceeding ſmallneſs and tenuity, go thro' every point of ſpace. Some of theſe enter our eyes, whoſe texture and conſtruction is properly adapted thereto, and do there by their motion excite vibrations in the optic nerves, by which we have the ſenſation of ſeeing; and by the wonderful ſtructure of our eyes, give us certain intelligence of objects exiſting without us, and their affections, ſuch as their ſize, form, colour, motion, and even their diſtance: ſo that viſion is not made at random, but is performed by certain uniform and ſteady laws.

It appears by experiment, that light conſiſts of parts both ſucceſſive and contemporary. For one may ſtop that which comes one moment, and let that paſs which comes the next. One may alſo ſtop it in one place, and let it paſs in another. And the part that is ſtopt cannot be the ſame with the part that is gone paſt.

As viſion is the moſt uſeful, as well as the moſt delightful ſenſe we have; ſo that ſcience which teaches the improvement and perfection of this ſenſe, muſt be the moſt valuable of any, and that is Optics, *which not only explains the manner of viſion, but from the* *ſame principles teaches how to ſupply a proper help for bad eyes, and to remedy the ſeveral defects of our ſight, by the help of proper inſtruments contrived according to the rules of this art, ſuch as ſpectacles, reading glaſſes, teleſcopes, microſcopes, &c. and to this art is owing the invention of ſeveral inſtruments, which by curious and ſurprizing operations, give us the moſt exquiſite delight; ſuch as the camera obſcura, the magic lanthorn, the perſpective views of countries, &c. ſo that it is a ſcience of infinite pleaſure as well as profit.*

Were there no other uſe of Optics, but the contrivance of ſpectacles, to help the eyes of old men and purblind people; yet that alone would be ſufficient to recommend the ſtudy of it. Of what infinite advantage are theſe to mankind, not one in a thouſand, but what has need of their help, at ſome time of life, eſpecially in old age. For as the ſight is the moſt noble and moſt extenſive of all our ſenſes; and as we can go about no concerns in life without making uſe of our eyes; certainly the inſtrument muſt be the moſt valuable, which relieves the decayed eyes, and cures their defects, and makes them uſeful, and without which, they would be almoſt uſeleſs. They are certainly in a miſerable condition that have the two windows of the ſoul ſhut up, and are obliged to live in perpetual darkneſs. And ſuch as have them partly obſcured, partake of the ſame miſery in ſome degree. How miſerable would the latter part of men's lives be, if there were no ſuch thing as ſpectacles; and how tedious muſt the lives of old people have been before their invention. Without them a mechanic grown old could not follow his buſineſs, ſuch as watchmakers, engravers, &c. nor a ſcholar converſe with his books, or with his friends by letter. So that this curious invention cannot be too highly valued.

Our natural ſight is confined to very narrow views, and tho' it is very extenſive, yet ſome objects at a diſtance are ſcarce perceptible, while others more remote, cannot be ſeen at all. Here then we have the advantage of teleſcopes, by which we can behold very remote objects very clearly; thus we can have a full view of the heavenly bodies, and by that means underſtand the wonderful mechaniſm of the world; without this inſtrument, all theſe charming ſcenes of nature, ſo far diſtant, would be hid from us. This noble inſtrument then can never be ſufficiently recommended, that has opened a way for our diſcoveries in the heavens, which could never be known by any other means. By this, the whole ſyſtem of nature is laid open to our view. By this we find the ſun to be a great globe of fire; that the planets revolve about him; and that from their different phaſes, they have no light but what they receive from him. By the dark ſpots upon their ſurfaces, are found the times of rotation round their axes. The ſame inſtrument has given us intelligence of the ſatellites of Jupiter and Saturn, which move round theſe planets, in their fixt periodic times; a thing utterly unknown to the ancients. And theſe are of great uſe for determining the longitude; and from thence the motion of light has been proved. Saturn's ring is another diſcovery owing to the teleſcope. The motion of all the heavenly bodies, their diſtances and magnitudes have been hereby found out. The immenſe diſtance of the fixt ſtars is proved by the teleſcope; and by this we diſcover almoſt an infinite number of ſtars more than can be ſeen with the naked eye. Likewiſe all that has been diſcovered of the comets is owing to the uſe of the teleſcope. And by it the times of the beginning and ending of eclipſes are exactly determined, with many other things of like nature, equally aſtoniſhing and curious. The advancement of aſtronomy is wholly from the uſe *of the teleſcope. Teleſcopes may be alſo uſefully employed upon the earth. The merchant may at a great diſtance diſcern his ſhips upon the ſea. The ſeaman may ſee the enemies ſhips at a diſtance; or he may ſee land from the maſt head, long before he comes at it. Generals may ſurvey the army of his enemy, or his own. And any man may take a view of diſtant towns, cities, buildings, trees, and other objects in the country, eſpecially when he is ſeated on a high place. Theſe are noble diſcoveries of the teleſcope; which, if it had been known to the ancients, would have put an end to all their doubts and diſputes about the ſolar ſyſtem.*

If a teleſcope be fixed in a certain poſition, and thro' it we ſee the top of a tower, &c. at noon, we ſhall ſee the lower part of it at night, or in the morning, owing to a greater refraction at morn and night.

The microſcope diſcovers to us another world of wonders. And here our contemplations may be endleſs, all bodies we look at, ſhewing us ſuch curious contexture of parts, and affording ſuch admirable appearances, and ſuch glorious colours, as cannot be well conceived without ſeeing them.

The very ſmall parts of bodies cannot be diſtinguiſhed by the eye, they ſubtend an angle ſo very ſmall; and if they are brought nearer to ſubtend a greater angle, they come without the limits of diſtinct viſion. For the eye cannot contract itſelf further than for an object at 4 inches diſtance to appear diſtinct. If it is brought nearer, it will appear confuſed; and the more confuſed, the nearer it is. This confuſion may be prevented either by looking thro' a hole in a thin plate of lead, whoſe aperture muſt be the leſs, the nearer the object is; or elſe by interpoſing a microſcope glaſs, between the object and the eye, to look thro'.

To mention in particular ſome diſcoveries made by the microſcope. The ſkin of a human body is compoſed *ed of ſeveral ſcales lying over one another; between theſe layers of ſcales the excretory ducts appear, thro' which we perſpire; there are about 500 of theſe ducts for one ſcale, and 250 ſcales are but equal to a grain of ſand. The particles of duſt flying out of the puff ball, are perfect ſpheres of an orange colour, and their diameters not above* $\frac{1}{50}$ *of an hair's breadth. The circulation of the blood is ſeen by the microſcope, in the fins and tales of fiſhes, and the feet of frogs, and the inſertion of the veins and arteries made viſible. Hence alſo may be ſeen the armour of the flea, the ſnout of the tick, the briſtles of the mite, the motion of the muſcles, and the pulſe in a louſe. Many imperceptible animals may be ſeen with this inſtrument, in green weeds, which grow in water, and in ſtanding water; in the hollow of the cabbage and teazle; and ſwarms of them in the water that drains from a horſe dunghill; ſmall animals lodged in the ſcabs of ſuch as have the itch. It would be an infinite labour to mention all the wonders that this little inſtrument ſhews us.*

Thus it appears, that Optics is not only a ſciense of the moſt extenſive uſe, but likewiſe of the greateſt pleaſure and delight. A great many things in philoſophy can neither be underſtood nor proſecuted without the knowledge of Optics. This gives an account of Light, Colours, Opacity, Tranſparency, Brightneſs, Faintneſs, &c. The ſame ſcience alſo accounts for the appearances of mæteors, the Rainbow, Coronas, Parahelion's, Paraſelene's; it gives account of the nature of ſeeing; without it nothing can be done in aſtronomy. Optics alſo brings forth Perſpective, in which every rule of it is founded.

Spectacles were not found out till the beginning of the 13th century. For the ancients had no knowledge of optic glaſſes, as is evident from their univerſal ſilence in this matter. Not one of them giving the *leaſt hint thereof. And without doubt an invention of ſuch univerſal uſe to the world could never be concealed. But the uſe of ſingle glaſſes was known 300 years before men underſtood how to combine them in teleſcopes. They knew nothing of the advantage the eye received thereby, by increaſing its power ſome thouſands of times beyond its natural abilities. For teleſcopes were not invented till about the year 1600, nor microſcopes till 10 or 12 years after.*

Many effects produced by optic glaſſes ſeem wonderful and ſurprizing. A multitude of objects ſeem diffuſed thro' a large ſpace where there is no ſpace; and objects ſeem hanging in the air, where upon trial we can feel nothing. Objects appear ſometimes greater, and ſometimes leſs than they are; and ſometimes pretty, and ſometimes deformed. And they that underſtand the principles of Optics and Perſpective, eaſily underſtand the reaſon of theſe phænomena. Optics alſo tends to free the mind from ſeveral prejudices it had imbibed; and when glaſſes are at hand to demonſtrate their effects by experiment, it gives a vaſt pleaſure to the mind, to ſee the Theory and Practice agree, and at once confirms us in the truth of the Principles. And theſe experiments inſinuate to us, the great affinity there is between reaſon and experience; a matter of the greateſt conſequence in all kinds of ſcience; as ſuch an agreement conduces more than any thing elſe, to the certainty thereof, by freeing the mind from all doubt, and fixing a firm aſſent therein.

From what has been ſaid, and many more things that might be ſaid, appears plainly the excellency of the ſcience of Optics, and its great uſe to mankind above all others; and how neceſſary it is for us to be acquainted with it, to let us into the ſecrets of nature, not only in regard to the grand fabric of the whole univerſe, but likewiſe to the moſt minute and imperceptible parts of it.

As to this Treatiſe, I have handled the three different parts of the ſcience diſtinctly; and treated of each of them as clearly as I could; and in as ſhort and conciſe a manner, as I could think on; and have demonſtrated all the principles in a ſhort way, and in a method very natural and plain; omitting nothing that I could think conducive to the advancement of this ſcience, or could find in the moſt approved authors. And in the laſt book have deſcribed ſeveral optical inſtruments, and demonſtrated their powers and effects, which I hope will be acceptable to the reader.

W. Emerſon.

A Synopsis, or ſhort View of the Contents of this Treatiſe.

BOOK I. Simple Optics.

Of direct viſion. Of the nature of light, and how the colours are generated by refraction thro' a priſm. The apparent places, magnitude, and brightneſs of objects ſeen directly. The colours of natural bodies how cauſed. Fits of eaſy tranſmiſſion and reflexion of the rays of light. The action of light and bodies upon one another. The motion of light is performed in time. The proportion of moon-light, &c. to day-light.

BOOK II. Catoptrics.

The reflexion of light from plane and ſpherical ſurfaces. A general theorem for finding the focus. Of images, their places, diſtances, poſitions, magnitudes, motions, and brightneſs. Burning concaves. The aberration of the rays.

BOOK III. Dioptrics.

The refraction of light thro' one or more refracting ſurfaces. The power of different bodies to refract light. Of finding the focus of rays refracted by plane and ſpherical ſurfaces. A general theorem for finding the focus of rays refracted by a ſpherical ſurface. General theorems for finding the foci, for all ſorts of lenſes. The conjugate foci of all lenſes determined. To find the focus, and the vertex of a lens by experiment. Images made by refraction their places, diſtances, poſitions, magnitudes, ſhapes, motions, and brightneſs. Burning glaſſes. Aberration of the rays.

The

BOOK IV. OPTICAL INSTRUMENTS.

The combination of glaſſes. Of viſion, and the ſtructure of the eye. Teleſcopes with 2, 3, 4, glaſſes; their magnifying powers, diſtinctneſs, brightneſs, and viſual angle. Table of their dimenſions. The ſolar teleſcope. Teleſcopic ſights, and taking angles. Teleſcopes without tubes. Reflecting teleſcopes; Gregory's, Caſſegrain's, Newton's; their magnifying powers, diſtinctneſs, brightneſs, and viſual angle. Tables of their dimenſions. To find the magnifying power of a teleſcope by experiment. A ſingle microſcope, its magnifying power and brightneſs. A double microſcope, its magnifying power, brightneſs, and viſual angle. A reflecting microſcope. The ſolar microſcope. The magic lanthorn. Portable camera obſcura, ſeveral kinds of them. Several machines for ſhewing perſpective views. Explanation of the colours of the rainbow.

OPTICS.

DEFINITIONS.

DEF. I.

OPTICS, is a ſcience which teaches the nature of viſion; and of every thing relating to the ſight.

DEF. II.

A *Ray* of light, is an extremely ſmall part of it moving in a right line, or otherwiſe.

DEF. III.

A *Beam* of light is a ſenſible part of it, conſiſting of a great number of rays.

DEF. IV.

A *Pencil* of rays, is a portion of light of a conical form, proceeding from a point, or elſe tending to a point.

DEF. V.

Parallel Rays, are ſuch as run in parallel directions; and *converging rays*, are thoſe that tend to a point; and *diverging rays*, are ſuch as proceed from a point.

DEF. VI.

A *Medium*, is whatever the rays of light paſs thro', as water, oil, glaſs, air, or any tranſparent body; and even a vacuum.

DEF. VII.

Reflexion, is when a ray of light flies off from the ſurface is falls upon, without entering into it.

DEF. VIII.

Refraction, is the bending or breaking of a ray of light, where it goes out of one medium into another.

DEF. IX.

The *Angle of Incidence*, is the angle made by the incident ray, and the perpendicular to that point of the ſurface on which the ray falls.

DEF. X.

The *Angle of Reflexion*, is the angle comprehended between the reflected ray, and the perpendicular on that point of the ſurface, on which it falls.

DEF. XI.

The *Angle of Refraction*, is the angle which the refracted ray contains with the perpendicular to the refracting ſurface, at the point of incidence. And the *refracted angle*, is that contained between the incident and refracted rays.

DEF. XII.

The *Focus*, is a point in which all the reflected or refracted rays meet together. The point from which the rays ſeem to diverge, is called the *virtual Focus*. The point to which parallel rays are reflected or refracted, is the *principal Focus*.

DEF. XIII.

The *focal Diſtance*, is the diſtance of the focus from the reflecting or refracting ſurface. This generally relates to the principal focus.

DEF. XIV.

Conjugate Foci, are two ſuch points, that either of them being the radiant point, the other will be the focus.

BOOK I.

Of DIRECT VISION, *and ſuch Properties of Light, as are deduced from Experiments.*

PROP. I.

WHEN the eye views many objects at great diſtances, it refers all their places to the ſurface of a ſphere, whoſe center is the eye itſelf.

For as far as an eye can ſee diſtinctly in any one direction, ſo far it can ſee in any other direction, if not hindered by intermediate bodies; and this it can do quite round. Hence, when the objects we look at, are at great diſtances, it is natural to ſuppoſe them at equal diſtances. For under equal circumſtances, nothing appears to make one diſtance to be judged greater than another. Therefore we take the ſimpleſt way, and ſuppoſe them all equidiſtant, or conſider them as ſo. Whence theſe diſtances being all equal or ſuppoſed equal, and all proceeding from one point, it it plain they are the radii of a ſpherical ſurface, whoſe center is that point. That is, the eye places all theſe objects in the ſurface of a ſphere whoſe center is the eye itſelf.

Cor. 1. *The eye is always placed in the center of its own view.*

Cor. 2. *Any portion of the heavens which falls under our view, appears as the portion of the concave ſurface of a ſphere.*

Cor. 3. *When we look at the ſun, moon, and ſtars; or even at the clouds; they appear to us to be ſituated in the ſurface of one and the ſame ſphere.*

For theſe objects being beyond the extent of diſtinct viſion, therefore the eye cannot judge of their true diſtances. Therefore they will appear to us, as all ſituated in the ſurface of the ſame ſphere, and ſo equally diſtant. Tho' they be really at different diſtances from us.

Cor. 4. *Hence if the eye was to have a full view, from any place on the ſurface of the earth; it could ſee no more than a hemiſphere.*

For the horizon cuts off the other hemiſphere; and the body of the earth below that, hinders us from ſeeing any thing at a great diſtance.

Cor. 5. *The eye, keeping a fixed poſition, can ſee no more objects than are contained in a hemiſphere lying directly before the eye, nor quite ſo much.*

For if the baſe of this hemiſphere is ſuppoſed to touch the pupil of the eye; no rays can enter below that baſe; nor even in the plane of this baſe or near it, to cauſe any diſtinct viſion.

PROP. II.

Such rays as enter the eye perpendicularly, have the greateſt power to cauſe diſtinct viſion.

This is matter of obſervation. And it is plain that rays that are oblique have leſs force to produce their effects; and may be ſo oblique as to produce no ſenſible effect at all. Therefore thoſe rays that are moſt direct, are moſt efficacious in cauſing viſion; as they will fall perpendicularly upon the retina, at the bottom of the eye. And by theſe the image of any object will be painted moſt diſtinctly upon the retina.

Cor. *Hence, we ſee objects the moſt diſtinctly, which are directly before our eyes.* Fig.

This is plain from this Prop. And is confirmed by experience, as is evident by our turning our eyes directly to the object; or from one part of an object to another, in order to ſee it clearer.

PROP. III.

An object appears greater or leſs, according as it is ſeen under a greater or leſſer angle. 1.

Let AC*a* be the axis of the eye; AB, DE two equal objects. To the eye at C, draw BC*b*, and EC*e*. Then the object BA appears under the angle BCA, or *bCa*, and ED under the angle ECD or *e*C*a*. But *e*C*a* is greater than *b*C*a*; and *ae* greater than *ab*. But *ae*, *ab* are the appearances or pictures of the objects DE, AB, upon the retina. Therefore the former appears greater than the latter.

Cor. 1. *The diſtance between two objects* A *and* B, *appears to be greater as the diſtance of the ſpectator is leſs.*

For at the diſtance CD, the diſtance of the objects appears under the angle ECD or *e*C*a*. And at the diſtance AC, it appears under the angle BCA or *b*C*a*.

Cor. 2. *The apparent linear magnitude of an object* AB, *is reciprocally as the diſtance* CA.

For AB appears no greater than DF. But DF : DE or AB : : CD : CA. That is, the apparent magnitude of AB : apparent magnitude of DE : : CD : CA.

Cor. 3. *The ſuperficial apparent magnitude of an object, is reciprocally as the ſquare of the diſtance.*

PROP. VI.

2. *The denſity of rays flowing from an object, or any part of an object; is reciprocally as the ſquare of the diſtance.*

Let the rays flow from A, and fall upon the plane BC; then taking the plane BC away, let the ſame light fall on any other plane DE, which is done by producing the lines AB, AC, to D and E. Then the area of the ſurface BC, is to the area of the ſurface DE; as BC^2 to DE^2; or as AB^2 to AD^2. Let s be any ſmall ſpace, l the quantity of light contained in DAE. Then $BC^2 : s :: l : \frac{ls}{BC^2} =$ the number of rays on the ſpace s, at the diſtance AB. And $DE^2 : s :: l : \frac{ls}{DE^2} =$ number of rays on the ſpace s, at the diſtance AD. And the denſity of light being as the number of rays; it will be, denſity of light at B : denſity of light at D $:: \frac{ls}{BC^2} : \frac{ls}{DE^2} :: DE^2 : BC^2 :: AD^2 : AB^2$.

Cor. 1. *The quantity or intenſity of light, flowing from a luminous object, is reciprocally as the ſquare of the diſtance.*

Cor. 2. *The brightneſs of an object, is the ſame at all diſtances from the eye; if none of the rays be ſtopt by the way.*

For the brightneſs is the ſame, when the ſame quantity of light falls upon the ſame quantity of the retina, in the eye; or when the denſity of light on the retina is the ſame. But the denſity of light falling on the pupil, or the quantity of light falling on it, is reciprocally as the ſquare of the diſtance. And (III. Cor. 3.) the apparent magnitude of the object is reciprocally as the ſquare of the diſtance. Therefore the quantity of light on the pupil, is as the apparent magnitude of the object, that is, as the ſpace taken up on the retina. Therefore the quantity of light being as the ſpace taken up, its denſity is given; and therefore the brightneſs is given, or remains the ſame. Fig. 2.

Cor. 3. *Hence the faintneſs of remote objects is owing to the denſity and opacity of the atmoſphere, which ſtops a great part of the light, in coming to the eye.*

PROP. VII.

If a room be made quite dark, and a ſmall hole in the window ſhut be made. The objects out of the houſe being ſtrongly illuminated; the rays proceeding from them, thro' the hole, will form their pictures on a paper, or on the oppoſite wall, in their proper colours; but in an inverted poſition. 3.

Let PQR be the dark room, F the hole in the window ſhut, ABCD an object without. The rays proceeding from the top A, in the right line AF, will form its image at *a* towards the bottom of the wall. And the rays proceeding from the bottom C, in the line CF, will paint its image at the top *c*. Likewiſe the rays proceeding from the left B, in the line BF, will make the image of B on the right hand at *b*. And thoſe proceeding from the right hand D, in the line DF, will form its image on the left at *d*. Likewiſe all the rays proceeding from the line AC, will form its image *ac*. And thoſe from BD will make its image *bd*. And the like for any other object expoſed to the hole F.

Fig. 3. Cor. 1. *If the hole* F *be very ſmall, the pictures will be very diſtinct, but hardly viſible for want of light. And if the hole be large, the pictures will be brighter, but not diſtinct. To remedy theſe things, a broad convex lens of a proper focus, muſt be placed in the hole made wide enough to receive it.*

Cor. 2. *If an object, too near the eye, appear confuſed; it may be ſeen clearly thro' a hole made in a piece of paper with a pin, and put cloſe to the eye. Then it will appear diſtinct, and in its true poſition.*

This is upon the ſame principle. For if the hole is ſmall, the rays flowing from all the points of the object, will fall upon as many diſtinct points of the retina; and make a diſtinct picture.

Cor. 3. *But if the paper with a ſmall hole in it, be held at a ſmall diſtance, and a needle held cloſe to the eye, or any way between the eye and the paper; it will appear inverted, and magnified.*

The reaſon of this is, that the rays croſſing one another at the hole, before they come to the object; the picture of the object muſt be painted the ſame way that it lies; contrary to the caſe of the 2d Corol. where the rays croſſing one another between the object and the eye, makes the picture inverted. But we know, that a picture inverted in the bottom of the eye, makes the object ſeem erect; conſequently, when the picture is erect in the bottom of the eye, the object will ſeem inverted.

42. Cor. 4. *From the ſame principle it follows, that if the ſun ſhines thro' a long narrow ſlit* AB *in a window ſhut; its image* CD *at a good diſtance from the window, will approach to a circle.*

For the cone of rays flowing from all parts of the ſun thro' the point A, will be projected into a circle at C; and thoſe flowing thro' B will be projected into a circle at D. And ſo rays flowing

ing thro' all points between A and B, are projected into innumerable circles between C and D; Fig. 42. each of theſe circles is the image of the ſun, and all together make up the oblong figure CD. In this figure the extreme circles C, D, grow larger at a greater diſtance, whilſt the diſtance of their centers remains the ſame, being equal to AB. So that as the diſtance increaſes, the length and breadth of the figure increaſes equally, and at a great diſtance, their difference vaniſhes as to ſenſe. At the place FG, the figure is narrow, having rectilinear ſides and circular ends. And after the ſame manner, if the ſun ſhines thro' a triangular hole or any other figure, the image at a great diſtance will be nearly circular, and the centers of the extreme circles will form a triangle or other figure, ſimilar and equal to the hole.

SCHOLIUM.

An apparatus made after the ſame manner deſcribed in this Prop. is called a *Camera obſcura.*

PROP. VIII.

The light of the ſun conſiſts of rays differently refrangible.

Ex. 1.

4. To prove this by experiment. In a very dark chamber, make a round hole F, about $\frac{1}{4}$ inch diameter, in the window ſhut EG. Place a glaſs priſm ABC very near the hole; ſo that a beam of light SF coming in at the hole, may paſs thro' the priſm and be refracted upwards, towards the oppoſite wall of the chamber, and there form a coloured image of the ſun PT. This priſm is triangular at each end, 3 or 4 inch long, and is poliſhed on the 3 ſides AB, BC, CA. It is here placed parallel to the horizon, and its axis perpendicular

Fig. 4. dicular to the beam SF. Turn the priſm ſlowly about the axis, and you will ſee the refracted light on the wall, or the coloured image of the ſun, firſt to deſcend, and then to aſcend. Between the deſcent and aſcent, when the image ſeems ſtationary, ſtop the priſm and fix it there. For in that poſture the refractions of the light on the two ſides CB, CA, are equal to one another.

So likewiſe in other experiments, when you would have the refractions on both ſides the priſm to be equal, you muſt note the place on the wall where the image ſtands ſtill, between two contrary motions; and make it faſt there. In this poſture of the priſm all the following experiments are to be made, unleſs ſome other poſture is deſcribed. The priſm remaining in this poſture, let the refracted light fall perpendicularly upon a ſheet of white paper at the oppoſite wall of the chamber, and you'll ſee the ſolar image formed on the paper to be oblong, and not oval, but terminated with two rectilineal and parallel ſides, and two ſemicircular ends; the ſides bounded pretty diſtinctly, but the ends very confuſedly and indiſtinctly, the light there decaying, and vaniſhing by degrees. Here the breadth anſwers to the ſun's diameter; at 18 ½ feet from the priſm, the breadth diminiſhed by ¼ inch, the diameter of the hole, is 1 ⅞ inch, and ſubtends an angle at the priſm of half a degree, which is the ſun's apparent diameter. The length 10 ¼ inches, when the refracting angle of the priſm C is 64 degrees. With a leſs angle of the priſm the length of the image will be leſs. If the priſm be turned about its axis, that way which makes the rays emerge more obliquely to the ſide AC; the image ſoon becomes an inch or two longer, or more. And if the priſm be turned about the contrary way, ſo as to make the rays fall more obliquely on the ſide CB, the image ſoon becomes

comes an inch or two ſhorter. And therefore in trying this experiment, one muſt be as careful as can be, in placing the priſm, ſo that the refractions on both ſides may be alike. Fig. 4.

This experiment tried with a priſm, whoſe refracting angle C is $62\frac{1}{2}$ degrees, the length of the image will be $9\frac{3}{4}$ or 10 inches, at the ſame diſtance. The magnitude of the hole in the window ſhut, the different thickneſs of the priſm, where the rays paſs thro', the different inclinations of the priſm to the horizon (or different hights of the ſun), make no ſenſible changes in the length of the image. Neither does the different matter of the priſm make any; for in a priſm made of plates of glaſs and water within, there is the ſame reſult.

Alſo the rays always go on in right lines from the priſm to the image. And therefore at their going out of the priſm, have all that inclination to one another, which cauſes the length of the image; that is the inclination of $2\frac{1}{2}$ degrees or more. From all which it is plain, the rays falling on P are more refrangible than thoſe falling on T. For if the rays were all alike refrangible, the image PT would be a circle, whoſe length is equal to its breadth; which is eaſily proved thus.

Let SKHP be a ray coming from the lower part of the ſun to the higher part of the image; and refracted by the priſm at K and H, SLIT a ray which comes from the higher part of the ſun, to the lower part of the image, and is refracted at L and I. Suppoſe the refraction on both ſides the priſm, are equal; we have, refraction at K = refraction at I, and refraction at H = refraction at L. Then by addition, the refractions at K and H = refractions at I and L. And therefore the two rays being equally refracted, have the ſame inclination to one another after refraction as before; that is, half a degree, equal to the ſun's apparent diameter.

Fig. diameter. So then PT would ſubtend an angle at
4. the priſm of half a degree, and therefore would be equal to the breadth, and conſequently the image would be round. But inſtead of that, it is five times longer than it is broad.

Likewiſe the image PT is coloured; for from T to P it has theſe different colours in this order, red, orange, yellow, green, blue, purple, violet; at the places, *r*, *o*, *y*, *g*, *b*, *p*, *v*, reſpectively. So that the moſt refracted part at P is violet, and the leaſt refracted at T is red. And the intermediate part, all the other colours.

Exp. 2. When a beam of light is let in at the hole F, of the window ſhut; hold the priſm in ſuch a poſition that its axis may be perp. to the beam. Then look thro' the priſm to the hole, and turning the priſm about back and forward, to find when the image is ſtationary; there ſtop the priſm, the refraction on both ſides being then equal. Then looking thro' it, at the hole; the length of the image will appear to be many times greater than the breadth. The moſt refracted part being violet; and the leaſt refracted, red. The middle parts blue, green, yellow in order.

And if the priſm be removed out of the ſunbeam, and then looking thro' it at the hole; you will have the ſame appearance. Now if all the rays were refracted alike, the hole would appear round when refracted thro' the priſm. So that at equal incidencies there is a conſiderable inequality of refraction.

5. *Exp.* 3. To try what is the effect of refracting the rays a ſecond time. Things being ordered as in the firſt experiment, place a ſecond priſm cloſe by the firſt in a croſs poſition as DH, that it may again refract the beam which comes thro' the firſt priſm. The firſt priſm refracts it upwards; and the ſecond, ſideways. By the firſt priſm alone the

beam is refracted to PT, and *qt* is the image made Fig. by the refraction of the two croſs priſms together. 5. Then it will appear by the refraction of the ſecond priſm that the breadth of the image is not increaſed, but the upper part P ſuffered a greater refraction in both priſms, and the lower part T, a leſs refraction in both priſms; and that without any dilatation of the image in breadth. Likewiſe as before, the upper part *q* appears violet; and the lower part *t*, red; and the reſt as before. So that here is no dilatation or ſplitting of rays.

And if a third and even a fourth priſm be placed in the ſame manner after the ſecond, the reſult will be the ſame; and the moſt refrangible rays will be moſt refracted, as before; and the leaſt, the leaſt refracted, whilſt their colours remain unchanged.

Exp. 4. If inſtead of one hole in the window, 6. there be made two holes, under one another, at a ſmall diſtance; and refracting the light upwards by two parallel priſms, placed one at each hole; and then refract the two images to the oppoſite wall, in ſuch manner that the two coloured images may be joined end to end in a right line. Then if both theſe refracted beams were again refracted ſideways by a third priſm placed acroſs the two firſt (as in Exp. 3.). Then the two beams would be refracted to the places *pt*, *mn*; which would not now lie in one right line with their ends touching as before; but be ſeparated and become parallel. The blue ends *m*, *p* being by a greater refraction tranſlated further from its former place, than the red ends *n*, *t*. And the ſame happens if the third priſm be placed at a good diſtance from the others.

Exp. 5. If a beam of light be reflected by one ſide of a priſm, and then refracted by another priſm, ſome of the rays will be more refracted, and others leſs. Or if it paſſes thro' one ſide of a priſm, Fig. 6. a priſm, and is reflected by another ſide, and made to paſs thro' the third ſide; and after this be refracted by another priſm; ſome rays will at laſt be more refracted than others. Or if a beam paſs thro' parallel ſurfaces of glaſs, and be then refracted; the conſequence will ſtill be the ſame. And therefore it is fully proved, that the ſun's light conſiſts of rays differently refrangible.

4. *Exp.* 6. Let a ſmall hole be made in the paper where the image PT is; and let each of the colours paſs ſucceſſively thro' this hole; and be refracted again by a priſm behind the hole. Then each of theſe colours will be found to continue the ſame as before without any manner of alteration. And the figure or ſpot of light on the wall, will, as to ſenſe, be circular. So that none of theſe colours are the leaſt changed by refraction. Likewiſe if ſmall bodies be placed in this new light, they will appear of the ſame colour as that light, red in the red light, green in the green light, &c. So that the colours are no way changed by reflexion. Likewiſe if you look at any of theſe ſpots of new light upon the wall, thro' a priſm, they will appear wholly of the ſame colour, and no way dilated or expanded in length. So that homogeneal light ſuffers no manner of alteration in any caſe. And in all theſe trials it will appear that thoſe rays, which were moſt refracted at firſt, are always afterwards moſt refracted. And thoſe that are leaſt at firſt, are always leaſt afterwards.

Cor. 1. *Every ray of light has a peculiar degree of refrangibility, which cannot be changed by any reflexions or refractions; but remains conſtantly and invariably the ſame.*

Cor. 2. *The confuſed viſion of any object ſeen thro' a refracting body or medium, ariſes from the different refraction of the light.*

For different rays flowing from the ſame point Fig. of an object, will be refracted to different points on the retina at the bottom of the eye.

Cor. 3. *The light reflected or emitted from all bodies, conſiſts of rays differently refrangible.*

For the light reflected from all bodies on the earth, is the ſun's light. And beſides, if one looks at any object thro' a priſm, that object will appear tinged with colours. Take a ſmall part of a body, illuminate it ſtrongly, and looking at it thro' a priſm, you'll have an oblong image like PT (*fig.* 4.) with all the colours. In like manner, a ſtar, a lamp, a candle, a burning coal, red hot iron, or any burning matter ſeen thro' a priſm, gives the ſame appearance.

Cor. 4. *Hence alſo the ſimple primary colours are ſeven; red, orange, yellow, green, blue, purple, violet. The red being leaſt refracted, and ſo in order to the violet, which is moſt. And the ſpaces taken up by theſe, are not equal; the orange and purple being little more than half each of the reſt, which are nearly equal.* 4.

This is known by meaſuring the parts of the image PT.

Cor. 5. *Hence objects ſeen thro' glaſſes in purified or homogeneal light will appear diſtinct; but very confuſed in heterogeneal or common light.*

This is plain from the 3d experiment; and may be further evinced by placing flies, or the letters of a book, or any minute objects in homogeneal light, where they will be perfectly ſeen, and their parts diſtinctly defined; but will appear confuſed in common light, and their ſmall parts cannot be diſtinguiſhed therein.

SCHOLIUM.

Sir J. Newton, by accurately meaſuring the ſpaces, that the ſeveral colours take up, found them as follows;

red,	orange,	yellow,	green,	blue,	indigo,	violet.
$\frac{1}{8}$,	$\frac{3}{40}$,	$\frac{2}{15}$,	$\frac{1}{6}$,	$\frac{1}{6}$,	$\frac{1}{9}$,	$\frac{2}{9}$.

Where the whole length of the image is 1.

PROP. IX.

Rays of light which differ in colour, differ alſo in degrees of refrangibility.

7. *Exper.* 1. Take an oblong piece of paper DGI, draw a line croſs it in the middle, to divide it into two equal parts. Paint one end DG with blue, the other GI with red, as ſtrong as you can. Lay it flat where there is light enough; look at it thro' a glaſs priſm, held parallel to it, and to the horizon; then if the refracting angle of the priſm be upwards, ſo that the paper may ſeem to be lifted upwards by refraction; the blue half FO will be lifted higher by refraction, than the red half OK. But if the refracting angle of the priſm be turned downwards, ſo that the paper may ſeem to be carried lower by refraction; its blue half will be carried lower than the red half. Therefore the light from the blue is more refracted, and therefore is more refrangible, than the light from the red.

Exp. 2. Take the ſame paper, whoſe two halves are painted with red and blue, and lap a thread of black ſilk ſeveral times about it, ſo that the parts of it may appear upon the colours like ſo many black lines drawn over them. Set this paper perpendicular againſt a wall in a dark room, or in the night; ſo that one of the colours may ſtand to the right, and the other to the left. Place a candle near the paper to illuminate it ſtrongly. Then take a lens of a large focal diſtance, and place it at ſuch a diſtance, that the rays coming from the ſeveral points of the paper may be collected to a focus, at about the ſame diſtance on the other hand; and ſo form the image of the coloured paper upon a white paper placed there. Then moving the paper to and from the lens, to find where the images of the blue and red parts of the paper appear moſt diſtinct; which is eaſily known by the images of the black threads of ſilk, which will be diſtinct there; and you'll find where the red half of the paper appears diſtinct, the blue half will appear confuſed; ſo that the black lines on it can ſcarce be ſeen. And on the contrary, where the blue half appears diſtinct, the red half will be confuſed, ſo that its black lines can ſcarce be ſeen. And therefore between theſe two places, where theſe ſeveral images appear diſtinct, there will be a remarkable diſtance, which will be about an inch and a half, if the diſtance of the candle or paper from the lens be about ſix feet. Therefore the focal diſtance of the red being greater, than that of the blue; proves the blue rays to be more refrangible than the red. Therefore rays that differ in colour, differ alſo in their degrees of refrangibility. The ſame appears by the experiments in the laſt Prop. Fig. 7.

Note, A lens is a ſolid piece of glaſs, the two ſides being the ſegments of ſpheres, and poliſhed.

Cor. 1. *Hence all light naturally conſiſts of heterogeneous rays, which exhibit different colours.*

Cor. 2. *Every ſort of homogeneal rays produces its proper colour, according to its degree of refrangibility. And thoſe that agree in colour, agree in the degree of refrangibility; and its colour cannot be changed by any reflexions or refractions.*

Fig. 7. Cor. 3. *There are as many ſimple colours as there are degrees of refrangibility, and therefore an infinite number.*

For tho' in Exp. 1. Prop. VIII. there are only ſeven ſimple colours mentioned; yet each of them conſiſts of an infinite number of degrees, and therefore are infinitely various.

Cor. 4. *The perfection of teleſcopes and microſcopes is prevented by the different refrangibility of the rays of light.*

This appears by the 2d experiment; for theſe inſtruments are compoſed of lenſes of ſeveral ſorts. No rays iſſuing from a point can be refracted exactly to a ſingle point.

Cor. 5. *Hence it is plain, if the ſun's light conſiſted but of one ſort of rays; there would be but one colour in the whole world, or all things would be of one colour. Nor would it be poſſible to produce any new colour by reflexion or refraction. And of conſequence the variety of colours depends upon the compoſition of light.*

SCHOLIUM.

4. If a large hole F be made in the window ſhut EG, and the priſm ABC placed near it, and cloſe to the priſm on the other ſide, a thin board DO, with a ſmall hole in it; and at 12 feet diſtance, the board MN be placed, having a ſmall hole in the middle of it; and cloſe behind it, the priſm RQ. Then if the priſm ABC be turned ſlowly to and fro about its axis, the image PT will move up and down on the board MN; and by this means, all the ſeveral parts of it may be made to paſs thro' the hole in it. And each of theſe paſſing thro', is refracted again by the priſm RQ, and ſo paſſes to the oppoſite wall. Then it will appear, that the light which is moſt refracted at ABC, is alſo moſt

refracted at RQ, and goes to the higher part of the wall at X. Fig. 4. And the light which is leaſt refracted at ABC, is alſo the leaſt refracted at RQ, and goes to the lower part of the wall at Y. And that the moſt refracted at X is violet, and the leaſt refracted at Y is red. And all this happens, whilſt the two boards and the priſm RQ remain unmoved; by which means the incidence of the rays upon the ſecond priſm, remains always the ſame. Therefore at the ſame incidence, ſome rays are more refracted than others; and thoſe that are more refracted at firſt, are more refracted afterwards, and have always the ſame colour annext; and the like for the reſt. So that it is evident, that thoſe rays that are endued with different colours, have different degrees of refraction. From the perſpicuity and evidence of this experiment, it is called the *Experimentum crucis.*

PROP. X.

All rays of light falling on a reflecting ſurface, in the ſame angle; are reflected in equal angles, if they be reflected at all.

Exper. Let F be a hole in the window ſhut, G a reflecting ſurface placed horizontally. 8. Let a beam of light from the ſun FG, come thro' the hole F, and fall upon the reflecting ſurface G, and then be reflected to H on the oppoſite wall; the figure of the reflected light at H will be circular like the hole F. And therefore there is no ſeparation of the rays as in refraction. And no colours are brought by reflexion.

Alſo if a paper be held any way between G and H parallel to the wall at H; the image received upon the paper will always be ſimilar to the image at H, and of a circular form. And therefore the rays that had the ſame incidence, will run parallel

Fig. to one another after reflexion; and conſequently
8. are reflected at equal angles.

Indeed the Beam FG grows wider as it is further from F, and GH grows bigger as it is further from G; but this is only owing to the apparent magnitude of the ſun's diameter. For from any one point of the ſun's diſc, the rays which fall parallel on G, will be reflected parallel to H.

9. Cor. *Hence there is no ſeparation of the rays of light by reflexion, any thing like what there is by refraction.*

Indeed the moſt refrangible rays will be the ſooneſt reflected, if they move out of a denſe medium into a rare one; as ſuppoſe a beam of heterogeneous light moves in direction FG; ſome of it is refracted along GD, and ſome along GC. Therefore if the angle FGA be diminiſhed; the way of the moſt refrangible rays, GC, will come to GB, and be reflected along GH. But this is nothing to the preſent Prop. For it is reflected, becauſe it is impoſſible it can be refracted any longer, as it cannot enter the medium; and in this ſenſe, the moſt refrangible rays are the moſt reflexible.

PROP. XI.

Whiteneſs, or the ſun's light, is compounded of all the primary colours mixed in a due proportion.

4. *Exp.* 1. A beam of light being refracted by a priſm, as in Ex. 1. Prop. VIII, and made to form the image P on a piece of paper. Take away the paper, and hold a lens in the refracted rays, ſo as they may all fall upon it. This will cauſe the coloured light, which diverged from the priſm to converge, and meet again at its focus. If you hold a paper behind the lens, the ſolar image will appear upon it intenſely coloured; and that by re-

ceding from the lens, thoſe colours will come more and more towards one another; and by mixing together, will more and more dilute one another continually; till at length the paper comes to the focus, where by a perfect mixture, all the colours will wholly vaniſh, and be converted into whiteneſs; the whole light appearing now upon the paper, like a little white circle. And afterwards by moving farther from the lens, the rays which converged and met at the focus, will now croſs one another there, and diverge from thence, and thereby make the colours to appear again, but in a contrary order, the red being now above, and the violet below. Here then the ſun beam SF, which at firſt was white, is by the refraction of the priſm ABC reſolved into its original colours; theſe colours, by the refraction of the lens, are again intimately mixt, and reduced to their original whiteneſs. Fig. 4.

Now it is evident this whiteneſs is made up of all the colours of the image. For if any of the colours be ſtopt at the lens, the whiteneſs will ceaſe, and degenerate into that colour which is made by the compoſition of the remaining colours. And if the intercepted colour be then let paſs, and fall upon that compound colour, it will immediately reſtore its whiteneſs by mixing with them. Thus if the blue rays be intercepted, the reſt will appear reddiſh, and then letting the blue paſs, the whiteneſs is reſtored. And if the red be ſtopt, the reſt will appear blueiſh; and letting the red paſs, the whiteneſs is again reſtored.

And in making this white, the ſeveral rays do not ſuffer any change by acting upon one another. For if the paper be held beyond the focus, and the red colour be ſtopt and then let paſs again, the violet colour on the paper ſuffers no change. Nei-

Fig. 4. ther will the red be changed by ſtopping and letting paſs the violet.

When the paper is held in the focus, and the white image there be viewed thro' a priſm, you'll have a coloured image like PT. And if any colour at the lens be ſtopt and let paſs by turns, it will diſappear and return as often; but the remaining colours will not ſuffer any change. One colour then depends on one ſort of rays, and another colour, on another ſort.

10. *Exp.* 2. Make an inſtrument XY in faſhion of a comb, whoſe teeth are 1½ inch broad, and diſtance two inches; and interpoſing ſucceſſively the teeth of this inſtrument near the lens MN, placed as in the laſt experiment, you will ſtop part of the colours by the teeth, whilſt the reſt go on, thro' the ſpaces between the teeth, to the paper DE, in the focus, where it makes a round image. This image appears white, when the comb is taken away; but the comb being interpoſed, the whiteneſs changes its colour to that which goes on thro' the comb. And the colour is perpetually ſo varied by the motion of the comb up and down, that in paſſing of all the teeth over the lens, the colours red, yellow, green, blue, purple, do always ſucceed one another, when the motion is ſlow. But if the comb be moved very quick, that the colours by their quick ſucceſſion cannot be diſtinguiſhed from one another, the appearance of the ſingle colours ceaſes, and from a confuſion of them all, there ariſes one uniform white colour. Therefore of the whole light which now appears white, by the mixture of all the colours, there is no part really white. One part is red, another is yellow, a third green, a fourth blue, a fifth purple. If the impreſſions of theſe ſeveral colours follow one another ſlowly, ſo that they may be ſeverally perceived,

there is made a diſtinct ſenſation of all the colours Fig.
ſucceſſively. But if the impreſſions follow one another 10.
ſo quickly, that they cannot be ſeverally perceived, there ariſes out of them all one common ſenſation, which is neither of this nor that colour, but is indifferent to them all; and this gives a ſenſation of whiteneſs. Therefore when all the colours are viewed perfectly mixt in the focus of the lens, this will much more ſtir up a ſenſation of whiteneſs.

Exp. 3. A beam of light being refracted by a 4.
priſm ABC, and the ſun's coloured image PT made to fall on the oppoſite wall. If a ſpectator ſtand at FG, and place another priſm cloſe by ABC, and in the ſame poſition, and look thro' it to the image PT; then the colours of it will appear to be contracted into a round ſpot perfectly white. Which therefore is made ſo by the mixture of all the colours together.

Exp. 4. Take a large circle or wheel ADF. Divide 11.
its circumference into 12 equal parts; ſet two of them from A to B; that is, make AB two of them, BC 1, CD 2, DE 2, EF 1, FG 2, GA 2. Draw lines to the center O. Then paint it, making DOE red, EOF orange, FOG yellow, GOA green, AOB blue, BOC purple, and COD violet; all of as bright colours as can be had. Then put an axis thro' the center O, and cauſe the wheel to revolve ſwiftly about its axis; and none of the colours will then appear ſingly; but the whole will appear to be white. And this white will be the fuller, as the particular colours are the brighter.

Exp. 5. Whiteneſs may be made by mixing coloured powders together. Thus, put together orpiment, a purple powder, biſe, and verdigreaſe, theſe being mixt well together, will compound a white colour; the quantities of them muſt be found out by trials, and leſs or more of each muſt be taken, Fig. taken, according to their brightneſs or dullneſs. But all powders of this kind, loſe or ſtifle a great deal of the light that falls upon them; and therefore a white cannot be made bright enough of theſe, but will appear duſky; to help this, it muſt be placed in a ſtrong light, and viewed at a diſtance. From all theſe experiments it appears, that whiteneſs is a due mixture of all other colours.

Cor. 1. *A mixture of colours may be made which ſhall be like the colour of homogeneal light. But it will differ in this, that the parts of it will be differently refrangible.*

For almoſt any of the priſmatic colours may be imitated by mixing the colours on each ſide. Thus red and yellow make an orange. Orange and green make a ſort of yellow. Yellow and blue make a green. Green and purple make a blueiſh colour. And blue and violet make a purple.

Cor. 2. *Hence one may know how to make a mixture of colours, that ſhall be like the colour of a given object.*

Look at the object thro' a priſm, to ſee the oblong coloured image of it, which divides the colour of it into all the ſimple colours it is compoſed of. Take ſuch a quantity of every ſeparate colour, you ſee in the image, as ſhall be proportional to its intenſity or degree of brightneſs. And ſuch as are faint or hardly appear, may be omitted. Then mix them together.

Cor. 3. *All colours in the world are either the colours of homogeneal light; or compounded of them.*

This is evident from this and the foregoing Propoſitions.

Cor. 4. *A body that reflects no colour, is black.*

Cor. 5. *Hence black is no colour, and white is all colours.* Fig.

PROP. XII.

If any body be placed in ſimple or homogeneal light of any colour; it will appear of the ſame colour.

Exp. 1. Suppoſe the beam of light SF to paſs thrō' the hole F, in the window ſhut, and being refracted by the priſm ABC, to caſt the image PT on the oppoſite wall, and to form all the original priſmatic colours. Take a bit of paper, and hold in the red light, and it will appear red. Hold it in the orange, yellow, green, blue, &c. ſucceſſively, and it will appear orange, yellow, green, blue, &c. ſucceſſively. 4.

Take a bit of coloured paper, and hold it in the red light, and it will appear red; alſo hold it in the orange, yellow, green, &c. and it will appear orange, yellow, green, &c. reſpectively. But it will appear moſt luminous when placed in the light of its own kind. The room muſt be dark in trying this experiment, as well as in the following.

Exp. 2. The ſame apparatus continuing. Take *cinnabar*, and place it in the yellow light, and it appears yellow; place it in the blue light and it appears blue, &c. But place it in the red light, and it is moſt luminous, this is its native colour. *Indigo* placed in the red light is red, in the orange it is orange, in the green it is green, &c. and placed in the purple, its own natural light, and it is a moſt reſplendent purple; and its ſplendor gradually diminiſhes, by moving it by degrees thro' the green and yellow to the red. Place a bit of blue ribbon in the red light it appears red; and in the orange, yellow, green, it appears orange, yellow, or green, but faint. Place it in the blue light, and it is a moſt bright blue.

Fig. 4. *Exp.* 3. The ſame things remaining. If *cinnabar* and *ultra marine* be held together in the red light; they will both appear red. But the cinnabar will appear of a ſtrong luminous reſplendent red; and the ultra marine of a faint obſcure and dark red. And if they be held together in the blue light, they will both appear blue; but the ultra marine will appear of a ſtrong luminous and reſplendent blue; and the cinnabar of a faint and dark blue. So that the cinnabar reflects the red light much more copiouſly than the ultra marine does; and the ultra marine reflects the blue light much more copiouſly than the cinnabar doth. The ſame may be tried with other bodies.

Cor. 1. *The colours of all bodies ariſe from hence, that ſome bodies reflect one ſort of rays, others another ſort of rays, more copiouſly than the reſt. And they become coloured, by reflecting the light of their own colours more plentifully, and that of all other colours more ſparingly.*

That this is the only reaſon of their colours, appears from theſe experiments; and further from this conſideration, that the colour of homogeneal light cannot be changed by the reflexion of natural bodies. And therefore they cannot appear coloured by any other means than by reflecting the light of their own colour.

Cor. 2. *'Tis light only that gives colour to bodies; and they have naturally no colour in themſelves.*

For a body cannot be ſeen in the dark, when there is no light. For there is nothing in the body but a diſpoſition to reflect this or that ſort of rays, which makes them be ſaid to be of that colour. And the rays properly ſpeaking are not endued with colours; for in them there is nothing but a certain power or diſpoſition to excite a ſenſation of this or that colour in the brain.

Cor. 3. *Liquors that reflect any colour, tranſmit the other ſorts.* Fig. 4.

For inſtance, a liquor that reflects the blue light, imbibes the red, which will paſs thro' it, if the veſſel is not too broad. And if there be two liquors a red and a blue; tho' either liquor be tranſparent apart, yet you'll not be able to ſee thro' both together. For if only the red rays paſs thro' one, and only the blue thro' the other; no rays can paſs thro' both. Now if bodies become coloured by reflecting, or tranſmitting this or that ſort of rays more copiouſly than the reſt; it muſt be conceived, that they ſtop and ſtifle within themſelves the rays which they do not reflect or tranſmit.

Colour of liquors varies with their thickneſs. Thus a red liquor in a conical glaſs, looks of a pale yellow at bottom; a little higher it grows orange; higher ſtill it becomes red; and where the red is deepeſt, the glaſs is broadeſt. And higher it will tranſmit no rays; they being all loſt by degrees in the body of the liquor. And thence it appears that red is the ſtrongeſt light, and will penetrate fartheſt into a fluid. And this is evident by the obſervations of thoſe that dive into the ſea; for the objects look the redder the deeper one goes; the other rays being reflected back.

Cor. 4. *Hence the red rays are the ſtrongeſt, are leaſt turned out of their way, and penetrate fartheſt into a reſiſting medium. And the reſt of the colours are weaker and weaker in their order; the violet being the weakeſt.*

Cor. 5. *We can make a body to be of any colour we pleaſe, by holding it in the light of that colour.*

Fig.

PROP. XIII.

The colours of natural bodies depend upon the ſize and denſity of the compounding particles.

12. *Exper.* 1. Take two object glaſſes belonging to two long teleſcopes, one a plano convex, the other a double convex. Lay the plane ſide of one AEB, upon the convex ſide of the other CED. Then looking perpendicularly upon them, you'll ſee a black ſpot in the middle where they touch; and ſeveral concentric rings of colours about it, at Z, F, G, H, I, A, &c. Theſe rings may be reckoned of ſeveral orders, by reaſon of the ſame colour coming over again. Theſe orders are as follows, 1. Z, black, blue, white, yellow, red. 2. F, violet, blue, green, yellow, red. 3. G, purple, blue, green, yellow, red. 4. H, green, red. 5. I, greeniſh blue, red, &c. Which colours grow weaker and weaker from the center; and alſo narrower and narrower.

But if the light be made to paſs thro' the glaſſes to the eye, the ſpot in the middle will be white; and the orders of colours, going from the center, quite contrary to the others. 1. White, yellowiſh red. 2. Black, violet, blue, white, yellow, red. 3. Violet, blue, green, yellow, red, &c. The black lines between the glaſſes, at, F, G, H, I, A, are diſtances in arithmetical progreſſion, being the places where the ſame colour returns, by reflected light, as at the firſt trial.

Meaſuring the diameters of the rings in the brighteſt part, their ſquares are found to be in arithmetical progreſſion of the odd numbers 1, 3, 5, 7, 9, 11, &c. And ſince one of the glaſſes was plane and the other ſpherical; their diſtances at theſe rings, that is, at Z, F, G, H, &c. muſt be in the ſame progreſſion. Alſo meaſuring the Fig. 12. diameters of the faint rings, in the middle between the bright ones; their ſquares, and conſequently the diſtances of the glaſſes there, appear to be in the arithmetical progreſſion of the even numbers, 2, 4, 6, 8, 10, 12, &c. And from the known radius of the lower glaſs, the diameter of any ring may be computed, and conſequently the diſtance of the glaſſes in that place. And hence the diſtance of the glaſſes in the middle of the firſt dark ring, was found to be $\frac{1}{89000}$ part of an inch. And half of it multiplied by 1, 3, 5, 7, &c. gives the diſtance of the glaſſes or the thickneſs of the included air, at the moſt luminous parts of all the bright rings, $\frac{1}{178000}$, $\frac{3}{178000}$, $\frac{5}{178000}$, $\frac{7}{178000}$, &c. and $\frac{2}{178000}$, $\frac{4}{178000}$, $\frac{6}{178000}$, &c. the thickneſs at the darkeſt parts.

Exp. 2. Wetting the objects with water; as the water crept in between them, the circular rings grew leſs. And by meaſuring their diameters, and from thence finding the diſtances of the glaſſes, theſe diſtances are found leſs than before in proportion as 3 to 4; that is, as the ſines of refraction out of water into air.

Exp. 3. Viewing theſe rings of colours in a dark room, in homogeneal light, they become more diſtinct. And trying all the priſmatic colours ſucceſſively, the red circles will appear to be bigger than the blue or violet. The diſtances of the glaſſes, at the red and violet circles, are as $14\frac{1}{4}$ to 9. Theſe rings were not now of various colours, but appeared only of that priſmatic colour, with which they were illuminated. And the light which fell on the deep ſpaces between the coloured rings, was tranſ-

Fig. 12. mitted thro' the glaſſes without any variation of colour. From hence the origin of theſe rings is manifeſt; viz. That the air between the glaſſes, according to its various thickneſs, is diſpoſed in ſome places to reflect, and in others to tranſmit the light of any one colour; and in the ſame place, to reflect that of one, where it tranſmits that of another.

The diameter of the 6th circle made by the yellow was $\frac{58}{100}$ of an inch.

Exp. 4. If a little ſoap be agitated in water till it diſſolve, and a bubble of this water blown up, and covered with a glaſs, to keep it from the air. You'll obſerve the bubble grow thin at the top, by the continual deſcent of the water. The bubble at laſt will grow ſo thin at top, as to tranſmit the light and appear black. Then obſerving the bubble by reflected light from the ſky, the black ſpot will be encompaſſed with the ſame coloured circles, and in the ſame order, as in the firſt experiment. As the water deſcends, and the bubble grows thinner, the rings continually grow bigger till the bubble burſts.

If the bubble be viewed by light tranſmitted thro' it, all the colours will be contrary. Therefore it appears that altering the thickneſs of a very thin plate, the colour is altered.

Exp. 5. Take very thin plates of iſinglaſs, talk, or muſcovy glaſs, that ſhow the colours; then by wetting theſe plates, the colours remain the ſame as before, only they become more faint and languid, eſpecially when wet on the under ſide. Therefore the thickneſs of the plate to produce any colour, depends only on the denſity of the plate, and not on the denſity of the incloſing medium. But the colours are more vivid as their denſities are different.

Exp. 6. Two object glaſſes being laid on one another as in experiment 1. If a ſpectator looks obliquely at them, the rings of colours will be increaſed in bigneſs. Hence the colour of ſome plates will change by changing the poſition of the eye. Fig.

From all theſe experiments it is plain, that the colours of bodies, depend upon the thickneſs and denſity of the particles that compoſe them.

Cor. 1. *Hence if the denſity or ſize of the particles in the ſurface of a body be changed; the colour is likewiſe changed.*

Cor. 2. *When the thickneſs of the particles of a body is ſuch, that one ſort of light or colour is reflected; another light or other colours will be tranſmitted. And therefore the body will appear of the firſt colour.*

This appears by Exper. 1. where the tranſmitted light is different from the reflected light.

Cor. 3. *The thickneſs of a plate of air, between two glaſſes, which reflects a deep yellow; muſt be* $\frac{1}{178000}$, or $\frac{3}{178000}$, or $\frac{5}{178000}$, &c. *parts of an inch. And that of a plate of water between two glaſſes, will be* $\frac{3}{4}$ *of it.*

This is plain by the 1ſt experiment, for the thickneſs of the plates of air; and by the 2d, for thoſe of water.

Cor. 4. *The thickneſs of any plate reflecting red, is to the thickneſs of the ſame plate reflecting violet; as* 19 *to* 12.

For by Exp. 3. they were as $14\frac{1}{4}$ to 9.

Cor. 5. *Hence the thickneſs of a plate may be ſuch, that it will reflect different colours.*

If r, v, be the thickneſſes of plates that reflect red and violet. Then (Cor. 4.) $12r$ and $19v$ will be thickneſſes of plates that reflect both.

Cor. 6. *The parts of bodies and their pores, muſt not be leſs than a certain bigneſs to make them coloured.*

For if a particle be thinner than $\frac{1}{178000}$ part of an inch, it can reflect no yellow (by Cor. 3.) and the like for others. In which caſes the light will be tranſmitted, and the body be tranſparent.

Cor. 7. *The particles of bodies reflect rays of one colour, and tranſmit thoſe of another. And this is the ground of all their colours.*

This appears from experiment the firſt.

Cor. 8. *The bigneſs of the compounding particles of natural bodies may be conjectured by their colours.*

We are let into this by Exp. 1. And from hence a table may be made of the ſeveral orders of colours, to form our judgment by. Such a table you'll find in Sir J. Newton's Optics, pag. 206.

Cor. 9. *The cauſe of reflection, is not the impinging of light on the ſolid parts of bodies. But it is done by a certain power, uniformly diffuſed over the whole ſurface of the reflecting body.*

For in paſſing of light out of glaſs into air, there is a reflexion as ſtrong as out of air into glaſs, or ſtronger; and far ſtronger out of glaſs into water. And air is not likely to have ſtronger reflecting parts than water, or glaſs. And the reflexion is as ſtrong or ſtronger in vacuo.

Again, if light paſſing out of glaſs into air, fall at a greater angle than 40 or 41 degrees, it is all reflected, if more perpendicular it is tranſmitted. Now it is not to be imagined, that light at one angle of incidence ſhould meet with pores enow to tranſmit it; and at another incidence little different, ſhould meet with nothing but parts to reflect it.

Likewiſe in the paſſing of light out of glaſs into Fig. air; at a certain incidence the blue is reflected, and the red tranſmitted.

Again, where the glaſſes touch, there is no ſenſible reflexion.

Likewiſe at particular thickneſs of plates, which are in arithmetic progreſſion, there is no reflection but a tranſmiſſion.

Laſtly the ſmall protuberances, left after poliſhing, being innumerable and infinitely ſmall; would throw the light in all directions. All theſe things ſhew, that the reflexion is not cauſed by impinging.

Cor. 10. *Bodies reflect and refract light by one and the ſame power.*

For when light goes out of glaſs into air, as oblique as it can; if its incidence be made more oblique, it is totally reflected.

Again experience ſhews, that light is alternately reflected and tranſmitted, by thin plates of glaſs for many ſucceſſions, according to the thickneſs of the glaſs.

Likewiſe thoſe bodies that have the greateſt refracting power, reflect the greateſt quantity of light.

PROP. XIV.

Every ray of light in its paſſage thro' any refracting ſurface, is put into a certain tranſient conſtitution or ſtate, which in the progreſs of the ray returns at equal intervals; and diſpoſes the ray at every return, to be tranſmitted, and between the returns to be reflected by it.

This is manifeſt by Exper. 1, 2, and 3d. By theſe experiments it appears, that one and the ſame ſort of rays at equal angles of incidence, on any thin tranſparent plate, is alternately reflected and

Fig. tranſmitted, according as the plate increaſes in progreſſion of the numbers 0, 1, 2, 3, 4, &c. ſo that at the thickneſs 1, 3, 5, 7, &c. the rays are reflected; and at the thickneſſes 2, 4, 6, 8, &c. they are tranſmitted; and thereby make the rings of light. And this (by Exp. 5.) depends on both the ſurfaces of the glaſs.

Cor. 1. *The reaſon why the ſurfaces of bodies reflect part of the light falling on them, and tranſmit the reſt, is becauſe ſome rays at their incidence, are in fits of eaſy reflexion, and others in fits of eaſy tranſmiſſion: calling the returns of theſe diſpoſitions of the rays, the fits of eaſy reflexion or tranſmiſſion.*

This appears from the ſeveral rings of colours, made by the object glaſſes; and far more of them are viſible thro' a priſm, and always where the plates are of the thickneſs 1, 2, 3, 4, &c. ſo that they return in equal times.

Cor. 2. *Thoſe ſurfaces which refract the rays moſt ſtrongly in fits of eaſy refraction; do reflect them moſt ſtrongly in fits of eaſy reflexion.*

This follows from the foregoing, and Cor. 9 and 10, of the laſt Prop.

Cor. 3. *When rays of a deep yellow paſs perp. out of any medium into air; the interval of the fits of eaſy reflexion are the* $\frac{1}{89000}$ *th part of an inch. And of the ſame length are the fits of eaſy tranſmiſſion.*

From Exp. 1. of Prop. XIII.

Cor. 4. *When a plate is thinner than* $\frac{1}{178000}$, *the rays are all tranſmitted.*

This appears from the black ſpot, Ex. 1. Prop. XIII.

Cor. 5. *The intervals of the fits are ſhorteſt in the* Fig. *moſt refrangible rays.*

By Exp. 3. Prop. XIII.

Cor. 6. *All bodies appear tranſparent, whoſe particles and pores are too ſmall to cauſe any reflexions.*

Cor. 7. *The opacity of bodies ariſes from the multitude of reflexions within them.*

Cor. 8. *The parts of bodies on which their colours depend, are denſer than the medium included in the pores.*

For where the thin body is rarer than the ambient medium, the obliquity of the rays cauſes the colour to change, ſo that at any obliquity all ſorts of rays will be reflected, which cauſe a whiteneſs. But if the thin body be much denſer, the colours are little changed by the obliquity, and ſo the colour will appear.

Cor. 9. *The conſtituent parts of bodies are the greateſt whoſe colours are moſt dilute.*

For then it indicates a higher order of the colours, as being more mixt, according to Exp. 1. Prop. XIII. where the farther you recede from the center, the more dilute and mixt theſe rings are. And a table may be made therefrom in order to judge by. Such a table you have in Sir J. Newton's Optics, pag. 206.

Cor. 10. *A body becomes tranſparent, by filling the pores of it, with a fluid of the ſame denſity with itſelf.*

For all the parts being of equal denſity, there can be no refraction within it, and therefore the rays will go ſtrait forward and paſs thro' it.

Fig.

PROP. XV.

If a beam of light paſſes into any tranſparent body, as glaſs; part of it will be reflected at its going in, and the reſt tranſmitted to the further ſurface, where ſome is tranſmitted there, and ſome reflected back to the firſt ſurface. And ſo a quantity of light will be continually reflected back and forward within it, for a vaſt number of times till it be all loſt. Whilſt at every point of incidence there is a part tranſmitted into the air.

Take a glaſs ſpeculum, ground concave on one ſide, and convex on the other, and quickſilvered on the convex ſide. And let a beam of light, coming thro' a ſmall hole in the window ſhut in a dark room, fall upon the ſpeculum. Then hold a paper at the center of the glaſs, ſo that the beam of light may paſs thro' a little hole made in the middle of the paper, to the ſpeculum, and ſo be reflected back to the hole. Then you'll obſerve ſeveral concentric rings of colours round the hole; like theſe before in the object glaſſes by tranſmitted light. Theſe rings grow fainter as they grow larger. Meaſuring the diameters of theſe rings, they are in the ſame proportion as thoſe tranſmitted thro' object glaſſes. Viewing them with homogeneal light, they appear more diſtinct, and to a greater number. The red are the largeſt, the violet leaſt, and ſo in order. Placing the eye where the rings appear plaineſt; the ſpeculum will be ſeen tinged all over with waves of colours. From all theſe things, it is plain, theſe rings are made by the light reflected back and forwards within the glaſs; whilſt ſome, being in the fits of eaſy reflexion, and ſome in the fits of eaſy tranſmiſſion, cauſed theſe rings of colours. When the quickſilver

ſilver was rubbed off the glaſs, there would be the ſame rings but fainter; and therefore the quickſilver only increaſes the light of the colours, by increaſing the reflexion. A metal ſpeculum tried, produced no colours; and therefore the colours depend on both ſurfaces of the glaſs, and the thickneſs between them. From what has been ſaid it is evident, that light undergoes many reflexions in ſucceſſion within a body, and many refractions out of it for many viciſſitudes. Fig.

Cor. 1. *The outward ſurfaces of glaſſes not only reflect and refract incident light, in a regular manner; but ſcatters a part of it in all manner of directions.*

That light is ſcattered from glaſſes in all directions; is evident from this, that the glaſſes can be ſeen in all directions; and therefore the light reflected from them, enters the eye, let it be placed where it will.

Cor. 2. *Hence the image of a candle may be ſeen in a glaſs, repeated a great many times; till by the weakneſs of the light it vaniſhes.*

PROP. XVI.

Bodies act upon light at ſmall diſtances, by attraction and repulſion.

Exper. 1. Make a hole with a pin in a thin piece of lead; and placing it in the hole of a window ſhut, let a beam of the ſun's light paſs thro' it, into a dark room. Then will the ſhadows of hairs, threads, pins, ſtraws, &c. be bigger than they would be, if the rays of light paſſed by them in right lines. And it is the ſame thing, whether the ſmall body be encompaſſed with air or water, &c.

Fig. The ſhadows of ſcratches in glaſs, and the veins in it, are broader than they ought to be.

13. Let X be a hair placed in the beam; ADG, BEH, KNQ, LOR, rays of light paſſing by the ſides of it, bent at X, and falling upon the paper GQ. TI, VS, the direct light. The light paſſing neareſt the hair, as at D and N, is the moſt bent, paſſing to G and Q, the fartheſt aſunder; thoſe that are further off, as at E and O, are leſs bent; and ſo on, to TI and VS. Therefore the force acting upon the light is ſtrongeſt at the leaſt diſtance, making the ſhadow GQ; and weaker at greater diſtances.

Exp. 2. The ſhadow of all ſorts of bodies are bordered with three parallel fringes of coloured light; the neareſt to the ſhadow, the brighteſt; and the furtheſt very faint. The order of the colours, reckoning from the innermoſt, are violet, blue, green, yellow, red. So by looking on the ſun thro' a feather or black ribbon, held cloſe to the eye, ſeveral fringes of colours will appear.

Exp. 3. Let the ſun ſhine thro' a hole, a quarter of an inch broad; place a ſheet of paſteboard two or three feet off, blackt over on both ſides, and in the middle of it, a hole $\frac{3}{4}$ of an inch ſquare, for the light to paſs thro'. Behind the paſteboard faſten the blade of a knife very ſharp, to ſtop part of the light, going thro' the hole. The knife and paſteboard are parallel, and both perp. to the rays. Let a part of the light, which paſſes by the knife edge, fall upon a white paper two or three feet diſtant; and there will be made two ſtreams of light; which will appear fairer, if the light paſſes thro' a hole in the paper, and fall on a black cloth behind it. One of theſe ſtreams ſubtends an angle of about twelve degrees; and is perp. to the edge of the knife. And placing ones eye in the ſtream, a line of light will be ſeen upon the edge of the knife; and even when the eye is placed without Fig. 13. the ſtream.

Exp. 4. Placing another knife, with its edge very near and parallel to the firſt. If they be diſtant the 400th part of an inch, the ſtream of light, paſſing between them, will part in the middle, and leave a ſhadow between them, quite dark. As the knives approach the ſhadow grows broader, and the ſtream narrower at the inner end. Hence the light that is leaſt bent goes to the inner end of the ſtream, and paſſes at the greateſt diſtance from the knives. And this diſtance is about the 800th part of an inch, when the ſhadow begins firſt to appear. And the light which paſſes at diſtances leſs and leſs, is more and more bent; and goes to that hand which is furtheſt from the direct light. A little before the ſhadow appears the fringes begin to appear, on either ſide; and as the knives approach, they grow diſtincter and larger, till they vaniſh, the utmoſt firſt, and the reſt in order.

Hence one may conclude, that the light of the firſt fringe paſſes by the knife, at a greater diſtance than the 800th part of an inch, the ſecond at a greater diſtance, and the third greater ſtill. And the ſtreams of light at a leſs diſtance than any of the fringes.

Exp. 5. Take two knives with ſtrait edges, and prick them into a board, ſo as to make an angle with each other, and faſten them ſo that the angle may be invariable; let the angle be near two degrees. Place theſe in a beam of light, in a dark room, about the 42d part of an inch broad; 10 or 12 feet diſtant from the hole; and let the light paſſing between their edges, fall very obliquely upon a ſmooth white ruler, at $\frac{1}{2}$ an inch or an inch diſtance. Then you'll ſee the fringes made by the two edges of the knives, run along the edges of their ſhadows, in lines parallel to theſe edges, without grow-

ing

Fig. 1. ing ſenſibly broader, till they meet in angles equal to that of the knives; and where they meet they end without croſſing. But when the ruler is held at a much greater diſtance, the fringes are narrower, the farther they are from the angular point, where they croſs, and then become much broader.

Hence, the diſtances at which the fringes paſs by the knives, are not increaſed or altered by the approach of the knives. But the angles, in which the rays are bent, are much increaſed by that approach. And the knife which is neareſt any ray, determines which way the ray ſhall be bent; and the other knife increaſes the bent. The diſtance of the edges of the knives (at the meeting of the dark lines between the firſt and ſecond fringes on each ſide) is the 160th part of an inch. Whence half the light paſſes by the edges of each knife, within the 320th part of an inch thereof, and falling upon the paper makes the fringes of the ſhadow of each knife. And the light which falls upon the paper where theſe dark lines meet, paſſes between the knives, where their edges are more than the 160th part of an inch diſtant.

The light which makes the fringes is not the ſame light at all diſtances of the paper from the knives. But when the paper is held near the knives, the light (that makes the fringes) paſſes at a leſs diſtance from the knives, and is more bent, than when the paper is held at a greater diſtance.

Exp. 6. If the paper be held at a great diſtance, perpendicular to the rays; the fringes of the ſhadows will be in the form of hyperbolas, of which there will be ſix for the three fringes on each ſide. In any one of theſe hyberbolas, one aſymptote is parallel to the edge of one knife, and its other aſymptote, is a line drawn perp. to the line which biſects the angle, and paſſes thro' the point where that direct ray of light cuts the paper, which ray Fig. paſſes thro' the angular point of the two knives. Therefore the angle of the aſymptotes of all the hyperbolas will be in that point, where a line drawn from the ſun, thro' the angle of the knives, cuts the paper. And theſe angles will be all obtuſe; being greater than a right angle by half the angle of the knives. And the three hyperbolic fringes on one ſide are equal to theſe on the other. All theſe may be found, by actually meaſuring the ordinates, &c. and ſo proved to be hyperbolas.

Exp. 7. Let the ſun ſhine thro' a ſmall hole made by a pin in a plate of lead; and place a priſm at the hole to refract the light, on the oppoſite wall. The ſhadows of all bodies held in the coloured light, are bordered with fringes of the colour of that light in which they are held; in the red they are red; in the blue, blue, &c. And the fringes made in the red light are largeſt, thoſe in the violet, leaſt; the green ones of a middle bigneſs. And they were ſo at all diſtances from the ſmall body, as a hair, &c.

So then the rays which made the fringes in the red light, paſſed by the hair at a greater diſtance, than thoſe which made the violet fringes. Therefore the hair in cauſing theſe fringes acted alike upon the red rays at a greater diſtance, as upon the violet at a leſs diſtance; and by theſe actions diſpoſed the red light into larger fringes, and the violet into ſmaller, and middle ones into a middle ſize, without changing the colours of the rays.

When a hair is held in a white beam of the ſun's light (Exp. 1 and 2.) and caſts a ſhadow, which is bordered by three fringes of coloured light; theſe colours ariſe from the various inflexions, by which the rays are ſeparated, and being ſeparated, produce each its own colour. In this laſt experiment, where they are ſeparated before the light

Fig. comes to the hair; the red or leaſt refrangible rays, were inflected at a greater diſtance, and the violet or moſt refrangible rays at a leſs diſtance, making three violet fringes at a leſs diſtance, whilſt the red makes three red fringes at a greater diſtance. And the other mean rays were inflected at mean diſtances, making three fringes of their proper colours, at mean diſtances, from the ſhadow of the hair. And in the white light, theſe colours are ſeparated by the various inflexions of the rays, and their fringes appear all together; and the innermoſt fringes being contiguous make one broad fringe, compoſed of all the colours in due order; the violet being next the ſhadow, and the red fartheſt off, and the reſt in their places. And in like manner the middlemoſt fringes compoſe one broad fringe, of all their colours; and the outmoſt fringes compoſe another broad fringe, like the reſt. And theſe are the three fringes of coloured light, with which the ſhadows of all bodies are bordered, as in Exp. 2.

From all theſe experiments it is obvious, that the rays of light are acted on by ſome force or other that turns them out of their direct road. And as this force bends the rays not into the ſhadow of the body that acts upon them, but from the ſhadow; in that reſpect it muſt be a repulſive force, and (by Exp. 1.) is ſtrongeſt at the leaſt diſtance.

Cor. 1. *It is probable, that the rays of light in the points of contact with bodies, and extremely near it, are acted on by an attractive force. And at greater diſtances with a repulſive one.*

For if the rays were acted on in all caſes and circumſtances by a repulſive force; the light would all be expelled at once out of the luminous body. But it is more probable, that theſe particles of light are kept in the body by ſome attractive force; till they be ſhaken off by the vibrating motion of the Fig. body, and thrown beyond the reach of attraction; the contrary power of repulſion then begins, and drives it away with amazing velocity. Light is acted on entirely by attraction, by refraction in a denſe medium, being made towards the perpendicular.

Cor. 2. *This power which acts upon light is infinitely ſtronger than the force of gravity.*

For the velocity of light being ſurpriſingly great, and the ſmall diſtance the active force extends to, makes the time of acting, ſmall beyond all comprehenſion. And therefore in ſuch an infinitely ſmall time, the finite force of gravity would have no manner of effect.

Cor. 3. *The force of repulſion acting on light, decreaſes very faſt in regard to the diſtance; ſo that in a very ſmall ſpace it becomes nothing.*

The force of gravity decreaſes as the ſquare of the diſtance. But this force being reduced to nothing in a very ſmall ſpace, ſhews that it decreaſes far faſter than in that ratio.

Cor. 4. *Hence bodies and light act mutually upon one another; bodies, upon light by emitting, reflecting, refracting, and inflecting it: And light upon bodies, by heating them, and putting their parts into motion.*

For action and re-action are always equal and contrary.

Cor. 5. *Black bodies, grow hot ſooner by light than any other bodies; and white bodies later.*

For the light falling on them is not reflected, but enters the bodies; and by being often reflected and refracted within it, excite a briſk motion among the internal parts of it. But white bodies reflect moſt of the light, and therefore are but little affected by it.

PROP.

Fig.

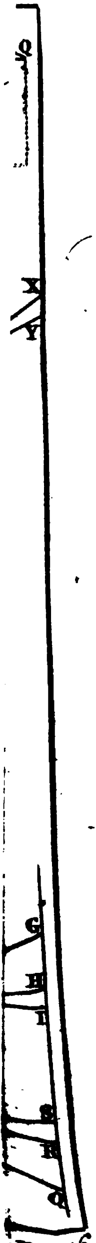
PROP. XVII.

The motion of light is not inſtantaneous, but is performed in time; and ſpends about eight minutes in paſſing from the ſun to the earth.

This is proved from the eclipſes of Jupiter's ſatellites; for when the earth is between the ſun and Jupiter, theſe eclipſes happen about eight minutes ſooner than they ought to do, by the rules of calculation. And when the earth is beyond the ſun, they happen eight minutes later than they ought to do. The reaſon is, the light has a leſs diſtance to move over, from the ſatellites to the earth, in the firſt caſe, than in the ſecond, by the diameter of the earth's orbit. And therefore the radius of the earth's orbit requires eight minutes of time for the light to paſs thro' it.

Cor. *It is very probable, that the rays of light are extremely ſmall bodies, projected with incredible velocity from the luminous body.*

For, 1. It is the property of a body to move thro' ſpaces which are proportional to the times, by a uniform motion; and light does this. 2. It is acted on by a body, when is is reflected, refracted, and inflected; and bodies can only act upon bodies. 3. Light cannot be increaſed or diminiſhed. It can only be increaſed in any place, by robbing other places of its light; and it can only be diminiſhed by conveying it to a different place. And all this may be done with bodies. So that we cannot increaſe it in one place, but we muſt decreaſe it in another. 4. Light conſiſts of parts both ſucceſſive in the ſame lines, and contemporary in ſeveral lines. For in the ſame place one may ſtop that which comes one moment, and let that paſs which



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which comes the next moment. And at the ſame time, one may ſtop it in one place and let it paſs in another. For that part of the light which is ſtopt, cannot be the ſame with that which is let paſs. And all theſe properties agree with what we find in bodies.

PROP. XVIII.

If light enters into a denſe medium, and in its motion in that medium, a part of it be loſt, ſo that only the part p gets thro' the ſpace a. Then the quantity of light which gets thro' the ſpace na in the ſame continued medium, will be p^n.

I take it for granted, that if any quantity of light paſſes into a denſe medium, and in moving thro' the ſpace a loſes any part, as $\frac{1}{10}$ thereof; if half the quantity of light enters it, it will loſe $\frac{1}{10}$ of that half in the ſame ſpace; or if a quarter of the light enters it, it will loſe $\frac{1}{10}$ of that quarter, and ſo on. Therefore there will always be a certain part of the incident light, get thro' equal ſpaces.

Suppoſe then $\frac{8}{9}$ the light, gets thro' the ſpace a; then $\frac{8}{9}$ of that, or $\frac{8}{9}$ of $\frac{8}{9}$ will get thro' the next ſpace a, that is, $\overline{\frac{8}{9}}\big|^2$ will get thro' $2a$, and $\frac{8}{9}$ of that or $\overline{\frac{8}{9}}\big|^3$ will get thro' $3a$; and ſo on. Therefore $\overline{\frac{8}{9}}\big|^n$ of the entring light will get thro' the ſpace na. And inſtead of $\frac{8}{9}$ putting p, the quantity of light getting thro' the ſpace na, will be p^n.

Cor 1. *If* $1-p$ *be the light loſt or deſtroyed in paſſing thro' the ſpace or depth a;* $1-p^n$ *will be the light loſt in paſſing thro' the depth na.*

Cor.

Fig. Cor. 2. *The light tranſmitted at the depth ma, is to the light tranſmitted at the depth na, as* p^m *to* p^n.

Cor. 3. *The light tranſmitted thro' the depth a, is to the light tranſmitted thro' the depth na, as* 1 *to* p^{n-1}.

For the lights tranſmitted are as p to p^n, that is, as 1 to $\frac{p^n}{p}$ or p^{n-1}; where n may be whole or fractional.

PROP. XIX.

The brightneſs of an object is nearly the ſame, at all diſtances of the eye from it.

For if the eye at any diſtance receive any quantity of light, at half that diſtance it will receive four times the light, and the picture upon the retina will be four times as big, whence the denſity or brightneſs continues the ſame. Alſo at a third part of the diſtance it receives nine times the light, and the picture in the eye will be nine times as big; and therefore the denſity is ſtill the ſame. Or in general at $\frac{1}{n}$ of the diſtance, n times the light comes into the eye; and the picture is n times as big, and the denſity the ſame as before, becauſe n times the light is ſpread over n times the ſpace.

PROP. XX.

The proportion of moon light to day light, or the light of the ſun, at full moon, is as 1 *to* 96 *thouſand, nearly.*

For the moon's radius is to the earth's radius as 1 to 3.65; and ſince the radius of the moon's orbit is 60 of the earth's radii; therefore the radius of the moon's orbit, or its diſtance from the earth is 60 × 3.65 or 219 of the moon's radii.

Now ſuppoſe equal ſpaces at the earth and moon to be equally illuminated; and that the whole ſurface of the moon was thus equally illuminated, and the light reflected therefrom all around, ſo far as the ſurface of a ſphere paſſing thro' the earth, deſcribed from the moon's center; then all this light is ſpread upon this ſurface; and conſequently its denſity will be leſs in the reciprocal proportion of theſe ſurfaces, or reciprocally as the ſquares of the diameters. That is, the denſity of the light at the moon (which is ſuppoſed the ſame with day-light) is to the denſity at the earth (reflected from the moon); as the ſquare of 219 to 1, or as 47961 to 1. But ſince in reality, there is not ſo much light falls on the moon, being no more than what falls on her diſk, and her diſk being but half her hemiſphere; the denſity of light at her ſurface, upon each hemiſphere will be but half as much. And it matters not whether any falls upon the oppoſite diſk or not, for (by the rule of proportion) that would all be reflected to the oppoſite hemiſphere, and does not concern us, who are oppoſed to the full moon. Therefore it will now be, as day-light, to moon-light; ſo is 47961 to $\frac{1}{2}$, or as 95922 to 1, or in round numbers as 96000 to 1.

Cor. 1. *Moon-light is to day-light; as half the ſquare of the moon's radius, to the ſquare of the moon's diſtance, when ſhe is full. And in the quadratures, as* $\frac{1}{4}$ *the ſquare of the moon's radius, to the ſquare of the moon's diſtance.*

After the ſame manner may be found the light of any other celeſtial body, compared with day-light; let it be *Venus.* Let d = ſun's diſtance from the earth, v = Venus diſtance from the ſun, a = her diſtance from the earth, r = her radius. Then if Venus and the earth were equally illuminated by the ſun, then is her light to day-light : :

Fig. $\frac{1}{4}rr : aa$. But Venus being nearer the ſun, is more enlightned in the ratio of $\frac{dd}{vv}$ to 1; therefore her light is to day-light :: $\frac{1}{4}rr \times \frac{dd}{vv} : aa :: \frac{1}{4}rrdd : vvaa$. But ſince in her quadratures, $d = a$ nearly, therefore, Venus's light is to day-light; as $\frac{1}{4}rr$ to vv. And that is as 1 to 804.000000, nearly. Hence,

Cor. 2. *The light of Venus in her quadratures, is to day-light; as* 1 *to* 800 *millions.*

This is ſuppoſing the radius of Venus, to her diſtance from the ſun, to be as 1 to 14200.

Cor. 3. *Hence the light of the moon is* 8000 *times as great as the light of Venus. Suppoſing they both of them reflect all the light that falls on them.*

BOOK

BOOK II.

CATOPTRICS; OR REFLECTED VISION.

DEFINITIONS.

DEF. I.

CATOPTRICS, treats about that part of Optics, which relates to ſeeing, or making the pictures of objects by reflected light. Fig.

DEF. II.

A *Speculum* or *Mirror* is any poliſhed ſurface, commonly of glaſs or metal, which reflects the rays of light; and may be either *plain*, *concave*, or *convex*. The glaſs ones are commonly quickſilvered over on the backſide; to make the reflexion ſtronger.

PROP. I.

If a ray of light be reflected from a plain ſurface; the angle of reflexion will be equal to the angle of incidence.

It is ſhewn in Prop. XVI. that the rays of light in being reflected or refracted, are acted on by the bodies they come near, by repelling or attracting them. And theſe forces, be what they will, muſt act in lines perpendicular to the ſurfaces of theſe bodys; and that force muſt be equal at equal diſtances from the reflecting or refracting ſurface.

Let

Fig. 14. Let AB be the incident ray, CD the reflected one. RS the reflecting ſurface, PQRS the infinitely thin ſpace, within which the repulſive force of the body is contained, AB*m*CD the progreſs of the ray. Draw an infinite number of lines parallel to PQ or RS, ſuch as *nt*, *oq*. Then ſince the repulſive force begins at B, it will continually bend or inflect the ray, into the curve B*om*, till it comes to its loweſt point *m*, where it will be parallel to the reflecting ſurface. Afterwards it will recede from the plane SR, deſcribing the curve *mq*C. And ſince the forces are equal at the equal diſtances *o* and *q*, *n* and *t*, &c. therefore the particle of the curve *mq*, will be equal and ſimilar to the particle *om*, and *qt* to *on*. And for the ſame reaſon, all the particles of the curve *mq*C will be equal and ſimilar to the correſponding particles of the curve *mo*B; and therefore the whole *mq*C will be equal and ſimilar to the whole *mo*B. And conſequently the angles at C and B will be equal; and the angle of reflexion equal to the angle of incidence, being their complements.

Cor. 1 *If a ray of light be reflected from any point of a curve ſurface; the angle of reflexion is equal to the angle of incidence.*

For it is the ſame thing as if it was reflected from a plane touching the curve in that point.

Cor. 2. *The ray of incidence and reflexion are both in a plane, perpendicular to the reflecting ſurface, in the point of incidence.*

For there is no force acting out of that plane.

Cor. 3. *A ray of light falling perpendicular to any ſurface, is reflected back in the ſame perpendicular. But moving parallel to it, will continue ſo.*

Cor. 4. *If the reflected ray at* O, *return back thro'* OS, *it will be reflected to* R. Fig. 14.

SCHOLIUM.

The curve B*omq*C deſcribed by the ray of light, is ſo extremely ſmall, that it may be looked upon as a ſingle point. And therefore the points B and C may be ſuppoſed to coincide.

PROP. II.

If rays proceeding from a radiant point R *fall upon a reflecting plane ſurface* PL; *the focus of the reflected rays, will be at* F, *in the line* RF *perp. to the reflecting ſurface; and as far on the other ſide of it. That is* PF = PR. 15.

For ſince RPF is perp. to PL; let RS be any incident ray; and SO the reflected ray. Produce OS backward to F. Then (Prop. I.) $<$PSR = $<$LSO=PSF. Therefore in the right angled triangles PSR, PSF; all the angles are reſpectively equal, and PS common; therefore RF equal PF. Therefore the reflected ray SO proceeds from F, in the perp. RF, ſo that PF = PR. And by the ſame reaſoning, if RN be any other incident ray, its reflected ray NH will proceed from F. And therefore F is the virtual focus of all the reflected rays, which therefore all diverge from F.

Cor. 1. *The diſtance of the focus* F *from the eye at* O, *is equal to the incident and reflected rays taken together*, OF = OS + SR.

For OF = OS + SF = OS + SR.

Cor. 2. *Rays converging to* F, *and reflected by the plane* PL, *converge to* R.

Cor. 3. *If rays proceeding from an object at* R *be reflected by the plane* PL *to an eye at* O; *the object will*

Fig. *will appear to be placed at* F, *the focus of the reflected*
15. *rays, where they laſt diverged from.*

For we always judge an object to be in that place, from whence the rays proceed directly to the eye; which here is from F.

PROP. III.

16. *If rays proceeding from a radiant point* R, *fall up-*
17. *on any point* S *of a reflecting ſpherical ſurface* VS, *and* F *be the focus of the reflected rays; it will be,* SR : CR : : SF : CF. C *being the center of the ſphere.*

For (*fig.* 16.) the angle of reflexion CSF being equal to the angle of incidence CSR (Prop. I.); therefore the angle RSF is biſected; and therefore (Geom. II. 25.) SR : SF : : CR : CF.

And (*fig.* 17.) draw FA parallel to RS, and produce CS to B; and if FSD be the reflected ray, then the angle FSA = BSD = (Prop. I.) BSR = SAF; therefore SF = AF. Then by the ſimilar triangles CRS and CFA, SR : CR : : AF or SF : CF.

Cor. 1. *If the radiating point* R, *and the focus* F, *be both on one ſide the reflecting ſurface. If the rays diverge from one, they converge to the other.*

Cor. 2. *If the radiating point* R *and focus* F *be on different ſides of the reflecting ſurface; if the rays diverge from one, they diverge from the other. But if they converge to one, they converge to the other, by reflection.*

Cor. 3. *If* R *is at an infinite diſtance, then* SF = CF.

For then CR = SR, and conſequently CF = SF.

Cor. 4. *If* R *is the radiant point; make the* $\angle$ RSC = CSF, *or* RSB = BSD, *and draw* SF, *or* DSF, *and* F *is the focus of the reflected rays.* Fig. 16. 17.

Cor. 5. *The focus of rays proceeding from any radiant point* R; *is in the line* RCV, *joining the point* R *and the center.*

For taking V*s* = VS, the focus is in the place where the rays meet, which are reflected from all points of the arch S*s*. Let RS, R*s* be two correſpondent ones, then the angles SRF, *s*RF, being equal, and alſo RSF and R*s*F; therefore SF, *s*F will meet ſomewhere in the line RCV, at F.

Cor. 6. *A ſmall object placed in the center, cannot be ſeen by reflexion. Or the eye placed in the center, can ſee nothing by reflexion.*

For all the reflected rays go alſo to the center.

PROP. IV.

If VA *be a reflecting ſpherical ſurface, whoſe center is* C. *And if a ray of light* RA (*or* GA) *parallel to* CV, *fall upon* A, *and be reflected to* F. *Draw the tangent* AT; *then will* TF = FC. 18.

For CAT being a right angle, and (Prop. I.) $\angle$ CAF being = angle CAR; we ſhall have

$$CAT = \frac{RAF + FAG}{2},$$

and

$$CAF = \frac{RAF}{2}$$

then by ſubtraction $FAT = \frac{FAG}{2} = TAG = ATF$;

therefore TF = AF = FC, becauſe $\angle$ CAF = CAR = ACF.

Cor. 1. *Hence* FC, FA, FT, *are all equal to one another.*

Cor. 2. *The tangent* AT *biſects the angle* FAG.

Fig. 18. Cor. 3. *If* $CQ = QV$, *then* Q *is the focus of the rays falling on* V, *or the principal focus.*

Cor. 4. *And* QF (= *half* TV) *is the ſpace on which all the rays fall that come from the whole ſurface* AV.

19. Cor. 5. *If* AV *be any curve, and a ray of light* RA (*or* GA) *be reflected into the line* AH. *If* VL *be drawn parallel to* AR, *and* AL *perp. to the curve at* A. *Then* $AH = HL$.

For produce AL to C, that AC may be the radius of curvature in A; draw CF parallel to VL, and produce AH to interſect it in F. Then F is the focus of the rays reflected by the circle, paſſing thro' A, and therefore of the portion of the curve at A that coincides with it. Therefore (Cor. 1.) $CF = AF$, and conſequently (by ſimilar triangles) $LH = AH$.

Cor. 6. *When parallel rays fall on the ſurface of a ſphere, thoſe neareſt the vertex, have their focus neareſt the center.*

PROP. V.

20. 21. *If* aVA *be a ſmall portion of a reflecting ſpherical ſurface,* C *its center,* V *its vertex;* R *the radiant point, from or towards which the rays flow;* F *the focus after reflexion at* A. *Then it will be* $VR : VF :: CR : CF$, *nearly.*

For (Prop. III.) If A be the point of incidence, it is $AR : AF :: CR : CF$. But the arch VA being extremely ſmall, $AR = VR$, and $AF = VF$, very near. Therefore $VR : VF :: CR : CF$.

Cor. *The points* R, F *are conjugate foci to one another; that is, if* R *be the radiant point,* F *will be the focus; or if* F *be the radiant point,* R *will be the focus. Where I uſe radiant points in a general ſenſe, for*

for the point, which the rays either diverge from, or converge to, before reflexion. Fig.

PROP. VI.

If aVA be a ſmall portion of a reflecting ſpherical ſurface, whoſe center is C, and V its vertex. And if parallel rays, or thoſe from an infinite diſtance, as RA or GA, fall upon it, and F be the focus of the reflected rays; then F is in the middle of the radius VC, or VF = FC. 22.

For (by the laſt Prop.) VR : CR : : VF : CF. But becauſe R is infinitely diſtant, VR = CR, therefore VF = CF.

Cor. 1. *Rays diverging from F, the middle point of the radius, or converging to F, and reflected by the ſpherical ſurface VA: will after reflection go parallel to one another.*

Cor. 2. *A luminous body placed in F the middle point of the radius, will caſt a light to a great diſtance, by reflexion from the concave ſurface.*

Cor. 3. *The principal focus of a reflecting ſpherical ſurface, is in the middle point of the radius at F. And the principal focal diſtance FV, is half the radius.*

PROP. VII.

Let Aa be a ſmall part of a reflecting ſpherical ſurface; C the center, V the vertex, P the principal focus, R the radiating point (or that which the rays diverge from, or converge to, before reflexion). And if F be the focus after reflexion, then will PR, PC or PV, and PF, be in continual proportion. 23. 24.

For (*fig.* 23), ſince VF — PF = PF + FC, therefore VF — FC = 2PF. But (Prop. V.) VR

: VE

Fig. 25. $d + r : r - f$. Whence $dr - df = df + rf$, and $dr = 2df + rf$; and reduced, $f = \frac{dr}{2d+r}$.

Cor. 1. *If d is infinite, or* R *is at an infinite diſtance, then r vaniſhes, and $f = \frac{1}{2}r$.*

Cor. 2. *If* R *is on the concave ſide, d is negative, and $f = \frac{-dr}{-2d+r}$, that is, $f = \frac{dr}{2d-r}$. And moreover, if r is infinite, as in a plane ſurface, $f = -d$.*

Cor. 3. *If r is negative, or the reflecting ſurface concave towards* R, *then $f = \frac{-dr}{2d-r}$, that is, $f = \frac{dr}{r-2d}$.*

Cor. 4. *If* AV *is concave towards* R, *and d infinite, then r is negative, and $f = \frac{-dr}{2d}$, that is, $f = -\frac{1}{2}r$.*

Cor. 5. *If it is a plane ſurface, then r is infinite, and $f = d$.*

And many other corollaries may be deduced.

SCHOLIUM.

It is eaſy to know whether the focus be real or virtual, by the direction and motion of the rays; that is, by their converging or diverging after reflexion.

From this general ſolution it is plain, that when the focal diſtance comes out affirmative, it is to be taken right forward; but if negative, it muſt be taken backward, from the vertex.

From this Prop. ſeveral other Problems may be reſolved. As if it was required to know, what muſt be the radius of a ſphere, to repreſent the given object R, at the given focus F. Here r is Fig. to be found from the equation $f=\frac{dr}{2d+r}$; multi- 25. ply by $2d+r$ and we have $2df+rf=dr$, and $dr-fr=2df$, whence $r=\frac{2df}{d-f}$, the radius of the ſphere. From this equation, it is plain, that if d be greater than f, the reflector will be convex towards the object. But if f be greater than d, r will come out negative, and the reflector will be concave to the object. If d be equal to f, r comes out infinite, and in this caſe, the ſpeculum muſt be a plane ſurface.

Again, if it be required to find the diſtance of an object R, from the given ſpeculum AV, to have the focus at the given place F. Here we want to find d, from the equation $f=\frac{dr}{2d+r}$; by multiplying, we have $2df+rf=dr$, and tranſpoſing, $dr-2df=rf$, whence $d=\frac{rf}{r-2f}$, the diſtance from a convex ſpeculum. But if the ſpeculum be concave to the object, then $d=\frac{rf}{r+2f}$. Hence in a convex ſpeculum if r be greater than $2f$, the object will be on the convex ſide. But if $2f$ be greater than r, it will be on the concave ſide. And if $r=2f$, the diſtance of the object will be infinite; and f will be the focal diſtance of parallel rays.

The principal focus of a reflecting ſpherical ſurface may alſo be found by experiments, as follows.

1. For a concave reflector. Hold it directly before the ſun, and hold a bit of paper before it to receive the reflected light. Move this paper back and forward till the white ſpot of light be the leaſt poſſible, and that is the focus.

Or,

Fig. 25. *Or thus*, cover it with paper having ſeveral pin holes made in it. Then hold a piece of paper before it, and you'll ſee ſo many white ſpots on it made by the light reflected thro' the pin holes. Move the paper back and forward, till they all coincide, and there is the focus.

2. For a convex reflector. Cover it with paper, having two pin holes made in it on contrary ſides. Then expoſe it to the ſun beams, and hold another paper before it, having a hole made in it ſo big as to let the ſun's rays paſs thro' to the two pin holes. Then you will ſee two white ſpots of reflected light on each ſide the hole. Move the paper back and forwards, till the diſtance of the ſpots be twice the diſtance of the holes in the cover, firſt marked upon the paper. Then the diſtance of the paper from the lens is the principal focal diſtance.

PROP. IX.

26. *If rays flow from one focus* R *of a ſpheroid or hyperboloid* BA; *they will be reflected by the curve ſurface, into the other focus* F.

For (Conics, B. I. Prop. X.) the lines AR, AF drawn to the foci of an ellipſis, make equal angles with the curve; and therefore the angle of incidence is equal to the angle of reflexion. And the ſame is true of the hyperbola (ib. B. II. Prop. IX.); therefore if RA be the incident ray, AF will be the reflected one.

Cor. *Hence* F *is the real focus in the ellipſis, and only a virtual focus in the hyperbola.*

PROP. X.

If rays RA, *parallel to the axis of a paraboloid* BA, *be reflected by the ſurface thereof; they will all be reflected into* F *the focus of it.* 27.

For (by Prop. VIII. Cor. 3. B. III. Conics), the line FA drawn to the focus, and RA parallel to the axis, make equal angles with the curve at A. Therefore if RA be the incident ray, AF will be the reflected one.

Cor. *Rays proceeding from the focus* F, *will be reflected by the ſurface of the paraboloid, parallel to the axis.*

PROP. XI.

Wherever the rays, which come from all the points of an object, meet again in ſo many other points, after they have been made to converge by reflexion; there they will make the picture of the object upon a white paper, or any white body.

For if PR be any object, A*a* a concave ſpeculum, the rays that diverge from any point Q, fall upon the ſpeculum A*a*, and are reflected to their focus *q*, where they all meet, and upon a white paper held there give *q* the image of Q. In like manner the light which diverges from P, will be reflected from the ſpeculum to their focus *p*. And all the light diverging from R, will be reflected to their focus *r*. And thus from every point in the object, the light will be reflected to ſo many correſpondent points in the focus; and will form there the picture *pqr*, of the object PQR, in its proper ſhape and colours; but ſometimes the picture will be inverted; as in this figure, where the upper end is downward, and the right to the left hand. 28.

Cor.

Fig. 28. Cor. 1. *The rays of light flowing from all the points of an object* PR, *paint the image thereof pr in the real focus q; and it may be ſeen there hanging in the air, by an eye at* O, *placed at a proper diſtance, and ſituation to receive the rays coming from it.*

Cor. 2. *The extreme point of the image, as r, is determined by the extreme ray* R*r drawn thro' the center* C.

PROP. XII.

The image of a right line made by a reflecting plane ſurface, is alſo a right line.

29. Let AB be a reflecting plane, PR a right line, Q the middle point of it. Draw P*p*, Q*q*, R*r*, perpendicular to the plane AB, cutting it in D, I, L. And make D*p* = DP, I*q* = IQ, and L*r* = LR, then (Prop. II.) *p*, *q*, *r*, will be the foci of P, Q, R; and *pr* the image of PR. Whence the figure DPQRL is ſimilar and equal to the figure D*pqr*L, by conſtruction. And therefore ſince PQR is a right line, *pqr* is alſo a right line.

Cor. 1. *Hence the image of a plain figure, ſeen in a reflecting plain ſurface, is alſo a plain figure, equal and ſimilar to the figure itſelf.*

For all the right lines drawn in it, are repreſented by right lines, equal to thoſe in the expoſed figure. And therefore the figure and its image are equal and ſimilar.

30. Cor. 2. *The image of any object, made by an upright reflecting plane, has its parts in a contrary poſition, the left ſide being to the right, and the right to the left hand; the difference being the ſame as the ſeal, and its print on the wax. But in a horizontal reflector, the object will appear upſide down; the parts lying to either hand, remaining the ſame as in the object.*

For if ABC be a perp. reflector; the object PQR, making the image *pqr*, where P on the left being neareſt the glaſs, will be repreſented by *p* on the right hand, alſo neareſt the glaſs. But ſtanding at D and viewing the upright object PQR, in the reflector ABC, laid horizontally; the loweſt point P, neareſt the glaſs, will be repreſented by the higheſt point *p*, alſo neareſt the glaſs; whilſt R *and* *r* are on the right, and Q, *q* on the left. Fig. 30.

Cor. 3. *What has been ſaid of the image of an object, is alſo true of the image of an image.*

Cor. 4. *Changing the ſituation of the eye, does not at all change the ſituation of the image; it will always appear in the ſame place, whilſt the object and reflector are fixt.*

Cor. 5. *When an object is ſeen by a plain ſpeculum; its image appears as far behind the ſpeculum, as the object is before it.*

PROP. XIII.

If AV*a be a ſpherical reflector, the object* PQR *an arch of a circle, having the ſame center* C, *its image* *pqr*, *by reflexion, will alſo be an arch of a circle, having the ſame center* C. *And its image will be direct or inverted, according as it is on the ſame or different ſide of the center* C, *with the object.* 31. 32.

Here *p* is the focus of P, *q* of Q, and *r* of R. Let *e*, *f*, *g*, be the principal foci, of the points A, V, *a*, of the reflecting ſurface. Then (Prop. VII.) we have *f*Q : *f*C (*f*V : *fq*,) in continual proportion, and *g*R : *g*C : *gr*; alſo *e*P : *e*C : *ep*, alſo in continual proportion. But the two firſt terms are the ſame in all, and therefore the laſt; that is *fq* = *gr* = *ep*. And ſince *f*C = *g*C = *e*C, the ſums or

remainders

Fig. remainders will be equal, $qC = rC = pC$, therefore
31 rpq is the arch of a circle whoſe center is C.
32. And ſince the rays croſs at the center C, if the image be on the ſame ſide of C as the object, it will be in the ſame poſition; but on different ſides, in a different poſition.

Cor. 1. *The length of the object* PR, *is to the length of the image* pr, *as* CP *the object's diſtance from the center, is to* Cp *the diſtance of the image from the center.*

Cor. 2. *On the contrary, if* pqr *be a circular object, its image* PQR *will alſo be a circular arch concentric with the object.*

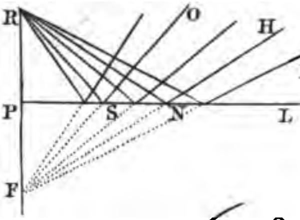
Cor. 3. *The image of a ſpherical ſurface, ſeen by reflexion in a ſpherical reflector parallel to it, will appear a ſpherical ſurface, alſo parallel to it.*

PROP. XIV.

33. *If a right line be placed directly before a concave*
34. *reflector, its image will appear concave; but in a convex reflector, the image will appear convex.*

To the center of the glaſs C, deſcribe the arches PQR, pqr; ſo as pqr may be the image of PQR. And let the expoſed lines be OQ or oq, tangents to the arches PQ, pq, in Q and q. Then if the arch PQ be an object, the arch pq will be its image. Let e, f, g, be the principal foci to the points A, V, a. Then (*fig.* 33.) we ſhall have (by Prop. VII.) $ep : eC : eP$, and alſo $eo : eC : eO$, continual proportionals; ſuppoſing o the image of O. Therefore $ep \times eP = eC^2 = eo \times eO$, whence $ep : eo :: eO : eP$. And by diviſion $ep : ep - eo\ (op) :: eO : eO - eP\ (OP)$, and alternately $ep : eO :: op : OP$, or

$\frac{ep}{eO}$



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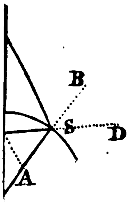

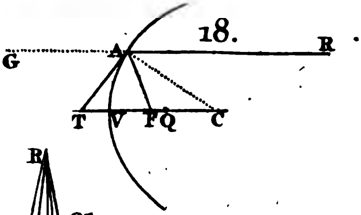

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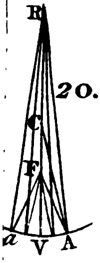

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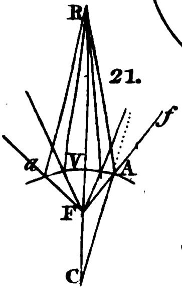

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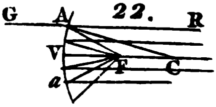

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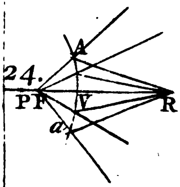

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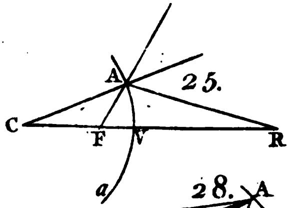

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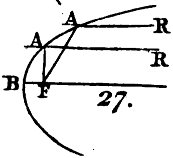

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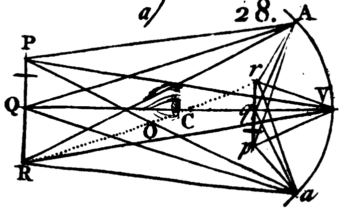

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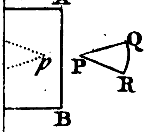

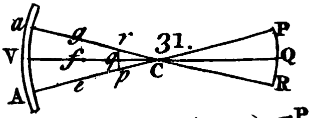

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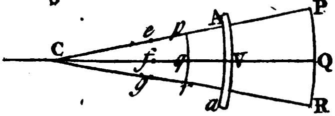


32.

$\frac{ep}{eO} = \frac{op}{OP}$. But if *qo* be alſo a right line, the Fig. 33. 34. figures *pqo* and PQO will be ſimilar; whence $\frac{Cp}{CP}$ would be $= \frac{op}{OP}$. But $\frac{op}{OP}$ is leſs than $\frac{Cp}{CP}$; for ſince *ep* : *e*C : : *e*C : *e*P, by diviſion *ep* : *e*C : : *e*C — *ep* (*p*C) : *e*P — *e*C (CP), therefore $\frac{ep}{eC} = \frac{Cp}{CP}$. But $\frac{ep}{eC}$ is greater than $\frac{ep}{eO}$; therefore taking their equals, $\frac{Cp}{CP}$ is greater than $\frac{op}{OP}$, or $\frac{op}{OP}$ leſs than $\frac{Cp}{CP}$. Therefore *oq* (the picture of OQ) will not be a right line, but *po* will be leſs; and therefore *oq* will be concave towards C.

On the contrary, if *oq* (concave towards C) be the object, the right line OQ will be the picture. But then a right line *oq* being an object, will not have the right line OQ for its picture. For as *o* approaches *e*, O will go farther from P. And therefore OQ (the picture of the right line *oq*) will be convex towards V, or concave the contrary way. So that when the image appears on the concave ſide of A*a*, it is always convex towards A*a*, or concave the contrary way, that is, towards the eye.

Again, (*fig.* 34.) ſince *pq* is the image of PQ an object; if QO becomes an object, then as O goes further from A, its image will advance from *p* towards *e*; and therefore the image of the right line OQ will be more curve than before, being convex towards AV.

On the contrary, if the arch *pq* be an object, the arch PQ will be its image; and therefore if the line *oq* be an object, where *o* approaches nearer A, P (the image) will alſo approach nearer A, and the image will then be more concave towards V,

or

Fig. or towards C. So that in all caſes, a concave re-
33. flector makes the image concave.
34.

Cor. 1. *The image of a right line is more curve when ſeen beyond any reflector; than when ſeen nearer to the eye, or on the ſame ſide.*

Cor. 2. *Any plane figure ſtanding directly before a concave reflector, will appear concave; and before a convex one, convex.*

Cor. 3. *The image of a ſhort right line or plane, is nearly a right line or plane figure.*

Cor. 4. *A right line at the center of a concave reflector, will appear a right line.*

For then $\frac{op}{OP} = \frac{Cp}{CP}$.

PROP. XV.

35. *The object and its image, ſubtend equal angles at*
36. *the vertex of the reflector:* $PVR = pVr$.

Let PR be an object, *pr* its image. Thro' the center C, draw QCV perp. to PR. Draw VP, VR, V*p*, V*r*, and P*p*A, R*ra*, paſſing thro' C. Then (Prop. V.) $VQ : Vq :: QC : qC ::$ (ſimilar triangles) $PQ : pq$. And ſince the angles at Q and *q* are right, the angle $PVQ = pVq$. In like manner $RVQ = rVq$; whence $PVR = pVr$.

Cor. *Hence the object and its image are equal to one another, where they meet together.*

Fig. 35. 36.

PROP. XVI.

The linear magnitude or diameter of the object PR, *is to the magnitude of the image* pr; *as the diſtance of the object from the vertex of the glaſs* VQ, *to the diſtance of the image from it* Vq.

For (laſt Prop.) the angles PVR and pVr being equal, and the triangles PVR and pVr ſimilar, we have $PR : pr :: VQ : Vq$.

Cor. 1. *The diameters or linear magnitudes of the object and image; are as their diſtances from the center of the reflector.* $PR : pr :: CQ : Cq$.
For $PR : pr :: VQ : Vq ::$ (Prop. V.) $CQ : Cq$.

Cor. 2. *If f be the principal focus; then the diameter or linear magnitude of the object, to that of the image; is as the principal focal diſtance, to the diſtance of the image from the principal focus; or as the diſtance of the object from the principal focus, to the principal focal diſtance.*
For we had $PR : pr :: VQ : Vq :: CQ : Cq :: VQ - CQ$ (or $CQ - VQ$) $: Vq - Cq$ (or $Cq - Vq$) $:: CV : 2fq :: fV : fq :: fQ : fV$.

Cor. 3. *Hence alſo it follows, that a convex reflector leſſens an object.*

Cor. 4. *Alſo a concave reflector leſſens an object, when it is farther from the glaſs than the radius of the reflector.*

Cor. 5. *But a concave reflector magnifies an object, when it is nearer to it, than the radius of the reflector; and then only.*

Cor. 6. *The image pr is always terminated at p, by the extreme ray* PA, *which paſſes thro' the center*

Fig. C *of the reflector. And if the object be at an immenſe*
35. *diſtance, it is terminated by the radius* CA, *drawn*
36. *parallel to the extreme ray* PV, *at the vertex.*

For when the object is infinitely diſtant, PV and PCA are parallel.

SCHOLIUM.

If it be required to find the place of an object, that its diameter ſhall be to the diameter of the image, in the given ratio of *m* to 1. It is eaſily done by the VIIIth Prop. For ſince $d:f::m:1$, therefore $\frac{d}{m}=f=\frac{dr}{2d+r}$, whence $2d+r=mr$, and $2d=mr-r$, therefore $d=\frac{m-1}{2}r$, for a convex ſpeculum; in which caſe *m* is always greater than 1. But if *m* be leſs than 1, then *d* comes out negative; that is, the object muſt be always on the concave ſide, to have the image magnified. In like manner, any two, *d*, *r*, or *m* being given, the third may be found.

PROP. XVII.

As the diſtance of the object from the principal focus of a reflector, is to its diſtance from the reflector; ſo the principal focal diſtance, to the diſtance of the image from the reflector. $fQ:VQ::fC:Vq$.

For f being the principal focus, we have (by Prop. XVI.) $VQ:Vq::PR:pr$. And (by Cor. 2. Prop. XVI.) $PR:pr::fQ:fV$; therefore $fQ:fV$ or $fC::VQ:Vq$.

Cor. *If the object and image be equidiſtant from the reflector; they will coincide either in the center or vertex of the reflector, and become equal.*

For if $VQ=Vq$, then $fQ=fC$ or fV, and Q falls upon C, or elſe upon V; and then $pr=PR$.

PROP.

Fig. 35. 36.

PROP. XVIII.

In a convex ſpeculum, the image always appears erect and on the other ſide of the glaſs, as in a plain ſpeculum. And ſo it does in a concave, when the object is between the glaſs and the principal focus. But in a concave ſpeculum, when the object is further from it than the principal focus, the image is on the ſame ſide of the glaſs, and inverted in every reſpect.

For *fig.* 36, let RP be placed where it will before the convex *a*A, its image *rp* will be on the contrary ſide. And if *rp* be an object, its image RP will be on the contrary ſide, till *rp* comes to *f*, and then RP will be infinitely diſtant. But when it is between *f* and C (*fig.* 35.) as at *q*, the image RP is on the ſame ſide. Or if RP be the object, then the image *rp* is on the ſame ſide, but inverted; becauſe the object and image are on different ſides of the center, where the rays croſs. And if the object PR be moved to an infinite diſtance, the image *pr* will be moved to *f*, ſtill on the ſame ſide of the glaſs.

Cor. 1. *If the center is between the object and image, they either both approach to it, or both fly from it. And if the ſpeculum be between them, they either both approach the ſpeculum, or both fly from it.*

For (*fig.* 35.) when C is between Q and *q*, the greater CQ is, the greater is C*q*. And (*fig.* 36.) when V is between them, the greater VQ is, the greater is V*q*.

Cor. 2. *As an object moves from the glaſs on the convex ſide to an infinite diſtance, its image moves from the glaſs to the principal focus. And as the object moves from an infinite diſtance on the concave ſide, to the center, the image moves from the principal focus*

Fig. 35. 36. *to the center alſo, where they meet. And further, if the object moves from the center to the principal focus; the image moves from the center, the contrary way, to an infinite diſtance. And as the object moves from the principal focus to the vertex, the image moves from an infinite diſtance on the convex ſide to the vertex alſo, where they meet again.*

For (*fig.* 36.) whilſt the object moves thro' VQ, the image moves thro' V*q* to *f*. And (*fig.* 35.) whilſt the object moves thro' QC, the image moves thro' *f*C. Alſo whilſt the object moves thro' C*f*, the image moves thro' CQ to an infinite diſtance; and (*fig.* 36.) whilſt the object moves thro' *f*V, the image moves from an infinite diſtance thro' QV.

Cor. 3. *Tho' the object be removed to an infinite diſtance, from a convex glaſs, yet the image will only recede to the diſtance of half the radius, and will there remain.*

Cor. 4. *The image is upright or inverted, according as the image and object are on the ſame or different ſides of the center.*

PROP. XIX.

To ſee the image of an object made by any ſpherical ſpeculum; the eye muſt be placed in the diverging rays, at a proper diſtance for diſtinct viſion, and facing the image.

For to ſee the image, one muſt look at it, as one would do at any viſible object. And therefore in order to be ſeen, the rays muſt diverge from it to the eye placed at a proper diſtance. Thus to ſee the image *pr* (*fig.* 35.) reflected from the concave ſpeculum A*a*, the eye muſt be placed ſomewhere in the line *q*CQ. But if PR was the image (of the object *pr*), the eye muſt be placed beyond Q. Again (*fig.* 36.) to ſee the image *rq*, reflected by the convex ſpeculum A*a*, the eye muſt be placed without the glaſs in the line VQ. But if *rp* is an object, and RP its image, then the eye muſt be placed in the line VC, on the concave ſide. The focus Q being only a virtual focus, to which the rays only tend to, but never reach. Fig. 35. 36.

Cor. 1. *If the eye be placed near the ſpeculum, in the converging rays, before they reach the image; it will perceive the image (of the object) beyond the glaſs, at the ſame diſtance nearly as the object is before it; and of the ſame magnitude.*

For the eye being near the glaſs, that ſmall part of it which reflects rays to the eye, may be taken for a plane, and therefore the image will appear, as in a plane ſpeculum, at the diſtance of the object behind it. But the rays that come to the eye placed beyond the image, do not fall upon the eye in the ſame manner; for they all diverge from points very near the eye.

Cor. 2. *If an image be looked at, with both eyes placed very near it, and in the diverging rays; (or elſe it cannot be ſeen at all); it will appear double.*

For the axes of the eyes cannot both be directed to an object extremely near; and the caſe is the ſame in all objects near hand, as well as this image.

Cor. 3. *Tho' an eye cannot ſee an image in the air, except it be placed in the diverging rays; yet if that image be received on a white paper, it may be ſeen in any poſition of the eye.*

For the rays reflected from the glaſs, to the image and beyond it, flow but in that one direction. But when the image is received on a white paper, the paper reflects them in all directions, wherever the eye is placed.

Cor. 4. *If the eye be moved whilſt it views the image, the image will appear to be moved.*

For

Fig. For rays from different points of the ſpherical
35. reflector, will come ſucceſſively to the eye.
36.

PROP. XX.

35. *If an object rp be placed in the principal focus of a concave reflector; its apparent magnitude to the eye, at any place whatever as* O, *will be invariably the ſame; and equal to the apparent magnitude to the naked eye, when ſeen from* C *the center of the ſpeculum.*

Here RP the image will be at an infinite diſtance. And (by Cor. 1 and 3, Prop. VI.) all the rays of the pencil *r*BV will be reflected parallel to one another, and to the axis *a*C paſſing thro' the center of the ſpeculum; and therefore the angle VOB, which is the apparent magnitude of the image to the eye at O, is equal to the angle VC*a* or VC*r*, which is the apparent magnitude of *rq* to an eye at C. And this is ſo, wherever O is, or the place of the eye.

Cor. 1. *Hence the apparent magnitude of a body placed in the principal focus, will always continue the ſame, however the eye is moved backward or forward from the reflector.*

Cor. 2. *The nearer the eye is to the ſpeculum, the more of the object appears; and the further off, the leſs.*

For rays parallel to C*r* or BO, that lye beyond O (in reſpect to C), cut the reflector in points beyond B, and at laſt will miſs the reflector, and then the point *r* cannot be ſeen there.

Cor. *If the object be nearer than the principal focus, its apparent magnitude grows leſs in going from the ſpeculum. If it be further off, it encreaſes.*

For if the object be nearer, the image is beyond Fig.
the ſpeculum, and then OB, Cr (converging to a 35.
point beyond the ſpeculum), makes the angle BOC leſs than rCq. But if the object is further off, than the principal focus, the image is on the ſame ſide; and then the rays converging to a point beyond O, makes the angle BOV greater than rCq.

SCHOLIUM.

Some other phœnomena of concave reflectors, are theſe that follow.

1. The apparent magnitude of an object will be invariable, wherever it is placed, when the eye is at the principal focus.

2. When the eye is fixt at a leſs diſtance than the principal focus; as the object is moved from the ſpeculum, the apparent magnitude decreaſes.

3. When the eye is fixt at a greater diſtance; the apparent magnitude of an object in going from the ſpeculum increaſes, till it comes to the conjugate focus (of the place of the eye), and then it is infinite, and all confuſion. And afterwards it diminiſhes again, and is inverted.

4. A man's face in a reflecting concave, in going from it, decreaſes to the principal focus, and then increaſes.

PROP. XXI.

The brightneſs of the image pr formed by the re- 35.
flecting concave AB, *from the luminous object* PR, *is as*
$\frac{AB^2}{Vq^2}$, *or as the area of the lens directly, and the ſquare of the diſtance of the image reciprocally.*

For if the brightneſs of the object be given, the quantity of light reflected from the ſpeculum to the image, will be as the magnitude of the object and of the reflector directly, and the ſquare of the

Fig. 35. diſtance reciprocally. Whence the light falling on the image is as $\frac{PR^2 \times AB^2}{QV^2}$; alſo the denſity of the light, or the brightneſs of the image is as the quantity of light directly, and the area of the image reciprocally, or as $\frac{PR^2 \times AB^2}{QV^2 \times pr^2}$. But ſince PR, pr are parallel, and (Prop. XV.) the angle PVR $=$ pVr, &c. PR : pr : : QV : qV, and $\frac{PR^2}{QV^2} = \frac{pr^2}{qV^2}$. Therefore the brightneſs of the image is as $\frac{pr^2 \times AB^2}{qV^2 \times pr^2}$ or $\frac{AB^2}{qV^2}$.

Cor. 1. *If the brightneſs of the object be increaſed in any ratio, which call* B; *the brightneſs of the image will then be as* $\frac{AB^2}{qV^2}B$.

Cor. 2. *In very remote objects, if* f *be the principal focal diſtance; then the brightneſs of the image will be as* $\frac{AB^2}{ff}B$.

For then Vq becomes equal to f.

Cor. 3. *The brightneſs of the image increaſes as the area of the glaſs increaſes; but its diſtinctneſs decreaſes upon that account.*

For the rays further from the vertex are reflected to different points of the axis, as appears from the different proportions in Prop. III. and V.

PROP.

PROP. XXII.

If a concave ſpeculum A*a be held directly to the rays of the ſun; and any combuſtible body be held in the principal focus f, it will be fired by the heat thereof.* 35.

For all the rays of the ſun are reflected into the focus, forming there, by reflexion, the image of the ſun. Which image muſt be of an intenſe heat, becauſe all the rays that fall upon the ſpeculum, are brought thither. And the broader the ſpeculum is, the more rays are contained in that ſmall ſpace; and may be ſo increaſed by the breadth of the glaſs, as to burn or melt all ſorts of bodies placed there; as is plain from numerous experiments.

Cor. 1. *The heat generated in the focus of the glaſs, is to the ſun's direct heat; as the area of the glaſs, is to the area of the image in the focus.*

For the rays that fell upon the glaſs are all brought into that ſpace, where their denſity is reciprocally as the areas they are contained in.

Cor. 2. *The degrees of heat in different reflectors, are as the ſquares of the diameters directly, and the ſquares of the focal diſtances reciprocally.*

For the denſity of the rays in each, are in that proportion.

SCHOLIUM.

Upon account of the rays meeting in that point, and burning bodies therein, it is called the *focus* or *burning point*; and the reflector itſelf a *burning concave.* For a reflector to burn the moſt intenſly, ſee Schol. Prob. V. B. IV.

Fig.

PROP. XXIII.

37. *If parallel rays* RA, CV, *are reflected from a concave (or convex) ſpherical ſurface* A*a*, *of a ſmall hight* VP. *Let* F *be the focus of the extreme ray* AF, Q *the principal focus. Then the aberration of the rays, in the length of the axis* VC, *is equal to half the hight of the ſegment*; QF $= \frac{1}{2}$VP, *nearly*.

For draw the tangent AT. Then by the nature of the circle CP : CV : : CV : CT. And dividing, CP : CV : : CV — CP (VP) : CT — CV (TV). But VP being very ſmall, CP is = CV nearly, therefore VP = VT. But (Prop. IV.) TF = FC, or CF = $\frac{1}{2}$ CT. And (Prop. VI.) VQ = QC, or CQ = $\frac{1}{2}$CV. Therefore CF — CQ or FQ = $\frac{CT - CV}{2} = \frac{TV}{2} = \frac{VP}{2}$. But (Cor. 4. Prop. IV.) QF is the ſpace on which all the rays fall that come from the whole ſurface A*a*, that is, QF is the aberration of the rays in length; therefore $\frac{1}{2}$VP is equal to that aberration.

Cor. 1. *The aberration* QF $= \frac{1}{2}$TV *exactly*.

Cor. 2. *The aberration* QF $= \frac{AV^2}{4VC}$, *nearly*.

For VP $= \frac{VA^2}{2VC}$, and $\frac{1}{2}$VP $= \frac{VA^2}{4VC}$.

Cor. 3. *The lateral aberration at the principal focus is* $= \frac{VA^3}{2VC^2}$, *nearly*.

For the triangles APF, DQF are ſimilar, whence PF : PA : : FQ : QD; that is, $\frac{1}{2}$CV : AV : : $\frac{VA^2}{4CV}$: QD $= \frac{VA^3}{2VC^2}$, nearly; becauſe AV is very ſmall.

Cor. 4. *The radius of the ſphere being given; the aberrations in length, are as the ſquares of the diameters; and the lateral aberrations, as the cubes of the diameters of the reflectors.* Fig. 37.

PROP. XXIV.

38. *If parallel rays* RA, SG, *are reflected from a concave (or convex) ſpherical ſurface* Aa; *the diameter of the leaſt circle of aberration of the rays, is* $\frac{1}{4}$ *the aberration made at the principal focus.* BI $= \frac{1}{4}$QD.

Let any reflected ray GB cut the axis in L, and the extreme ray AF in B. Draw BI perp. to VQ, Q being the principal focus. Then ſuppoſing the ray AFD to be fixed; as the point of incidence G moves from the vertex V, the perpendicular BI, will firſt increaſe, becauſe the angle VLG continually increaſes; and afterwards it will decreaſe, becauſe the line FL continually decreaſes. And when BI is the greateſt, it is plain, all the rays, on the ſame ſide, will paſs thro' it; and then the circular ſection of all the rays, in that place I, will be the leaſt ſection poſſible. To find the point I, or the radius BI.

By ſimilar triangles, LI : BI :: LP or FP : PH,
and BI : FI :: AP : PF.
and ex equò. LI : FI :: AP : PH.
and compounding LI + FI (FL) : FI :: AP + PH (AH) : PH.
Again QF : QL :: $AP^2 : PH^2$ (by Cor. 4. Prop. 23.)
and by diviſion, QF : QF — QL (FL) :: $AP^2 : AP^2 - PH^2$ ($AH \times Ha$).
but we had FL : FI :: AH PH
therefore QF : FI :: $AP^2 \times AH : PH \times AH \times Ha$.
or QF : FI :: $AP^2 : PH \times Ha$.
but we had FI : BI :: FP : AP
therefore QF : BI :: $AP^2 \times FP : AP \times PH \times Ha$
or QF : BI :: $AP \times FP : PH \times Ha$.

But

Fig. 38. But QF, AP, FP are given; therefore BI is the greateſt when PH × Ha is the greateſt, that is, when PH = Ha, $= \frac{1}{2}$AP, and then PH × Ha $= \frac{1}{4}AP^2$. And ſince AP^2 : PH × Ha ($= \frac{1}{4}AP^2$) : : QF : FI ($= \frac{1}{4}$FQ), and conſequently BI $= \frac{1}{4}$QD.

Cor. 1. *If* R *be the radius of the reflector,* B = *half its breadth; then the diameter of the leaſt circle of aberrations cauſed by the figure of the ſpherical reflector, will be* $\frac{B^3}{4RR}$.

For (Cor. 3. Prop. 23.) $QD = \frac{VA^3}{2RR} = \frac{B^3}{2RR}$, and 2BI, or $\frac{1}{2}QD = \frac{B^3}{4RR}$.

Cor. 2. *The diſtance* FI *of the leaſt circle of aberrations from* F *the focus of the extreme ray, is* $\frac{1}{4}$ FQ, *or* $\frac{1}{4}$ *the diſtance from the principal focus.*

Cor. 3. *If this circle of aberrations be ſet in any other place, all the rays will not paſs thro' it.*

Cor. 4. *And therefore the circular ſection cutting all the rays, will be greater at any other place, beſides* I.

Cor. 5. *The area of the circle of aberrations, is to the area of the glaſs; as* VA^4 *to* $64R^4$. R *being the radius of concavity.*

For the area of the glaſs, is to the area of the circle; as VA^2 to BI^2, or as VA^2 to $\frac{B^6}{64R^4}$; or as $64R^4$ to B^4; B being = VA.

Fig.

PROP. XXV.

Let R *be the radiant point*, F *the focus of the rays falling at* A, *f the focus of thoſe falling at* V. C *the center of the reflector. Then will* $FC = \frac{CR \times CA^2}{AR^2 - CR^2}$, *and* $fC = \frac{CR \times CA}{VC + 2CR}$. *And the aberration in length is* $= FC - fC$. 39.

Let radius CA or CV $= r$, AR $= a$, CR $= d$, AF $= z$, FC $= y$, Then (Geom. II. 25.) $a : d :: z : y$, and $dz = ay$, and $z = \frac{ay}{d}$. Alſo (Geom. II. 26.) $rr + dy = az$, and $z = \frac{rr + dy}{a} = \frac{ay}{d}$. Whence $drr + ddy = aay$, and $\overline{aa - ad} \times y = drr$, and $y = \frac{drr}{aa - dd}$.

Again (Prop. V.) VR $(r + d)$: Vf $(r - fC)$ :: CR (d) : Cf. And by compoſition, $r + 2d : d :: r : Cf = \frac{dr}{r + 2d}$. And the aberration $= \frac{drr}{aa - dd} - \frac{dr}{r + 2d}$.

Cor. 1. *On the contrary, if* F *be the radiant point*, R *the focus of rays falling on* A, *r the focus of rays falling on* V; C *the center of the reflector. Then will* $CR = \frac{CF \times CA^2}{AF^2 - FC^2}$, *and* $Cr = \frac{CF \times CA}{VC - 2CF}$. *And the aberration* $Rr = Cr - CR$.

For from the equations $dz = ay$, and $rr + dy = az$, we have $a = \frac{dz}{y} = \frac{rr + dy}{z}$, and $dzz = rry + dyy$. Whence d or CR $= \frac{rry}{zz - yy}$.

Fig. 39. Again (Prop. V.) FV ($r - y$) : FC (y) : : Vr ($r + Cr$) : Cr. By diviſion, $r - 2y : y : : r :$ Cr $= \frac{ry}{r - 2y}$. Therefore Cr — CR $= \frac{ry}{r - 2y} - \frac{rry}{zz - yy}$ = Rr.

Cor. 2. *If* R *and* F *are conjugate foci, for rays falling at* A; *then* R *and* f, *alſo* r *and* F, *will be conjugate foci for rays near the axis at* V. *And* f *and* r, *will be further from the vertex than* F *and* R.

That f and r are further from the vertex, will appear thus. AC + CR is greater than AR; that is, VC + CR or $r + d$ is greater than a, and $rr + 2rd + dd$ greater than aa, and $rr + 2rd$ greater than $aa - dd$. Whence $\frac{drr}{aa - dd}$ is greater than $\frac{drr}{rr + 2rd}$ or $\frac{dr}{r + 2d}$; that is, FC is greater than fC.

Again, AF + FC is greater than AC or VC, or VF + FC, and AF greater than VF; that is, z is greater than $r - y$, and zz greater than $rr - 2ry + yy$, and $zz - yy$ greater than $rr - 2ry$. Therefore $\frac{rry}{rr - 2ry}$ or $\frac{ry}{r - 2y}$ is greater than $\frac{rry}{zz - yy}$; that is, Cr is greater than CR.

Cor. 3. *Therefore if* R, f *be conjugate foci to the vertex* V; R *and* F, *will be conjugate foci to the point* A. *And if* F *and* r *be conjugate foci to the vertex* V; *then* F *and* R *will be conjugate foci to the point* A.

Cor. 4. *Let* F, r, *be the conjugate foci to the vertex* V; *then the aberration* Rr $= \frac{\text{VP} \times r\text{C} \times 2\text{CF}}{\text{CV}^2 - 2\text{CF} \times \text{CP}}$; *for rays falling at* A.

For let $x = \mathrm{VP}$, $v = r - 2y$; and $rr = yy + zz + 2y \times \mathrm{FP}$; and $zz = rr - yy - 2y \times \overline{r - y - x}$, and $zz - yy = rr - 2ry + 2yx. = rv + 2yx$. Fig. 39.

Then $\mathrm{R}r = \frac{ry}{r - 2y} - \frac{rry}{zz - yy} = \frac{ry}{v} - \frac{rry}{rv + 2yx}$

$$= \frac{ry}{v} - \frac{ry}{v + \frac{2yx}{r}} = \frac{vry + 2yyx - vry}{v \times \overline{v + \frac{2yx}{r}}} =$$

$$\frac{2yyx}{v \times \overline{v + \frac{2yx}{r}}} = \frac{x \times \frac{y}{v} \times 2y}{v + \frac{2yx}{r}} = \frac{x \times \frac{ry}{v} \times 2y}{rv + 2yx} =$$

$$\frac{x \times \frac{ry}{r - 2y} \times 2y}{rr - 2ry + 2xy} = \frac{\mathrm{VP} \times \mathrm{C}r \times 2\mathrm{CF}}{\mathrm{CV}^2 - 2\mathrm{CF} \times \mathrm{CP}}.$$

Cor. 5. *If* T *be the principal focus. And let* TF : TC : : 1 : n; *then the aberration* $\mathrm{R}r =$

$$\frac{\overline{n - 1} \times \mathrm{VP} \times \mathrm{VT}}{\frac{\mathrm{VC}}{n - 1} + \mathrm{VP}}.$$

For by division TF : TC — TF or FC : : 1 : $n - 1$; that is, $\frac{1}{2} r - y : y : : 1 : \overline{n - 1} =$

$$\frac{y}{\frac{1}{2}r - y} = \frac{2y}{r - 2y}, \text{ and } \frac{r \times \overline{n - 1}}{2} = \frac{ry}{r - 2y}. \text{ And}$$

$$\text{(Cor. 4.) } \mathrm{R}r = \frac{x \times \frac{ry}{r - 2y} \times 2y}{rr - 2ry + 2xy} = \frac{x \times \frac{r \times \overline{n - 1}}{2} \times 2y}{rr - 2ry + 2xy}$$

$$= \frac{\overline{n - 1} \times \frac{1}{2} rx}{\frac{rr - 2ry}{2y} + x} = \frac{\overline{n - 1} \times \frac{1}{2} rx}{\frac{r}{n - 1} + x}; \text{ becauſe } n - 1$$

$$= \frac{2y}{r - 2y}, \text{ and } \frac{r}{n - 1} = \frac{rr - 2ry}{2y}.$$

PROP. XXVI.

Fig. 40. *If* R *be the radiant point,* F *the focus of rays falling on* A, *and reflected by the convex ſpeculum* AV, *whoſe center is* C, *f the focus of rays falling on* V. *Then will the aberration* $Ff = \frac{RC \times CA^2}{RC^2 - RA^2} - \frac{RC \times CA}{2RC - CV}$.

Draw RH parallel to the reflected ray AG, and RO perp. to CA produced. Then $\angle H = FAC = CAS = RAH$, and conſequently $RH = RA$, and $AO = OH = \frac{1}{2}AH$. Let CA or $CV = r$, $CR = d$, RA or $RH = a$, $CF = y$, $AF = z$, $AO = v$, $VP = x$.

Then in the triangle RCA, (Geom. II. 22.) $dd = aa + rr + 2vr$, and $dd - aa = rr + 2vr = \overline{r + 2v} \times r$. And by the ſimilar triangles CFA, CRH, CR (d) : CF (y) : : CH $(r + 2v)$: CA (r) : : $\overline{r + 2v} \times r : rr : : dd - aa : rr$. Whence $y = \frac{drr}{dd - aa} = CF$.

Alſo (Prop. V.) VR $(d - r)$: Vf $(r - Cf)$: : CR (d) : Cf. And compounding, $2d - r : d : : r : Cf = \frac{dr}{2d - r}$. And $Ff = CF - Cf$.

Cor. 1. *If* R, *f be the conjugate foci, to the vertex* V; *then for rays falling on* A, *the aberration* $Ff = \frac{Cf \times VP}{\frac{CV^2}{2Cf} - VP}$; *f being the focus of rays falling at* V.

For (Geom. II. 23.) $CP = \overline{r - x}$, and C being an acute angle, we have in the triangle RAC, $aa = rr + dd - 2d \times \overline{r - x}$, and $dd - aa = 2dr - 2dx$

$2dx - rr$. Therefore $Ff = \frac{drr}{2dr - rr - 2dx} -$ Fig. 40.

$$\frac{dr}{2d - r} = \frac{drr}{2dr - rr - 2dx} - \frac{drr}{2dr - rr} =$$

$$\frac{2dr - rr - 2dr + rr + 2dx}{\overline{2dr - rr - 2dx} \times \overline{2dr - rr}} drr =$$

$$\frac{2drx}{\overline{2dr - rr - 2dx} \times \overline{2dr - rr}} dr = \frac{\frac{dr}{2d - r} \times x}{\overline{2dr - rr - 2dx}}$$

$$\times 2d = \frac{\frac{dr}{2d - r} \times x}{\frac{2d - r}{2d} r - x} = \frac{\frac{dr}{2d - r} x}{\frac{2d - r}{dr} \times \frac{1}{2} rr - x} = \frac{Cf \times x}{\frac{rr}{2Cf} - x}.$$

Cor. 2. *Let* T *be the principal focus, and* CT *or* TV $= t$. *And let* TR : TV :: m : 1. *Then the aberration* $Ff = \frac{\frac{m + 1}{m} tx}{\frac{m}{m + 1} \times 2t - x}$.

For TR $(d - \frac{1}{2} r)$: TV $(\frac{1}{2} r)$ :: m : 1, or $2d - r : r :: m : 1$, and by addition $2d : r :: m + 1 : 1$. Hence $m = \frac{2d - r}{r}$, and $m + 1 = \frac{2d}{r}$, therefore $\frac{m + 1}{m} = \frac{2d}{2d - r}$, and $\frac{m + 1}{2m} r = \frac{dr}{2d - r}$

$= Cf$. Whence $\frac{Cf \times x}{\frac{rr}{2Cf} - x} = \frac{\frac{m + 1}{2m} rx}{\frac{rr}{2} \times \frac{2m}{\overline{m + 1} . r} - x} =$

$$\frac{\frac{m + 1}{m} \times \frac{1}{2} rx}{\frac{m}{m + 1} \times r - x} = \frac{\frac{m + 1}{m} tx}{\frac{m}{m + 1} \times 2t - x}$$

PROP. XXVII.

41. *Let R be the radiant point, RA the incident ray falling upon the ſpherical ſurface AV, AD the reflected ray, C the center of the reflector; draw CB, CD perp. to RA, AD. And let T, t, be the middle points of AB, AD. Then if F be the focus of the reflected ray, it will be as RT : TA : : TA or tA : tF. Where F muſt lie the ſame way from t, as R lies from T.*

Let R*a* be another ray infinitely near RA; *a*F*d* its reflected ray; draw C*b*, A*n* perp. to R*a*; and C*d*, A*s* perp. to F*a*. Then the right angled triangles A*as*, and A*an* are equal and ſimilar; for $\angle naA = FaV = SaA$, and A*a* is common, therefore $As = An$. Alſo in the equal right angled triangles ACB, ACD; and alſo *a*C*b*, *a*C*d*; $CB = CD$, and $Cb = Cd$, therefore $Bb = Dd$. And by the ſimilar triangles RB*b*, RA*n*; and FA*s*, FD*d*; RB : RA : : B*b* or D*d* : A*n* or A*s* : : FD : AF. And by compoſition and diviſion, RA + RB : RA − RB : : AF + FD : AF − FD. And

$$\frac{RA+RB}{2} : \frac{RA-RB}{2} : : \frac{AF+FD}{2} : \frac{AF-FD}{2}.$$

That is, RT : TA : : A*t* : *t*F.

Cor. 1. *This Prop. holds true of any curve* AV, *whoſe radius of curvature at* A, *is* AC.

Cor. 2. RA : RB : : AF : FD.

Cor. 3. T *and t are the foci of rays parallel to* RA, FA.

Cor. 4. *The middle point of* CA *is the focus of rays parallel to* CA.

Cor 5. *If rays fall on the convex ſide, it is only putting* R *on the other ſide of* AV; *and the ſame proportion holds good.* Fig. 41.

SCHOLIUM.

Altho' it appears by Prop. IX, and X. that a conic ſection will reflect all rays iſſuing from one focus, accurately to the other focus; and therefore mathematicians have endeavoured to grind their glaſſes into the form of ſome of the conic ſections, yet they have never ſucceeded in giving them a true figure, nor has any method yet been found out to grind them accurately into that form. And if they could, it would not anſwer their expectation; for neither the conic ſections, nor any other curve, will reflect the rays iſſuing from any point in the collateral parts of an object, accurately to a ſingle point. So that if glaſſes were truly made in the form of any of the conic ſections, they would not perform much better than thoſe of a ſpherical form. For near the vertex of a conic ſection, it nearly coincides with a ſphere of a certain radius, and therefore in a ſmall ſegment, they will nearly reflect alike. And ſpherical glaſſes are eaſily ground into a true form. And therefore conſidering all things, a ſpherical ſurface is the beſt curve, and the moſt commodious for optical uſes of any.

BOOK III.

DIOPTRICS, *or refracted Viſion.*

DEFINITIONS.

DEFIN. I.

Fig. *DIOPTRICS* is that part of Optics, which treats about refracted light, and every thing that relates to ſeeing thro' different mediums, and the pictures of objects made by refraction through them.

DEF. II.

43. *A Lens* is a ſolid tranſparent body as AB, generally of glaſs, being of ſuch a ſhape, that the ſides of it are either flat, or the ſegments of ſpherical ſurfaces. Of theſe there are ſeveral ſorts, as, 1. *Plain* on both ſides. 2. *Plano convex*, being plain on one ſide and convex on the other. 3. *Plano concave*, or plane on one ſide and concave on the other. 4. *Double convex*, being convex on both ſides. 5. *Double concave*, being concave on both ſides. 6. A *Meniſcus*, being convex on one ſide, and concave on the other.

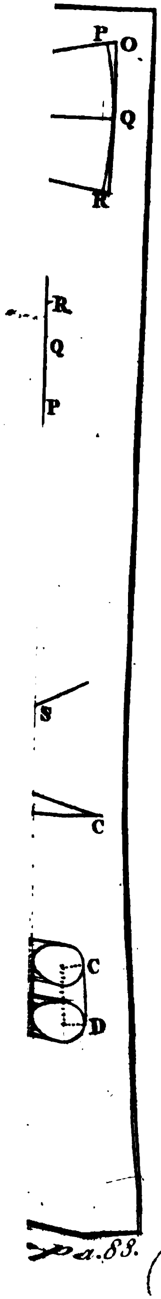
DEF. III.

The *Axis* of a lens, is a right line perpendicular to both the ſurfaces, or paſſing thro' both their centers.

DEF. IV.

The *Vertex* is the point where the axis cuts the ſurface of the lens.

PROP.



pa.83.

Fig.

PROP. I.

If a ray of light AC, *paſs into a refracting medium*; 44. *the ſine of incidence* ACD, *is to the ſine of refraction* ECF, *in a conſtant ratio.*

Let the ray BC falling moſt obliquely on the refracting medium MS, be refracted in the line CN. And the ray ACI be refracted into the line CE. Draw AP perp. to MS, and make, CH = CP, and CL = CM. And draw HIE and LN perp. to CS. And let the equal lines BC, AC, repreſent the equal motions of the rays B and A. Then ſuppoſing BC parallel or nearly coinciding with the refracting plane; let the motion AC be reſolved into the two motions, AP, PC; AP being perpendicular, and PC parallel to the refracting ſurface.

Then (by Prop. XVI. B. I.) bodies act upon light by the forces of attraction and repulſion; and theſe forces are directed in lines perp. to the ſurface of the refracting body MS. Therefore the motions of the rays BC, PC which are parallel to MS, ſuffer no alteration by the refraction of the medium. Therefore in the very ſame time that the rays A and B move to C, they would move to I and L without refraction, where CL = CB = CA = CI. But by the refraction of the medium, that from B goes to N, and that from A goes to E, in the perpendiculars LN, HE, becauſe the parallel motion is the ſame as before. But (by Cor. 3. Prop. XX. Mechan.) the velocities of the rays A, B, which were equal before refraction, will alſo be equal after; and therefore CN = CE.

Now in the right angled triangle CLN, we have S.CLN (MCD) : : S.CNL (NCF) : : CN : CL : : CE : CI : : (in the triangle CEI) S. CIH (ACD) :

Fig. 44. (ACD) : S.CEH (ECF). That is, S.ACD the angle of incidence : S.ECF the angle of refraction : : rad : S.NCF, or in a given ratio.

Cor 1. *If the refracted ray* EC *be turned directly back to the point of incidence; it will be refracted into the line* CA, *before deſcribed by the incident ray.*

For the forces that before accelerated its motion, will now equally retard it.

Cor. 2. *The angles of incidence and refraction lye both in one plane, which is perp. to the refracting ſurface at the point of incidence.*

For the refraction being made in lines perp. to the refracting ſurface, there is no force to cauſe the rays to deviate from that plane.

Cor. 3. *Heteregeneal rays of light have different degrees of refraction; but in any one conſidered by itſelf, the ſine of incidence is in a given ratio the ſine of refraction.*

Cor. 4. *If* AC *be the incident ray, and* CE *the refracted ray; and if* AC *produced cut the perpendicular* HE *in* I. *Then* CE : CI : : *as ſine of the angle of incidence : to the ſine of the angle of refraction.*

Cor. 5. *The velocity of light in the firſt medium* ADC, *is to its velocity in the ſecond medium* CFN; *as the ſine of the angle of refraction, to the ſine of the angle of incidence.*

For it is as AC *or* CI *to* CE.

Cor. 6. *If parallel rays paſs out of one medium into another, they will alſo be parallel after refraction: the refracting ſurface being a plane.*

For the rays being parallel, they are equally refracted.

PROP.

PROP. II.

If heterogeneal light be refracted out of glaſs into air; the ſine of incidence, is to the ſine of refraction of the leaſt, and moſt refrangible rays; as 50 to 77 and 78.

This is proved by experiment thus; take a glaſs prism BCD, with a very acute angle C, and place it ſo, that a ray of the ſun AF coming thro' a ſmall hole in the window ſhut, near the priſm, may fall perpendicularly upon the ſide DC, and be refracted by the oppoſite ſide at Y; the moſt refrangible rays to P, and the leaſt refrangible to T. Then if EYG be perpendicular to the ſide CB, by meaſuring the angles AYE, TYG and PYG, the ſines of theſe angles will become known. 45.

For example, if the angle of the priſm BCD be 31° 36′; then ſince the triangles CFY and CYE are both of them right angled, the angle BCD = FYE the angle of incidence at Y = 31 36, whoſe ſine is .524. And the angles TYG and PYG will be 53 36, and 54 44½; whoſe ſines are .805 and .8165. But .524 and .805 and .8165 are nearly to one another as 50, 77 and 78.

Here the angle of incidence on the plane BC is always equal to the angle of the priſm BCD; but it may be difficult to meaſure the angles TYG and PYG truly, for want of drawing the perpendicular YG exactly. Therefore to avoid any error, take the altitude of the ſun, and at the ſame time mark the points P, T. Then to the altitude of the ſun, add the angle of the priſm, which was ſhewn to be equal to FYE; the ſun gives the elevation of the perpendicular YG above the horizon. Then if you take alſo the elevation of TY, PY above the horizon; theſe taken from the elevation of GY, will give the angles TYG, PYG.

Cor.

Fig. 45. Cot. 1. *The angle* PYT, *which the coloured image ſubtends, is about* 1° $8'\frac{1}{2}$, *or* 1° 9′, *ſubtracting the ſun's diameter.*

Cor. 2. *In the coloured image* PT, *the lengths of the ſeveral colours, red, orange, yellow, green, blue, purple, violet, are found to be* $\frac{1}{16}$, $\frac{3}{80}$, $\frac{1}{15}$, $\frac{1}{12}$, $\frac{1}{12}$, $\frac{1}{18}$, $\frac{1}{9}$; *it is eaſy to divide the angle* PYT *in that proportion; and then there will be had the ſines of refraction at the confines of all the colours; that is,* 77, $77\frac{1}{8}$, $77\frac{1}{5}$, $77\frac{1}{3}$, $77\frac{1}{2}$, $77\frac{2}{3}$, $77\frac{7}{9}$, 78.

For the image is divided after the manner of a muſical cord, as Sir J. Newton has found. And in ſmall arches, the differences of the ſines are nearly as the differences of the arches.

Cor. 3. *If light paſs out of glaſs into air; the ſine of incidence, is to the ſine of refraction of the mean rays; as* 11 *to* 17, *or as* 20 *to* 31; *which is a greater refraction than as* 2 *to* 3.

For the mean refrangible rays are thoſe at the confines of the blue and green, or nearer the green, whoſe refraction is $77\frac{1}{4}$ or $77\frac{1}{2}$.

SCHOLIUM.

By a like experiment, the ſine of incidence and refraction of the leaſt refrangible rays out of water into air is as 3 to 4, or as 81 to 108; and of the moſt refrangible, as 81 to 109.

PROP.

PROP. III.

If A, B, C, *be three refracting mediums, contained between parallel ſurfaces; and if a ray of light* DE *paſſing thro' them be refracted at* E *and* F, *and paſs thro' the third in direction* FG. *It will ſuffer the ſame refraction by paſſing thro'* A, B, C, *as if it went immediately out of* A *into* C. 46.

For let the medium C approach the medium A in direction FE, and the medium B grow thinner and thinner, till F fall upon E, and the medium B be vaniſhed; all this makes no difference in the poſition of the line FG. And therefore the refraction into the medium C will be the ſame whether the ray paſs thro' B, or goes immediately from A to C, without coming into B at all. And the ſame likewiſe appears from experiments.

Cor. 1. *Hence if there be never ſo many refracting mediums contained between parallel ſurfaces; the whole refraction of a ray is the ſame at laſt, in paſſing thro' all of them; as it would be in paſſing immediately out of the firſt into the laſt.*

Cor. 2. *If ſeveral mediums* A, B, C, *terminated by parallel planes, be encompaſſed on all ſides by ſome other medium; any ray* RD *moving out of this other medium, and paſſing thro' all the reſt, will emerge parallel to itſelf, at coming into this other medium again; that is,* GS *will be parallel to* RD.

For the refraction is the ſame as if all the mediums A, B, C were away; and GS has the ſame direction as RD.

PROP.

PROP. IV.

47. *If* A, B, C *be three mediums, and the ſines of incidence and refraction out of* A *into* B, *be a and b; and out of* A *into* C, *be a and c; then thoſe out of* B *into* C, *will be b and c.*

Let the mediums B and C be encompaſſed by the medium A; and let RD be a ray refracted at D, F, and G, and proceeding to K. Then (Prop. III. Cor. 2.) GK will be parallel to RD. At D, F, G, let fall the perpendiculars LDO, NFP, IGH. Then by reaſon of the parallels GK, RD, and GH, DL, the angle HGK = LDR. Whence a = S.RDL, b = S.ODF = S.DFN = S. incidence of the ray DF on the medium C. Alſo a = S.HGK, and conſequently c = S.FGI = S.GFP = S. refraction of the ray into the medium C. Therefore the ſines of incidence and refraction out of B into C, are b and c.

Example. Let A be air, B water, and C glaſs. And let the ſines of incidence and refraction out of air into water be as 4 to 3; and out of air into glaſs as 17 to 11, or as 4 to $\frac{4 \times 11}{17}$; then $a = 4$, $b = 3$, $c = \frac{4 \times 11}{17}$. Therefore the ſine of incidence and refraction out of water into glaſs will be 3 and $\frac{4 \times 11}{17}$, or as 3 × 17 to 4 × 11, or as 51 to 44.

Cor. *If* B, C, D, *be three mediums; and the ſine of incidence and refraction out of* B *into* C, *be as b to c; and out of* C *into* D, *as r to s. Then thoſe out of* B *into* D *will be as br to cs.*

Fig. 47. For by this Prop. the ſines of incidence and refraction out of C into B are as c and b; and out of C into D, as r to s, or as c to $\frac{cs}{r}$. Therefore thoſe out of B into D, will be as b to $\frac{cs}{r}$, or as br to cs.

Examp. If the ſines of incidence and refraction out of glaſs into air, be as 20 to 31; and out of air into water, as 4 to 3. Then the ſines of incidence and refraction out of glaſs into water, will be as 20 × 4 to 31 × 3, or as 80 to 93.

PROP. V.

The forces of bodies to refract light, are nearly as their denſities; excepting that unctious and ſulphureous bodies refract more than others of the ſame denſity.

In uniformly accelerated motion, (by Prop. V. Cor. 2. Mechan.) if the body is given, the velocity is as the force and time; and the ſquare of the velocity, as force × time × velocity; that is, (by Cor. Prop. VI. ib.) as the force × ſpace deſcribed, and that is (when the ſpace is given) as the force. Let AF be the refracting ſurface, and let 49. it be divided into an infinite number of equal parts terminated by parallel planes paſſing thro' A, B, C, D, &c. Put a = AB or BC, &c. f, g, h, &c. for the forces acting in AB, BC, CD, &c. which may be eſteemed uniform. Then the velocities acquired in falling from reſt, thro' AB, BC, CD, will be as $\sqrt{f}$, $\sqrt{g}$, $\sqrt{h}$; and their ſquares as f, g, h. But (by Cor. 3. Prop. XX. Mechan.) the ſquare of the velocity at B being f, the ſquare of the velocity at C is = ſquare of the velocity at B + the ſquare of the velocity in paſſing thro' BC from reſt = $f + g$. And the ſquare of the velocity at D = ſquare of the velocity at C +

Fig. $C +$ the ſquare of the velocity in paſſing thro'
49. $CD = f + g + h$. And ſo on, therefore the velocity at $F = \sqrt{\overline{f + g + h}}$ &c. $=$ ſquare root of the force.

48. Let AB be the refracting ſurface, IC the incident ray, infinitely near AC, and CR the refracted ray. Let $CB = CA$, and BR perp. to CB. Then CB ($= CI = 1$) is the parallel motion of the ray, and BR the perpendicular motion, which is generated by the refraction. And therefore BR is as the ſquare root of the refracting force, as was ſhewn before; and the refracting force is as the denſity, therefore the ſquare of BR is as the denſity of the medium.

Let the ſines of incidence and refraction be as m to n. Then we have $m : n :: CR : CB = 1$, and $CR = \frac{m}{n}$, and $BR^2 = CR^2 - CB^2 = \frac{mm}{nn} - 1$. Therefore in the following table, againſt the names of the bodies in the firſt column, you have the ſines of incidence and refraction in the ſecond, the ſquare of BR or the refracting power in the third, their denſity or ſpecific gravity in the fourth, and their ratio in the fifth column.

Fig. 48.

Bodies	Refraction		Refracting power	Denſity	the Ratio
	m	n			
Pſeudo topaz	23	14	1.699	4.27	0.39
Air	3201	3200	0.000625	0.0012	.52
Glaſs of Antimony	17	9	2.568	5.28	.48
A Selenitis	61	41	1.213	2.252	.54
Glaſs	31	20	1.4025	2.58	.54
Rock Cryſtal	25	16	1.445	2.65	.54
Iſland Cryſtal	5	3	1.778	2.72	.65
Sal Gemmæ	17	11	1.388	2.143	.64
Alumn	35	24	1.1267	1.714	.65
Borax	22	15	1.1511	1.714	.67
Niter	32	21	1.345	1.9	.70
Vitriol	303	200	1.295	1.715	.75
Ol. Vitriol	10	7	1.041	1.7	.61
Rain Water	529	396	0.7845	1.	.78
Gum Arabic	31	21	1.179	1.375	.85
Spt. Wine	100	73	0.8765	0.866	1.01
Camphire	3	2	1.25	0.996	1.25
Oil Olive	22	15	1.1511	0.913	1.26
Linſeed Oil	40	27	1.1948	0.932	1.28
Spt. turpentine	25	17	1.1626	0.874	1.32
Amber	14	9	1.42	1.04	1.36
Diamond	100	41	4.949	3.4	1.45

This table of Sir J. Newton's, made from a great many experiments, ſhews that theſe bodies that differ from a conſtant ratio, are ſuch as abound more or leſs with oily ſulphureous particles. And that ſuch oily ſulphureous ſubſtances have a greater refractive power than others, in proportion as they are of a more ſulphureous nature.

Cor. 1. *The ſquare of* BR *is as the denſity of the medium.*

Fig. 48. Cor. 2. *Refraction out of a thinner medium into a thicker, is made towards the perpendicular; and the contrary.*

Cor. 3. *When light paſſes out of a thicker into a thinner medium, if the obliquity of the incident ray be ſo great, that the ſine of its incidence has a greater proportion to the radius, than the ſine of incidence to the ſine of refraction; that is, when the ſine of refraction would be greater than the radius, which is impoſſible. Then the light will not paſs into that medium, but be reflected back. And therefore light will not paſs out of glaſs into air, above 42 degrees of incidence; but is reflected back into the glaſs.*

PROP. VI.

50. *Let* AGFO *be any denſe medium,* AE *perp. to* AO, HO *an incident ray, making the angle* HOF *infinitely ſmall,* OB, OC *the refracted rays, being the leaſt and moſt refrangible. Draw* B*c* *parallel to* AO; *cutting* OC *in c, thro' c draw ace parallel to* AE; *then in any other denſe medium* AGH, *if the leaſt refrangible rays (of the incident ray* HO*) paſs to* D; *then drawing* D*e* *parallel to* AO *to cut ae in e, the moſt refrangible rays will paſs thro' e.*

Let the ray OE cut D*e* in *r*, and if HO or OA be the velocity of the incident ray, then (Prop. I. Cor. 5.) OB, OC, OD, OE will be the velocity of the rays going to B, C, D, E; and (Prop. V. Cor. 1.) AB^2 : AD^2 : : denſity of the firſt medium : denſity of the ſecond : : AC^2 : AE^2. And AB : AC : : AD : AE; whence AC : CB : : AE : ED. And by ſimilar triangles, B*c* : AO : : (CB : CA : : ED : EA : :) D*r* : AO; whence D*r* = B*c* = D*e*, and *r* falls upon *e* in the line *abe*. Therefore in all mediums, where the angle of incidence is a right angle, the extreme refracted rays will cut the pa-

rallels AE, *ae*, in the points B, *c*, and D, *e*, mak- Fig.
ing B*c*, D*e* alſo parallel to one another, and to the 50.
refracting ſurface OA.

But this does not hold good when the angle of incidence is not a right angle.

Cor. 1. *If the medium is ſuch, that the mean refrangible rays, are refracted at an angle of 45° with* QA, *then the rays will be the moſt diſperſed by refraction.*

For the angle DOE will be greateſt when AO makes 45 degrees with a line drawn from O, biſecting D*e*.

Cor. 2. *It does not hold that the ſines of refraction of the heterogeneous rays, are proportional,* S.B : S.C : : S.D : S.E.

For that proportion in inconſiſtent with this, AB : AC : : AD : AE.

Cor. 3. *Neither does this conſtruction hold good, in one and the ſame medium, for different incidencies.*

For if the heterogeneous rays at incidence 90, paſs thro' B, *c*; at leſs incidencies they will not paſs thro' D, *e*; the line D*e* growing perpetually leſs, as the angle AOD grows bigger. For in this caſe, it is, S.B : S.C : : S.D : S.E, which is inconſiſtent with the proportion AB : AC : : AD : AE, on which the equality of B*c*, D*e*, depends.

SCHOLIUM.

If the denſe medium AGHA be glaſs, then if OA be 40, A *a* will be 1; as may be calculated from having OA given, and the ſines of incidence and refraction. And this may ſerve for a ſtandard, whereby to meaſure the refractions of other mediums.

PROP. VII.

52. *If homogeneal rays be refracted by a priſm* ABC, *the inclination of the incident and emergent rays is leaſt, when the refraction is equal on both ſides*, APH = BSR.

51. In order to prove this, let the angle *b*I*g* = *g*I*d*, and let *b*I, *g*I, *d*I, be three homogeneal rays, paſſing out of a denſer into a rarer medium, and refracted at I, in the lines IB, IG, ID; draw the ſines *bn*, *go*, *dp*, and BN, GO, DP. Now it is known, that in ſmall arches, theſe arches are nearly as their ſines, and the increaſes of the arches as the increaſes of the ſines; but as the arches increaſe, that increaſe has a greater ratio to the increaſe of the ſine; and ſtill greater and greater as the arches become bigger. Now ſince the ſines *bn*, *go*, *dp* have a given ratio to BN, GO, DP, and their increments to their increments. And ſince AB is greater than *ab*, therefore, putting *inc.*OG for increaſe of OG, &c. we have $\frac{dg}{inc.og}$ greater than $\frac{bg}{inc.nb}$ or $\frac{dg}{inc.nb}$, and *inc.nb* greater than *inc.og*. And $\frac{DG}{inc.OG}$ much greater than $\frac{BG}{inc.NB}$, that is, $\frac{DG}{inc.og}$ much greater than $\frac{BG}{inc.nb}$, and *inc.nb*×DG much greater than *inc.og* × BG. Divide by *inc.nb*, and *inc.og* (which are not in ſo great a ratio), and then DG is greater than BG. Whence the ſum of the refractions AB, AD, is greater than twice the refraction AG, into the rare medium.

52. Now let the ray PS be refracted into the lines PH, SR making the ∠ APH = BSR. Alſo let

the ray PQ be refracted into PI and QL. Draw Fig. 52. PON perp. to CA, and QO, SN perp. CB. Produce RS, HP to V, and IP, LQ to T. Then $< OPQ + OQP = C = < NPS + NSP = 2NPS$, for O and C, or N and C, make two right angles. Therefore the angles OPQ, OPS, and OQP have equal differences, and their angles of refraction are OPT, OPV, and OQT. Therefore, by what was ſhewn before, $OPT + OQT$ is greater than $2OPV$ or $NPV + NSV$; and taking away the equal angles $OPQ + OQP = NPS + NSP$, and there remains $QPT + PQT$ greater than $SPV + PSV$, that is, DTQ greater than FVS, and therefore the inclination RVF is leſs than any other inclination LTD.

Cor. 1. *The greater the obliquity of the ray* IP *entering into the priſm, the greater will be the inclination of the incident and emergent rays,* DTL.

This appears from what was proved in *fig.* 51, where the greater the arch is, the greater is the ratio thereof to the ſine.

Cor. 2. *If a priſm be placed near the hole in a window ſhut, for a beam of the ſun to paſs thro', ſo as to throw the image upwards; the loweſt place of the image where it appears ſtationary (in turning the priſm ſlowly about), is when the incident and emerging rays make equal angles on each ſide the priſm.*

Cor. 3. *There are two poſitions of the priſm, where the image will have the ſame place, and that is where the ray paſſing thro' it, cuts the contrary ſides at the ſame angles.* 53.

For if IPQD be the progreſs of the ray, and if $< CGF := < CPQ$, and $CFG = CQP$. Then the ſum of the refractions of the ray FG, will be equal to the ſum of the refractions of the ray PQ. Therefore if the priſm be turned about the axis, Fig. 53. till the angle IPA be equal to DQB; then the ray will paſs along FG and go to D, as before it did in going along PQ.

52. Cor. 4. *When the angles of incidence and refraction are equal on both ſides the priſm; the angle of refraction* NPS *is equal to half the angle* C *of the priſm.*

For we proved before that C is equal to 2NPS.

PROP. VIII.

54. *Heterogeneous rays being refracted by a priſm* ABC, *the more oblique the incident ray* IP *is, the leſs will be the ſeparation of the rays* SL, Q*l*.

Inſtead of a more tedious method of demonſtration, I ſhall compute the progreſs of the rays trigonometrically, the angle C being 60°, and the ſines of incidence and refraction out of air into glaſs being as 106 to 68 and 69.

1. Let the angle of incidence API be infinitely ſmall, then it will be

106	2.0253059	106	2.0253059
68	1.8325089	69	1.8388491
Rad.	10.	Rad.	10.
S.39 54	9.8072030	S.40 37	9.8135432
the angle OPQ.		the angle OPS.	

Theſe angles taken from (C) 60, leave 20 6, and 19 23 for the angles of incidence at Q and S. Therefore again,

68	1.8325089	69	1.8388491
106	2.0253059	106	2.0253059
S. 20 6	9.5361286	S. 19 23	9.5209899
	11.5614345		11.5462958
S. 32 23½	9.7289256	S. 30 39	9.7074467

30 39 = the angle of diſſipation between
dif. 1 44½
the rays SL, Q*l*.

2. Let *l*Q be the incident ray; then ſince the most refrangible ray will return thro' *l*QP to I, making the angle API infinitely ſmall. Therefore we have nothing to do, but trace the leaſt refrangible ray thro' *l*QRH. Therefore Fig. 54.

106 —— 2.0253059
69 —— 1.8388491
S. 32 $23\frac{1}{2}$ — 9.7289256
11.5677747
S. 20 $24\frac{1}{2}$ — 9.5424688
the angle RQO.

Then 20 $24\frac{1}{2}$ taken from 60 leaves 39 $35\frac{1}{2}$ for the angle of incidence at R, or QRO, therefore

69 —— 1.8388491
106 —— 2.0253059
S. 39 $35\frac{1}{2}$ — 9.8043520
11.8296579
S. 78 15 — 9.9908089

And 78 15 taken from 90, leaves 11 45 for the angle made by RH and PI, for the ſeparation of the rays at P or R, far exceeding that at Q or S.

3. Let the angles at P, Q be equal, being each equal to 60; whence the angle of incidence is 30. Then

68 —— 1.8325089	69 —— 1.8388491
106 —— 2.0253059	106 —— 2.0253059
S.30 —— 9.6989700	S.30 —— 9.6989700
11.7242759	11.7242759
S.51 $12\frac{1}{2}$ — 9.8917670	S.50 11 — 9.8854268

51 $12\frac{1}{2}$
50 11

diff. 1 $1\frac{1}{2}$

And 1 $1\frac{1}{2}$ doubled gives 2 : 03 the whole ſeparation by both ſides of the priſm, when the angles

Fig. at P and Q are nearly equal, and the rays come on
54. any ſide.

So that when the rays fall very obliquely, the ſeparation of the heterogeneous rays is only 1 44½. When their incidence and emergence are about equal, it is 2 03. And when their incidence is ſo near a perpendicular, that their emergence is the moſt oblique, it is 11 45.

PROP. IX.

55. *If* R *be the radiant point,* RP *perpendicular to the refracting plane ſurface* PA. *Then if* RA *be the incident ray,* AH *the refracted ray, cutting* RP *in* F; *then* F *will be focus, ſo that* FA : RA : : S.*incidence* : S.*refraction.*

For the angle PRA = angle of incidence, and PFA = angle of refraction, and in the triangle ARF, FA : RA : : S.FRA : S.PFA : : S.angle of incidence : S.angle of refraction.

Cor. 1. *As the ſine of refraction : to the ſine of incidence : ſo the incident ray to the refracted ray.*

Cor. 2. *If rays diverging from* R, *diverge from* F, *after refraction; then rays converging to* F *will converge to* R *after refraction.*

Cor. 3. *Parallel rays falling on a refracting plane, are refracted into parallel lines.*

For R and F being at an infinite diſtance; all rays proceeding from R will be parallel to one another, and all rays proceeding from F will alſo be parallel.

Cor. 4. *If rays diverging from* R, *diverge from* F *after refraction; then changing the mediums, the rays diverging from* F, *will diverge from* R, *after refraction.*

Cor. 5. *If rays* RA, R*a*, *falling near* P, *be refracted* Fig.
from the focus F; *then* FP : RP : : S.*incidence* 56.
: S.*refraction.*

For then FA = FP, and RA = RP, nearly.

Cor. 6. *If* R *is an object under water, and the* 57.
eye moves from C *to* D *and then to* H, *&c. the object will ſeem to riſe in the perp.* RP, *till at laſt it vaniſhes in* P. *Hence water looks deepeſt when looked at perpendicularly; but it appears not then to be ſo deep as it is, by a fourth part.*

For when the angle of incidence ARP is 48 : 40, the angle of refraction AFP becomes 90. And at C, the apparent place of R will be at F, ſo that FP : RP : : 3 : 4, or PF = $\frac{3}{4}$ PR.

PROP. X.

If AV *be any refracting ſpherical ſurface,* C *the* 58.
center, R *the radiating point,* F *the focus after refraction;* 59.
then RA × *ſine of incidence* : CR × *ſine of refraction* 60.
: : AF : CF. 61.

Let RA be the incident ray, AG the refracted one. Draw FH parallel to CA, cutting RA in H. Then < RHF = RAC = angle of incidence, and AFH = FAC = angle of refraction. Put m, n, for the ſines of incidence and refraction. Then in the triangle AFH,

S.RHF or AHF (m) : S.AFH (n) : : AF : AH.
And
by ſimilar triangles, AR : CR : : AH : CF.
and ex equò, m × AR : n × CR : : AF : CF.

Cor. 1. *On the contrary, if the rays go the contrary way in the line* GA *or* FA, *they will be refracted to* R; *and* R *and* F *will be conjugate foci.*

Cor.

Fig. Cor. 2. *If rays diverging from* R, *converge to* F; *then changing the mediums, rays converging to* R *will diverge from* F. *Or if rays diverging from* R, *diverge from* F, *then changing the mediums, rays converging to* R, *will converge to* F, *after refraction. Which ſaves the labour of drawing more figures.*

PROP. XI.

58. *If* AV*a be a very ſmall part of a refracting ſphe-*
59. *rical ſurface,* C *the center,* R *the radiating point or*
60. *object,* F *the focus after refraction;*
61. *Then the diſtance of the object and vertex* RV × *ſine of incidence* :
Diſtance of the object and center RC×*ſine of refraction* ::
Diſtance of the vertex and focus VF :
Diſtance of the center and focus CF, *nearly.*

Let RA be the incident ray, AG or AF the refracted ray; then ſince A is very near the vertex V, RA = RV, and AF = VF nearly. And (Prop. X.) putting m, n for the ſines of incidence and refraction, $m \times AR : n \times CR :: AF : CF$; that is, $m \times RV : n \times CR :: VF : CF$.

Cor. 1. *If* m, n, *be the ſines of incidence and refraction, then* $m \times VR \mp n \times CR : m \times VR :: CV : VF$.

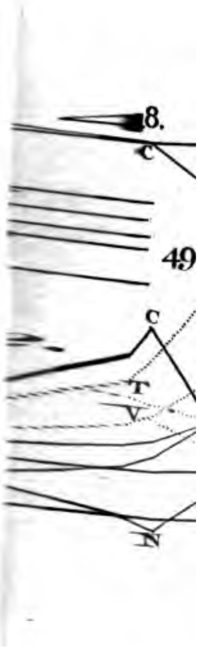
This follows from the Prop. by compounding or dividing.

Cor. 2. *If the rays be turned back again in the line* GA, *they will be refracted to the focus* R; *which ſupplies the place of more figures.*

Cor. 3. *If* R *and* F *lye on the ſame ſide of the refracting ſurface; rays diverging from* R *will diverge from* F; *if they lie on different ſides, they converge to* F.

PROP.

15.



PROP. XII.

parallel rays RA, RV *fall upon any refracting* 62.
cal ſurface AV, *whoſe center is* C; F *will be* 63.
cus of the refracted rays, ſo that, the ſine of in-
ce : ſine of refraction : : AF : CF.

raw the radius CA, then RAC or ACF is the
e of incidence (or its ſupplement), and FAC
angle of refraction. Let m, n, be the ſines of
dence and refraction. Then in the triangle
F, S.ACF (m) : S.CAF (n) : : AF : CF.

Cor. 1. *If the rays be turned back in the line* GA,
d refracted at A, *they will emerge parallel.*

Cor. 2. *If* AV *be any refracting curve,* AC *perp.*
the curve in A; *and* CF *parallel to* RA; *and if*
A *be the incident ray,* AF *the refracted ray; then*
F : FC : S.*incidence* : S.*refraction.*

PROP. XIII.

If parallel rays RA, RV, *fall on a very ſmall part* 62.
of a refracting ſpherical ſurface Aa, *whoſe center is* 63.
C; *and if* F *be the focus, then ſine of incidence : ſine of refraction* : : VF : CF, *nearly.*

For as AV is a very ſmall arch, AF = VF nearly, therefore (Prop. XII), S.incidence : S.refraction : : AF (VF) : CF.

Cor. 1. *If* m, n, *be the ſines of incidence and refraction; then* $m - n : m ::$ *radius* CV : VF *the principal focal diſtance from the vertex.*

For $m : n$: VF (VC + CF) : CF; and by diviſion, $m - n : m ::$ VC : VF.

Fig. Cor. 2. *The focus of parallel rays refracted by a*
62. *convex ſpherical ſurface of glaſs is diſtant from the*
63. *vertex near three times the radius.*

For m is to n, nearly as 3 to 2, and $m - n$ to m as 1 to 3.

Cor. 3. *As* $m - n$ *the difference of the ſines : to* n *the leſſer : : as* CV *the radius : to* VF — VC, *or* CF *the diſtance of the focus from the center.*

PROP. XIV.

64. *If* AV *be a ſmall portion of a refracting ſpherical*
65. *ſurface,* C *the center,* t, T *the principal foci, or the foci of parallel rays coming out of one medium into the other. And if* R *be the radiating point,* F *the focus; then it will be,* Rt : RV : : RC : RF; *or the contrary, changing the focus for the radiating point.*

Let m, n be the ſines of incidence and refraction, $d = \mathrm{RV}$, and $r =$ radius CV. Then (Cor. 1. Prop. XIII.) $m - n : m :: r : \frac{rm}{m-n} = \mathrm{VT}$. And on the contrary $n - m : n :: r : \frac{rn}{n-m} = \mathrm{V}t$, or rather $\mathrm{V}t = \frac{rn}{m-n}$, and $\mathrm{VT} - \mathrm{VC}$ or $\mathrm{CT} = \frac{rm}{m-n} - r = \frac{rn}{m-n}$, therefore $\mathrm{V}t = \mathrm{CT}$, and $\mathrm{VT} = t\mathrm{C}$.

But (Prop. XI. Cor. 1.) $md - n \times \overline{d + r} : md :: r : \mathrm{VF}$ (in fig. 64.), or $md - nd - nr : mr :: d : \mathrm{VF}$; and dividing by $m - n$, $d - \frac{nr}{m-n} : \frac{mr}{m-n} :: d : \mathrm{VF}$; that is, $\mathrm{RV} - \mathrm{V}t$ or $\mathrm{R}t : \mathrm{VT}$ or $t\mathrm{C} :: \mathrm{RV} : \mathrm{VF}$. And compounding, $\mathrm{R}t : \mathrm{R}t + t\mathrm{C}$ (RC) $:: \mathrm{RV} : \mathrm{RV} + \mathrm{VF}$ (RF). And (in fig. 65.)

it is $md - nd + rn : md :: r : VE$. Whence $RV + Vt : VT$ or $Ct :: RV : VF$, &c. Fig. 64. 65.

Cor. 1. $Rt : tC$ *or* $VT :: RV : VF$.
From the demonſtration.

Cor. 2. $Rt : tC :: CT : TF$.
For (Cor. 1.) $Rt : tC :: RV : VF :: RV - Rt$ (tV or CT) : $VF - tC$ or $VF - VT$ (TF).

Cor. 3. $Rt : tV :: VT : TF$.
For (Cor. 1.) $Rt : TV :: RV : VF :: RV - Rt$ (tV) : $VF - VT$ (TF).

Cor. 4. *If* T, *t be the principal foci, of rays paſſing out of one medium into the other; then* $CT = Vt$.
From the demonſtration.

PROP. XV.

If AV *be a ſmall portion of a refracting ſpherical ſurface;* C *the center,* R *the radiating point or the focus of the incident rays,* F *the focus of the refracted rays. And if* m = S.*incidence,* n = S.*refraction,* $RV = d$, $CV = r$, $VF = f$; *then* $f = \frac{mdr}{\overline{m - n} \times d - nr}$; *where if the rays converge, or* AV *be concave; d or r will be negative.* 66.

For ſince the arch AV is very ſmall, $RA = RV = d$, $AF = VF = f$. Draw FH parallel to CA, cutting RA in H. Then by ſimilar triangles RC ($d + r$) : CF ($f - r$) : : RA or RV (d) : AH $= \frac{df - dr}{d + r}$. Then in the triangle AFH, S.AHF or CAH (m) : S.AFH or CAF (n) : : AF or VF (f) : AH $\left(\frac{df - dr}{d + r}\right)$. Whence $nf = \frac{df - dr}{d + r} m$,

and

Fig. 66. and $ndf + nrf = dmf - dmr$, or $mdf - ndf - nrf = mdr$, therefore $f = \frac{mdr}{md - nd - nr}$.

Cor. 1. $f = \frac{mdr}{m - n . d - nr} = \frac{mdr}{md - n \times \overline{d + r}}$.

Cor. 2. *If rays fall on a concave refractor*, $f = \frac{-mdr}{m - n . d + nr}$.

Cor. 3. *If the rays converge to* R, $f = \frac{mdr}{m - n . d + nr}$.

For $f = \frac{-mdr}{m - n \times -d - nr}$.

Cor. 4. *If it be a plain ſurface*, $f = \frac{-md}{n}$.

For it is $f = \frac{mdr}{-nr} = -\frac{md}{n}$, r being infinite.

Cor. 5. *If the rays be parallel*, $f = \frac{m}{m - n} r$.

For then d is infinite, and $f = \frac{mdr}{m - n . d}$.

SCHOLIUM.

From this general ſolution it follows, that when f the focal diſtance comes out affirmative, it muſt be taken directly forward from V, but when it is negative, it muſt be taken backward towards R.

In the ſame manner that f is found from having d, and r, given; either d or r may be found from having the reſt given. For example, let r be ſought. Multiply the equation, $f = \frac{mdr}{md - nd - nr}$,

by the denominator $md - nd - nr$, and we ſhall Fig.
have $mdf - ndf - nrf = mdr$; and tranſpoſing, 66.
$mdr + nfr = mdf - ndf$, whence $r = \frac{mdf - ndf}{md + nf}$.
And this ſhews that when m is greater than n, r comes out affirmative, and that the refractor is convex towards the object. But when n is greater than m, r will be negative, and therefore 'tis concave to the object.

Again, let d be ſought, having f, r, given. From the former equation, $mdf - ndf - nrf = mdr$; by tranſpoſing, $mdf - ndf - mdr = nrf$, and $d = \frac{nrf}{mf - nf - mr}$. And from hence it appears, that if mf be greater than $nf + mr$, then d is affirmative; that is, R is on the convex ſide of the refractor. But when $nf + mr$ is greater than mf, then d comes out negative, and R is on the concave ſide of the refractor. And if mf be $= nf + mr$, then d will be infinite, and F is the focus of parallel rays. 67.

It may be further obſerved, that in all caſes where the rays converge to F, if a circle BD be deſcribed with any radius from the center F, to cut the refracted rays AF; theſe rays, paſſing thro' the meniſcus AVDB, will go to the ſame focus F. For theſe rays, paſſing perpendicularly thro' the refracting ſurface BD, go right forward. And for the ſame reaſon, rays diverging from F, will go to R as before.

PROP. XVI.

If parallel rays RA, *tV fall upon a ſmall portion of a ſphere, and be refracted by the firſt ſurface to* T; *they will be refracted by the ſecond ſurface to* F, *the middle of* DT. 68.

Let the rays RA, FG meet in H; then ſince the cord AG cuts the curve at A, G in equal angles,

Fig. gles, the refractions will be equal at A and G;
68. whence the triangle AHG is equiangular, and
equilateral. But FT being parallel to AH, the triangle GFT is ſimilar to AHG; whence GF = FT. But GF being = DF very near, therefore DF = FT.

Cor. *If parallel rays fall upon any part of a ſphere; and* T *be the focus after the firſt refraction;* F *will be the focus after the ſecond, ſo that* GF = FT.

PROP. XVII.

69. *In any lens* BH, *if two radii* CA, *ca, be drawn*
70. *parallel to one another, from the centers* C, *c, of the*
two ſurfaces; and A*a be drawn cutting the axis in* D. *Then every ray that paſſes thro'* D, *ſhall have its incident and emergent rays parallel.*

For by reaſon of the parallels CA, *ca*, the triangles CAD, *cad*, are ſimilar; therefore CD is to *c*D, as CA to *ca*, that is in a given ratio; and therefore the point D is invariable. And further the angles at A and *a* being equal, the line A*a* cuts the two ſurfaces in the ſame angle, and therefore if the ray A*a* be ſuppoſed to go both ways, the refractions will be equal and contrary; that is, they make equal angles with A*a*, and lie contrary ways, and therefore are parallel.

Cor. 1. *Hence in double convex and double concave lenſes, the point* D *lies within the lens; in a plano convex or plano concave, it is at the convex or concave ſurface; but in a meniſcus, it lies out of the lens, nearer to the greateſt curvity. And in a ſphere, it is at the center.*

Cor. 2. *The courſe of a ray, paſſing thro'* D, *very near the axis, may be taken for a ſtrait line, without ſenſible error.*

Cor. 3. *The point* D *is ſo placed, that* CG : *c*F :: Fig.
DG : DF. 69.
For CA or CG : *ca* or *c*F :: CD : *c*D :: CG — 70.
CD (DG) : CF — CD : (DF).

Cor. 4. *As the ſum of the radii (or the difference in a meniſcus) : to the leſſer radius :: ſo the thickneſs of the lens : to the diſtance of* D *from the vertex of greater curvature.*

For ſince CG : *c*F :: DG : DF, by compoſition (or diviſion) CG ± *c*F : *c*F :: DG ± DF (FG) : DF.

Cor. 5. *If* A*a be any ray paſſing thro'* D; *then the tangents to the lens at* A *and a, will be parallel.*

For the angles CA*a* and *ca*A being equal, their complements DAG and D*a*F are equal; and therefore the tangent at A is parallel to the tangent at *a*. And hence

Cor. 6. *For every point* A *of the lens, there is ſome ray or other* A*a, whoſe incident and refracted rays will be parallel; and that is the ray paſſing thro'* D.

For the tangents at A and *a* being parallel, it is the ſame thing as if the ray was tranſmitted thro' a medium contained between two parallel ſurfaces, in which caſe (by Cor. 2. Prop. III.) the incident and emergent rays will be parallel.

SCHOLIUM.

The point D is by optic writers called the *Center* of the lens.

PROP. XVIII. *Prob.*

*Having the lens a*V, *and the radiant point* R 71.
given; to find the focus F, *after the refraction of* 72.
both the ſurfaces.

Let *m*, *n*, be the ſines of incidence and refraction, and make *m* × R*v* — *n* × R*c* : *m* × R*v* ::

Fig. the radius $vc : vf$, and f is the focus after the firſt
71. refraction at av. Again $vf - vV = Vf$; then
72. make as $m \times Cf - n \times Vf : n \times Vf ::$ radius $CV : VF$, and F is the focus after the ſecond refraction at AV. And thus you may proceed to find the focus after refraction by a third, or fourth ſurface, &c.

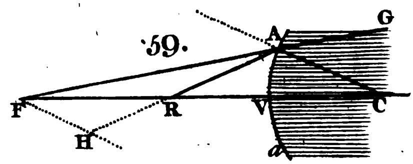
For ſince $m \times Rv - n \times Rc : m \times Rv :: vc : vf$, by diviſion, $m \times Rv : n \times Rc :: vf : vf - vc$ or cf. And therefore (by Prop. XI.) f is the focus of the rays refracted at av. Alſo ſince $m \times Cf - n \times Vf : n \times Vf :: CV : VF$; therefore, $n \times Vf : m \times Cf :: VF : CV + VF$ or CF. And $Vf : Cf :: \frac{VF}{n} : \frac{CF}{m} :: m \times VF : n \times CF$. And therefore (Cor. 2. Prop. XI.) rays converging to f and refracted by AV, will converge to the focus F. This method of demonſtration is general, and is applicable to any caſe.

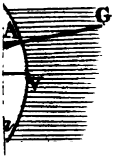
Cor. *If the incident rays are parallel; make as* $m - n : m :: vc : vf$, *and* f *is the focus after refraction at the firſt ſurface. And find* F *as before, for the focus after the ſecond refraction.*

For in this caſe Rv and Rc are equal.

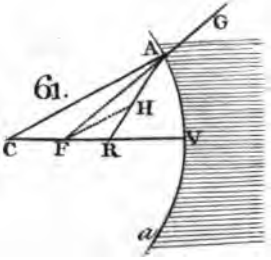
PROP.

59.





61.



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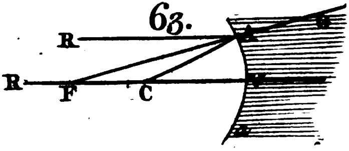

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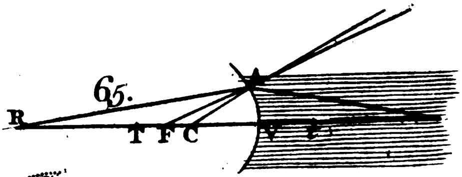
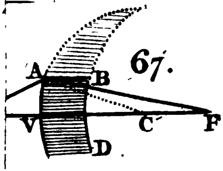
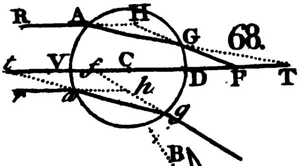


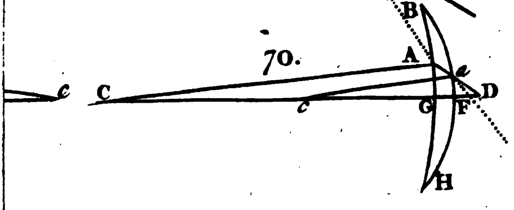
67.



68.



70.



Pl.V. pa.114.

UNI

Fig.

PROP. XIX.

Let parallel rays Sa, Rv, *be refracted by the lens* Av; *ſuppoſe the ſines of incidence and refraction as* m *to* n; *and put the radius of the ſegment towards the object* $cv = r$, *the radius of the ſegment from the object* CV $=$ R, *the thickneſs of the lines* vV $= t$, *the principal focal diſtance* VF $= f$. *And* $p = \frac{n}{m-n}$, $q = \frac{m}{m-n}$; *therefore* $mp = nq$. *Then* 71. 72.

$$f = \frac{Rn \times \overline{qr - t}}{m \times \overline{R + r - t} \times \overline{m - n}};$$ *wherein if* av, *or* AV *be convex the contrary way, then* r *or* R *is negative.*

For (Prop. XVIII. Cor.) $m - n : m :: vc\ (r) : vf = \frac{mr}{m-n} = qr$; and $Vf = qr - t$. Again $m \times Cf - n \times Vf : n \times Vf :: CV : VF$; that is, $m \times \overline{R + qr - t} - n \times \overline{qr - t} : n \times \overline{qr - t} :: R : f$

$$= \frac{Rn \times \overline{qr - t}}{mR + mqr - mt - nqr + nt} =$$

$$\frac{Rn \times \overline{qr - t}}{mR + \overline{m - n} \,.\, qr - \overline{m - n} \,.\, t} =$$

$$\frac{Rn \times \overline{qr - t}}{mR + \overline{mr - t} \times \overline{m - n}} = \frac{\overline{qr - t}}{q \times \overline{r + R - t}}\, pR.$$

And a like calculation may be made for the concave, fig. 72.

Cor. 1. *If* $t = 0$, *or inconſiderable, then* $f = \frac{pRr}{R + r}$, *and is the ſame on either ſide of the lens.*

For then $f = \frac{nqRr}{m \times \overline{R + r}} = \frac{mpRr}{m \times \overline{R + r}} = \frac{pRr}{R + r}$.

Fig. 71. 72. Cor. 2. *If* $t = 0$, *and* av *be a plane ſurface, then* $f = \frac{n}{m-n} R$.

For then $r =$ infinity, and $f = \frac{pRr}{r} = pR = \frac{n}{m-n} R$. And

Cor. 3. *If* $t = 0$, *and* AV *a plane ſurface,* $f = \frac{n}{m-n} r$.

Cor. 4. *If* $t = 0$, *and* $R = r$, *then* $f = \frac{n}{m-n} \times \frac{1}{2} r$.

Cor. 5. *The principal focal diſtance made by one refraction, is* $\frac{mr}{m-n}$.

For by the demonſtration $vf = \frac{mr}{m-n}$.

Cor. 6. *If a luminous body be placed in the principal focus of a lens, it will caſt a light to a great diſtance.*

For the rays emerging nearly parallel, will not unite till at a great diſtance.

SCHOLIUM.

In glaſs, $m = 3$, and $n = 2$ nearly; whence $p = 2$, and $q = 3$. Therefore from this Prop and its corollaries, may be deduced the following rules peculiar to glaſs lenſes.

1. In a double convex glaſs of equal radii, the principal focus is diſtant, the radius of the ſphere; or the focal diſtance is equal to the radius

2. In a ſphere, the principal focus is diſtant half the radius; of an hemiſphere $\frac{4}{3}$ the radius, when

the convex ſide is expoſed to the rays; and twice Fig.
the radius, when the plane ſide is expoſed. 71.

3. In a plano convex, the diſtance of the prin- 72.
cipal focus is equal to the diameter, when the plane ſide is expoſed to the rays; but to the diameter — $\frac{2}{3}$ the thickneſs, when the convex ſide is expoſed.

4. In a double convex glaſs of different ſpheres, the diſtance of the principal focus is $\frac{2Rr}{R+r}$.

5. In double concaves of equal radii, the principal focus is diſtant the radius of the ſphere, and is virtual.

6. In a plano concave, the principal focus is diſtant twice the radius, and is virtual.

8. In a double concave glaſs of different ſpheres, the diſtance of the principal focus is $-\frac{2Rr}{R+r}$, and is virtual.

9. In a meniſcus, the diſtance of the principal focus is $\frac{2Rr}{r-R}$, the concave ſide being expoſed to the rays.

Fig.

PROP. XX.

71. *If rays issuing from the radiant point* R, *be refrac-*
72. *ted by the lens* Av; *let* m, n = *sines of incidence and refraction,*

$d = \mathrm{R}v$, *the distance of the radiant point.*
$r = cv$, *the radius of the nearer surface.*
$\mathrm{R} = \mathrm{CV}$ *the radius of the farther surface.*
$t = \mathrm{V}v$ *the thickness of the lens.*
$f = \mathrm{VF}$ *the focal distance.*

$$p = \frac{n}{m-n},\ q = \frac{m}{m-n},\ pm = qn;\ \textit{then}$$

$$f = \frac{mpdr\mathrm{R} - nd\mathrm{R}t + npr\mathrm{R}t}{m.d \times \overline{r+\mathrm{R}} - mpr\mathrm{R} - dt \times \overline{m-n} + nrt}$$

wherein, if the rays converge, or if av, AV, be concave the contrary way, then d, r, or R *is negative.*

Or at short, $f = \dfrac{\mathrm{D}-t}{\mathrm{D}-t+q\mathrm{R}}\,p\mathrm{R}$, *where* $\mathrm{D} = \dfrac{qdr}{d-pr}$.

For (by Prop. XVIII.) $m \times \mathrm{R}v - n \times \mathrm{R}c : m \times \mathrm{R}v :: vc : vf$; that is, $md - n \times \overline{d+r} : md :: r : vf = \dfrac{mdr}{md-nd-nr} = \mathrm{D}$ ſuppoſe, and $\mathrm{D} - t = \mathrm{V}f$. Again $m \times \overline{\mathrm{R}+\mathrm{D}-t} - n \times \overline{\mathrm{D}-t} : n \times \overline{\mathrm{D}-t} :: \mathrm{R} : f$, or $m\mathrm{R} + .\,\overline{m-n} \times \mathrm{D} - \overline{m-n}\,.\,t : n\mathrm{D} - nt :: \mathrm{R} : f$, therefore $f = \dfrac{n\mathrm{DR} - n\mathrm{R}t}{m\mathrm{R} + \overline{m-n}\,.\,\mathrm{D} - \overline{m-n}\,.\,t}$.

$$\text{But } \mathrm{D} = \frac{mdr}{md-nd-nr} = \frac{\frac{m}{m-n}dr}{d - \frac{n}{m-n}r} = \frac{qdr}{d-pr},$$

therefore $f = \dfrac{\dfrac{nqdr\mathrm{R}}{d-pr}}{}$

Fig. 71. 72.

$$\frac{\frac{nqdrR}{d-pr} - nRt}{mR + \overline{m-n} \times \frac{qdr}{d-pr} - \overline{m-n} \times t} =$$

$$\frac{nqdrR - ndRt + nprRt}{mdR - mprR + \overline{m-n}\,.\,qdr - \overline{m-n} \times \overline{dt-prt}}$$

$$= \frac{mpdrR - ndRt + nprRt}{mdR - mprR + m \times dr - \overline{m-n} \times dt + \overline{m-n} \times prt}$$

$$= \frac{mpdrR - ndRt + nprRt}{md \times \overline{r+R} - mprR - dt \times \overline{m-n} + nrt};$$

becauſe $\overline{m-n}\,.\,q = m$, and $\overline{m-n}\,.\,p = n$. Or at ſhort $f = \frac{D-t}{D-t+qR}\,pR$, where $D = \frac{qdr}{d-pr}$. And a like computation may be made for the concave, fig. 72.

Cor. 1. *If t be nothing or inconſiderable, then* $f = \frac{pdrR}{d \times \overline{r+R} - prR}$, *and is the ſame on both ſides the lens, whilſt the diſtance d continues the ſame.*

Cor. 2. *If the lens be of glaſs, then* $f = \frac{6drR - 2dRt + 4rRt}{3d \times \overline{r+R} - 6rR - dt + 2rt}$; *or when* $t = 0$, $f = \frac{2drR}{d \times \overline{r+R} - 2rR}$.

Cor. 3. *In a plane convex, if $t = 0$, and AV a plane ſurface; then* $f = \frac{ndr}{\overline{m-n}\,.\,d - nr}$.

For r vaniſhing, $f = \frac{pdrR}{dR - prR} = \frac{dr}{\frac{d}{p} - r} = \frac{ndr}{\frac{nd}{p} - nr} = \frac{ndr}{\overline{m-n}\,.\,d - nr}$.

Fig. 71. 72. Cor. 4. *If* $t = 0$, *and the lens has equal convexities; then* $f = \frac{ndr}{\overline{m-n} \times 2d - nr}$. *And in glaſs,* $f = \frac{dr}{d-r}$.

Cor. 5. *When* $t = 0$, *in a double concave,* $f = \frac{pdrR}{-d \times \overline{r+R} - prR}$. *And when the radii are equal,* $f = -\frac{dr}{d+r}$, *for glaſs.*

Cor. 6. *In a plano concave, expoſed with the concave ſide to the rays,* $f = \frac{-pdR}{d+pR}$.

Cor. 7. *In a meniſcus, the convex ſide turned to the rays,* $f = \frac{-pdrR}{d \times \overline{r-R} + prR}$.

Cor. 8. *The focal diſtance made by a ſingle refraction is* $\frac{mdr}{\overline{m-n} \,.\, d - nr}$.

From the demonſtration.

Cor. 9. *If the radiant point and the focus be equidiſtant from a lens; they will each of them be diſtant twice the principal focal diſtance.*

For put $d = f = \frac{pdrR}{d \times \overline{r+R} - prR}$, then $d \times \overline{r+R} - prR = prR$, or $d \times \overline{r+R} = 2prR$, whence $d = \frac{2pRr}{r+R}$ = twice the principal focal diſtance, by Cor. 1. Prop. XIX.

Cor. 10. *If the radiant point be nearer the lens, than the principal focus; the rays after refraction*

will diverge; if it is in the principal focus, they will Fig.
emerge parallel; and, if it is further off, they will 71.
converge. 72.

SCHOLIUM.

Hence if all the quantities *m*, *n*, *d*, *r*, R, *f*, except one be given, that one may be found, by the reduction of the equation in Cor. 1.

PROP. XXI.

If P, *p*, *be the principal foci of a ſphere or lens* 73.
A*v*; R *the radiant point*; F *the focus by refraction*; 74.
D *the center of the ſphere or lens*; *then* RP, RD, RF, *are continual proportionals, where* P *and* F *muſt lie the ſame way from* R.

Let the focal diſtance DP or D*p* = F, the reſt as in the laſt Prop. Then (Cor. 1. Prop. XIX.) Vp or $DP = \frac{prR}{r+R} = F$, and $prR = F \times \overline{r+R}$. And (Cor. 1. Prop. XX.) VF or DF =

$$\frac{dprR}{d \times \overline{r+R} - prR} = \frac{dF \times \overline{r+R}}{d \times \overline{r+R} - F \times \overline{r+R}}$$

$$= \frac{dF}{d-F} = \frac{RD \times DP}{RP},$$ and RP × DF = RD × DP, whence RP : RD : : DP : DF; and compounding, RP : RD : : RP + DP (RD) : RD + DF (RF). And a like computation may be made for the concave, fig. 74.

Cor. 1. RP : RD : : DP : DF; *or* $DF = \frac{RD \times DP}{RD - DP}$. From the demonſtration.

Cor. 2. RP : PD : : DP : *p*F; *or* RP × *p*F = PD^2.

For

Fig. 73. 74. For (Cor. 1.) RP : RD :: DP : DF; and by division RP : RD − RP (PD) :: PD : DF − DP (*p*F).

Cor. 3. *p*F *is always reciprocally as* RP; *and they lie contrary ways from p and* P.

Cor. 4. R *and* F *are conjugate foci to one another; and when they are on the same side of the lens, the incident and refracted rays diverge both from* R *and* F, *or else both converge to* R *and* F. *And the contrary when on different sides.*

PROP. XXII.

73. 74. *If* A*v be a lens or sphere,* F *the focus of the refracted rays which flow from the radiant point* R, *p the principal focus. Then* RF : RD :: DF : D*p*, *the principal focal distance.*

For (Prop. XXI.) RF : RD :: RD : RP, and by division, RF : RF − RD (DF) :: RD : RD − RP (PD or *p*D).

Cor. 1. *In a convex, the principal focal distance*

$$Dp = \frac{RD \times DF}{RD + DF}.$$

Cor. 2. RF : DF :: DF : *p*F, *or* $pF = \frac{DF^2}{RF} = \frac{DF}{RF} \times DF$.

For since RF : RD :: DF : D*p*, by division RF : RF − RD (DF) :: DF : DF − D*p* (*p*F.)

Cor. 3. *Hence, the focus* F *may be found geometrically thus. Let* P *be the principal focus; with the center* D *and radius* DP, *describe the arch* PG, *cutting*

ting any incident ray RA, *in* G. *Join* DG, *and draw* Fig.
AF *parallel to it, cutting the axis* RD *in* F, *then* F 73.
is the focus. 74.

For RF : RD :: DF : DP; that is, RF : RA :: DF : AG, therefore the triangles RGD, RAF are ſimilar, and AF parallel to GD. And all rays proceeding from G (or *g*) will emerge parallel to GD.

SCHOLIUM.

In the firſt Corol. we have a good practical rule for finding the principal focus of a lens, having firſt found any other focus by trial. And in Cor. 2. we have another rule tending to the ſame purpoſe, which finds the exceſs of any other focal diſtance found by trial, above the principal focal diſtance. Both theſe are very eaſy in practice.

PROP. XXIII.

The diſtance of the radiating point and its focus, 73.
made by a convex lens, that is, RF *is the leaſt poſſible, when they are equidiſtant from the lens.*

For (by Cor. 2. Prop. XXI). RP : PD :: PD : *p*F. Now if RP = *p*F, then the ſum of the extremes RP, *p*F is equal to the ſum of the means 2PD, which is conſtant. But in any other caſe (Propor. Prop. IX.) the ſum of the extremes RP, *p*F is greater than the ſum of the means, that is, greater than the ſum of the extremes when they are equal. And therefore RP + *pf* is the leaſt when RP = *p*F, and adding P*p*, the whole diſtance RF is the leaſt when RP = *p*F, and conſequently when RD = DF.

Cor. *The radiating point and its focus is at the leaſt diſtance, from each other, when each of them is diſtant from the lens, twice the principal focal diſtance.*

For

Fig. 73. For then RD = RP + PD = 2PD, and DF = Dp + pF = 2Dp = 2DP.

PROP. XXIV.

75. *An oblique pencil of rays* SAGC, *has its focus* G *a little nearer the lens, than a direct pencil* RAFC.

For if DS = DR, then the angle CSA will be ſomething leſs than the angle CRA; and upon that account the rays SA, SC, will be ſooner brought to interſect at G, than the rays RA, RC, do at F. But further, the rays RA, RC, falling very nearly perpendicular to the ſides of the lens AC, HN, are leſs bent, than the rays SA, SC, which fall obliquely on both ſides the lens. For the more oblique they are, the more they are bent, and turned out of their direct way. And conſequently the oblique rays SA, SC will ſooner interſect in G, than thoſe at F. But ſince the angle RDS is generally very ſmall, the difference between DF and DG will likewiſe be very ſmall.

Cor. 1. *The focus of an oblique pencil, by a ſingle refraction is nearly at the ſame diſtance as of a direct pencil.*

For in both caſes the extreme rays will interſect the ray which paſſes thro' the center of the ſurface AC, at the ſame diſtance.

Cor. 2. *This Prop. holds good in a concave lens, and alſo in a meniſcus, as well as in a convex one.*

For in all caſes the more oblique the rays are the more they are bent, and the ſooner they will interſect a direct ray paſſing thro' D the center of the lens, by Cor. 2. Prop. XVII. But I endeavour to avoid drawing a multiplicity of figures.

PROP.

Fig. 75.

PROP. XXV.

A convex lens collects the rays nearer together, and a concave one diſperſes them.

For rays that are refracted at a convex lens, are bent at the points of refraction ſo, that they incline more towards one another than before; but in concaves the rays after refraction, being bent the contrary way, are made to ſeparate more from one another. And this will be ſo whether there be one refracting ſurface, or two ſuch; or if there be more. For every ſucceeding refraction, adds to the effect, by making the rays either converge or diverge ſo much the faſter.

Cor. 1. *Converging rays are made more converging by a convex lens; and diverging rays are made more diverging by a concave.*

Cor. 2. *A meniſcus has the properties of a convex glaſs, when the inner radius is the greater. And of a concave, when the inner radius is the leſſer. Except when the two ſurfaces are concentric; and then it belongs to neither; for the rays will emerge parallel.*

Cor. 3. *A lens or ſpectacle glaſs proper for an old perſon, muſt be ſuch as to make the image of an object appear at that point where he can ſee that object diſtinctly with his naked eye; and the ſame is true of a concave, in reſpect of a ſhort-ſighted perſon.*

SCHOLIUM.

From what has been before laid down, the properties of all ſorts of ſingle lenſes may be deduced for practice as follows.

Let R, F, be conjugate foci; P, *p* the principal foci; then, 76.

1. Pa-

Fig. 1. Parallel rays falling on one ſide of a convex
79. lens A; will be refracted to P or p on the other ſide.

2. Parallel rays falling on one ſide of a concave lens A; will by refraction, diverge from P or p on the ſame ſide.

3. In a convex lens A, rays diverging from P or p; will emerge parallel on the other ſide.

4. In a concave lens A, rays converging to P or p; will emerge parallel, going out of the lens on the ſame ſide.

76. 5. In convex lenſes, rays diverging from R beyond P; will converge to F beyond p. And the contrary.

77. 6. In convex lenſes, rays diverging from R which is nearer than P; will diverge from F, on the ſame ſide with R.

78. 7. In convex lenſes, rays converging to F, will by refraction converge to R, nearer than P or p.

79. 8. In concave lenſes, rays converging to R, beyond P, will diverge from F, beyond p, and the contrary.

80. 9. In concave lenſes, rays converging to F,
81. nearer than p; will by refraction converge to R, beyond F.

82. 10. In concave lenſes, rays diverging from R, beyond p; will after refraction diverge from F nearer than p.

83. 11. In concave lenſes, rays diverging from R, nearer than p, will diverge from F, nearer ſtill than p.

Note, I reckon a meniſcus, convex or concave, according as the center of its inner ſurface is further off, or nearer than the center of the outward ſurface.

And in all theſe caſes, the conjugate foci F f, Fig.
are ſo ſituated, that it will always be; 76.

$$RA : PA \text{ or } pA :: RF : AF :: AF : pF$$ to

$$\text{or } RP : AP :: RA : AF :: Ap : pF.$$ 83.

$$\text{or } RP : RA :: RA : RF :: PA : AF.$$

Any of theſe rules will ſerve to find the conjugate focus. All this appears from what has been before demonſtrated; particularly from Cor. 4 and 5. Prop. XX. where the rules for diverging rays for convex glaſſes is $\frac{dr}{d-r}$ = focal diſtance; and for concaves, $-\frac{dr}{d+r}$ = focal diſtance. And it is the ſame thing whether the rays diverge for convexes, or converge for concaves; all the quantities are the ſame in both, and have the ſame ſituation. Therefore if a convex and a concave have the ſame principal focal diſtance; and if a ray diverge from any focus in the convex, and another ray converge to the ſame focus, thro' the concave (put in its place); then will both have the ſame conjugate foci. And in their motions, they will deſcribe ſeparate parts of the ſame lines.

The properties of lenſes, or refracting ſurfaces, demonſtrated in the foregoing propoſitions, hold good in regard to the foci, if the mediums be changed; provided you make the rays of light go contrary ways thro' the foci, to what they do in theſe propoſitions. That is, if you ſuppoſe the rays going out of a denſe medium into a rare one, or to be refracted out of a denſe medium thro' lenſes of air, or ſome rare medium; you muſt ſuppoſe the rays to converge inſtead of diverging, and to diverge inſtead of converging; and then the focus will be found by the foregoing Rules.

PROP.

PROP. XXVI. *Prob.*

To find the principal focus of any lens.

1 *Way.*

Hold a convex lens perp. againſt the rays of the ſun, hold a white paper behind it, to receive the refracted light, which will make a round white ſpot upon the paper. Move the paper back and forward till this ſpot, which is the ſun's image, be the leaſt poſſible; there fix it. Then the diſtance of the paper from the lens, is the focal diſtance required, or the burning point.

2 *Way.*

Cover one ſide of a convex lens with paper, having ſeveral holes made in it with a pin; and placing the glaſs directly againſt the ſun beams; the light paſſing thro' the holes will make ſo many white ſpots upon a paper held behind it. Move the paper back and forward till all theſe ſpots coincide in one; there is the focus. And the diſtance of the paper from the glaſs being meaſured, gives the focal diſtance.

3 *Way.*

Place a convex lens ſo, between any object (as a window in the day, or a candle in the night) and a white wall or paper, that the image of the object may be painted diſtinctly thereon, which may be found by moving the lens back and forward. Then meaſure the diſtance of the lens from the object, and alſo from the image. Multiply them together, and divide their product by their ſum, the quotient will give the focal diſtance, by Cor. 1. Prop. XXII. Or (Cor. 2.) divide the ſquare of the diſtance of the obſerved focus, by the diſtance of the object from this focus, gives the exceſs above Fig. the principal focal diſtance. If the lens be in the midway between the object and image; then half the diſtance of the lens from either of them, is the focal diſtance.

4 *Way.*

Cover one ſide of a concave lens with paper, having two holes made with a pin, diametrically oppoſite to one another. The lens being expoſed to the ſun beams, and a paper held behind it; the light paſſing thro' the lens will make two white ſpots upon the paper. Move the paper back and forward, till the diſtance of the two ſpots on the paper, be twice the diſtance of the two holes in the cover; then the diſtance of the white paper from the lens is the focal diſtance.

PROP. XXVII. *Prob.*

To find the vertex or center of a lens.

1 *Way.*

Hold the lens at a proper diſtance from the eye, and obſerve the two reflected images of a candle, made from the two ſurfaces. Move the lens till the two images coincide, and that point is the vertex. And if this be in the middle of its ſurface, the lens is truly centered, otherwiſe not.

2 *Way.*

Draw two black lines AB, KL, croſs one another upon a white wall, or rather on paper. Take the lens CDEF, and holding it directly between your eye and the two lines, and parallel to the wall. Move it to the right and left, and you'll ſee the part *gb* of the line AB, which appears thro' the glaſs, to move back and forward. Hold the 84.

Fig. 84. glaſs at laſt, ſo that *gb* may coincide with AB; then does AB paſs thro' the vertex or center of the glaſs. Repeat the ſame thing with the line KL and the part *no*; moving the glaſs up and down, till they coincide. Mark the point Z on the glaſs, where AB, KL interſect, whilſt *gb*, *no*, coincide with them, and that interſection Z is the vertex.

Note, it may be as well to fix the glaſs, and move your eye, till A*gb*B, and K*no*L, be two continued right lines, whoſe interſection gives the vertex.

PROP. XXVIII.

Wherever the rays, which come from all the points of an object, meet again in ſo many other points, to which they converge by refraction; there they will make the picture of the object upon any white body on which they fall.

85. Let PR be any object, AB a lens. The rays that diverge from any point Q fall upon the lens, and are refracted to their focus *q*, where they all meet, and make the image of Q upon a paper held there. In like manner the rays which diverge from P, are refracted to their focus *p*, and there give the image of P; and the rays diverging from R, are refracted to *r*, and make the image of R. And thus, from all the intermediate points of the object, the rays proceeding to the lens, are refracted to as many correſpondent points in the focus; and will all together make there the image *pqr* of the object PQR, in its proper ſhape and colours. But then the picture is inverted or turned upſide down, and the right ſide to the left.

Cor. 1. *Rays flowing from all the points of an object, as* PR, *paint the image thereof pr, in the real focus q, and may be ſeen there upon a white paper; or without the paper it may be ſeen there hanging in the*

the air, by an eye at O, placed at a proper diſtance, Fig. 85.
as 6 or 8 inches, &c. behind the focus.

Cor. 2. *An object ſeen by refraction, appears in that place, from whence the rays after their laſt refraction diverge to the ſpectator's eye.*

Cor. 3. *Rays flowing from any point of an object, muſt be ſo refracted, as all to meet again in ſome other point, to give the image of that point.*

For if they ſhould fall upon different points, it would breed nothing but confuſion, for different points cannot repreſent that one point from which the rays come. All the rays flowing from the ſeveral points, muſt go to as many correſpondent points, to make the compleat image of the body. And this will appear on a paper held there; or if there be no paper, the rays will diverge from that image, as from the body at firſt; and therefore may be ſeen by an eye placed behind it, for the ſame reaſon that a body is viſible by the rays flowing from it at firſt.

PROP. XXIX.

The image of a right line immerſed in water, or any fluid, is concave towards the eye; and is ſituated in a plane paſſing thro' the eye.

Let the line AB in the fluid be parallel to the ſurface RS, and the eye at E directly over it. Then (by Cor. 6. Prop. IX.) the focus of the point C is at G, ſo that DG = $\frac{3}{4}$ DC. But (by the ſame) as the eye moves towards R or S, G will ſeem to riſe; or which is the ſame thing, as G moves further off to F or H, it will ſeem to riſe up, till it get to the furface RS; therefore RF and SH continually grow leſs and leſs. That is, the focus of A and B are nearer the ſurface, than G or the 86.

Fig. focus of C; and conſequently the image FGH is
86. concave towards RS.

If AB is not parallel to the ſurface, yet as one part is raiſed as much as the other is depreſſed, the image will ſtill be concave, and that if the eye be moved out of the perpendicular ED. And a plain that at firſt paſſed thro' EFABHG, will alſo paſs thro' EFGH, when the eye is moved laterally.

Cor. 1. *If the eye at E was in a denſe medium, and the line* AB *in a rare one. The points* A *and* B, *at a diſtance from* C, *would ſeem to deſcend; and the image of* AB *would be convex towards the ſurface* RS.

For here G would be the object and C the image, and if FGH be reduced to a ſtrait line, ACB will be reduced to a curve convex towards G. Hence therefore

Cor. 2. *A right line ſeen thro' any medium terminated by parallel planes, will appear a right line.*

For the two refractions correct one another, having contrary effects.

Cor. 3. *A plane ſurface immerged deep in water, or any fluid, will appear concave to the eye.*

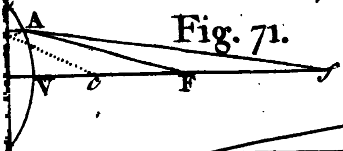
Cor. 4. *The image of a ſhort right line immerged in water, does not ſenſibly differ from a right line.*

PROP. XXX.

The image of a right line made by refraction thro' a lens, is concave towards the lens.

87. Let AB be a lens, PR a right line directly expoſed to the lens. Then (by Prop. XXIV.) if P, Q, R, were equally diſtant from the lens, r and

Fig. 71.



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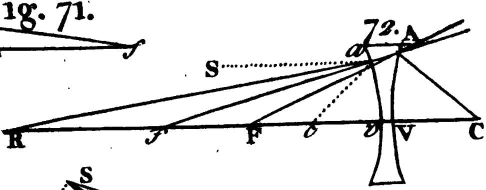

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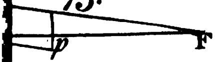

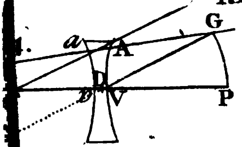

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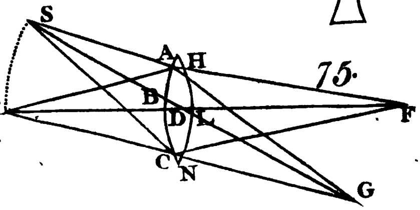

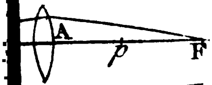

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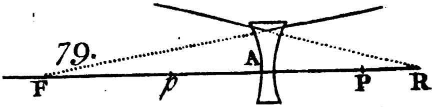

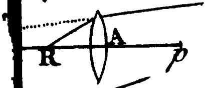

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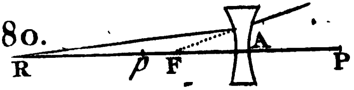

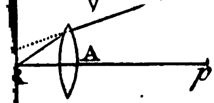

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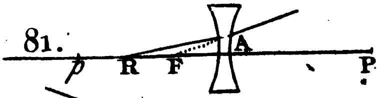

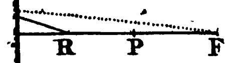

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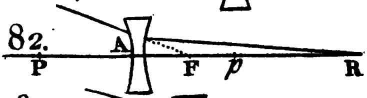

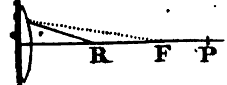

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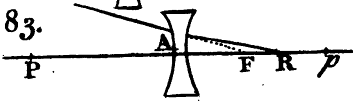

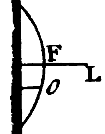

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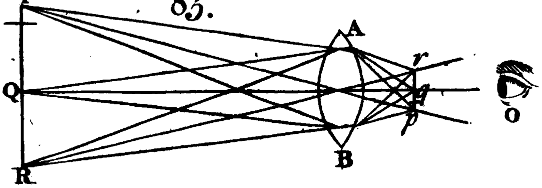


Pl. VI. *pa 132.*

and *p* would be nearer the lens than *q*, and therefore *rqp* is a curve line. But PQR being a right line, P and R are further off than Q, and that brings the focus *p*, and focus *r* nearer the lens ſtill; and conſequently *rqp* is more curve than before, and is concave towards the lens. And (from Cor. 2. Prop. XXIV.) the ſame holds good in a concave, and meniſcus. Fig. 87.

Cor. 1. *The image of a right line made by one refraction of a ſpherical ſurface, is alſo a little concave towards that ſurface.*

For ſuppoſing *rpq* made only by the firſt refraction. Then if P, Q, R be equidiſtant from the lens; (by Cor. 1. Prop. XXIV.) *p*, *q*, *r* would alſo be equidiſtant. But ſince P and R are ſomething further than Q; *r* and *p* will be ſomething nearer than *q*; or *rqp* will be a curve line.

Cor. 2. *The image of a ſhort right line, made by refraction thro' a lens, or thro' a ſingle ſurface, does not ſenſibly differ from a right line.*

For the deviation of the image of a very long line, from a right line, is but very little curved, and ſcarce perceptible.

Cor. 3. *The image of a circle refracted thro' a lens, to which it is concentric, will be nearly a circle alſo concentric to the lens. Or thro' a ſingle ſurface.*

For being concave to the lens, a ſmall part of it does not ſenſibly differ from a circle; whether it be thro' a lens or thro' a ſingle ſurface.

Cor. 4. *The image of a plane figure, made by refraction thro' a lens, is concave towards the lens; tho' in a ſmall figure it does not ſenſibly differ from a plane. And the ſame thro' a ſingle ſurface.*

PROP. XXXI.

The object and its image made by a lens, ſubtend equal angles at the lens.

88. 89. For let D be the center of the lens AB, PQR the object, *pqr* its image. Then (Cor. 2. Prop. XVII.) PD*p* is nearly a ſtraight line, and likewiſe RD*r* is alſo a ſtreight line; conſequently the angle PDR ſubtended by the object, is equal to the angle *p*D*r* ſubtended by the image, the angular point being at the center of the lens D.

Cor. 1, *Hence the object and image are equal, when they meet together.*

64. Cor. 2. *The object and image made by a ſingle refraction, ſubtend equal angles at the center of the refracting ſurface.*

For the unrefracted rays flowing from the extremes of the object, paſs thro' the center; and the object and image are contained between theſe.

PROP. XXXII.

88. 89. *The length of an object, is to the length of the image, made by a lens; as the diſtance of the object from the center of the lens, to the diſtance of the image from it.*

For (Prop. XXXI.) the angles PDR and *p*D*r* being equal, the triangles PDR, *p*D*r* are ſimilar; Q*q* being perp. to both PR and *pr*. Therefore PR : *pr* : : PD : *p*D : : DQ : D*q*.

Cor. 1. *The magnitude of the object (as to its area), is to the magnitude of the image; as the ſquare of*

of the object's diſtance, is to the ſquare of the image's Fig.
diſtance, from the lens. 88.
For the areas are as the ſquares of the diameters. 89.

Cor. 2. *If* T *be the principal focus of the lens; then the diameter of the object, is to the diameter of the image; as* QT : *is to* TD.

For PR : *pr* : : DQ : D*q* : : (Cor. 1. Prop. XXI.) QT : TD.

Cor. 3. *Hence a convex magnifies an object, when it is nearer than twice the principal focal diſtance: but if further off, it leſſens it. That is, in the firſt caſe, the image is bigger, and in the ſecond, the object is bigger.*

Cor. 4. *In all caſes a concave lens diminiſhes an object, or the image is leſs than the object.*

Cor. 5. *The length of the object, and the length of its image made by a ſingle refraction; are as their diſtances from the center of the refracting ſurface.*

For (Cor. 2. Prop. XXXI.) they make equal angles at that center.

Cor. 6. *A plain glaſs, eſpecially if it be thick,* 93.
magnifies a body.

For CD, EF being parallel, AC is parallel to EO, and BD to FO. Therefore AB, thro' the glaſs CF, appears of the magnitude *ab*.

Cor. 7. *If the object be cloſe to the lens it is very little magnified thereby: or if the eye be cloſe to the lens.*

For every part of it appears nearly, in the place it is really in.

Cor. 8. *The apparent magnitude of a body, is as the angle which the image appears under to the eye in any place.*

PROP. XXXIII.

88. 89. *If* T *be the principal focus of a lens; then the diſtance of the object* PR *from the lens, is to the diſtance of the image pr; as* QT *to* TD.

For (Cor. 2. Prop XXXII.) QT : TD : : diameter of the object : diameter of the image : : (Prop. XXXII.) DQ : D*q*.

Cor. *When* QT = TD, *the image is equal to the object.*

PROP. XXXIV.

88. 89. *If the object and its image be both on the ſame ſide of the lens; the image will be erect; if they be on different ſides, the image will be inverted.*

For when the rays keep the ſame poſition till they come at the image, then the image will have the ſame poſition as the object, and that is when they are both on one ſide of the glaſs. But when the rays croſs one another, before they come at the focus, or where the image is, and that is when they paſs thro' the glaſs; here the rays from the upper part of the object go to the lower part of the image, and the right to the left, &c. that is, the image will then be inverted. Thus P and *p*, alſo R and *r*, lie contrary ways by the convex lens; and the ſame way by the concave one.

90. Cor. 1. *When the object and image are on different ſides the lens, as the object approaches the lens the image recedes from it; or if the object recedes, the image approaches it. In general, if the object* Q *from an infinite diſtance approaches* D, *the image flies from the principal focus* t *to* q; *and when* Q *arrives at the principal focus* T, q *is infinitely diſtant.*

For (by Cor. 3. Prop. XXI.) QT and *tq* are Fig. reciprocally proportional.

Cor. 2. *If the object and image be both on one ſide* 91. *of the lens; if the object moves towards the lens; the image alſo moves towards the lens. And if the object moves from it, the image moves from it.*

For if *t* is the principal focus, *tq* is reciprocally as *t*Q, by Cor. 3. Prop. XXI.

Cor. 3. *In a convex lens, if the object be beyond* 90. *the principal focus, its image will be on the other ſide of the glaſs inverted. But if the object is nearer than the principal focus, the image will be on the ſame ſide of the glaſs, erect.*

For in the motion of Q to T, *tq* becomes infinite at T, and whilſt Q goes from T towards D, *q* comes up on the ſame ſide of the glaſs with Q, and then the image becomes erect.

Cor. 4. *In a concave lens, the object and image are always on one ſide of the lens.*

Cor. 5. *The diſtance of the image may be increaſed at pleaſure, and alſo its magnitude; by bringing the object nearer the principal focus.*

PROP. XXXV. *Prob.*

Having a lens whoſe principal focal diſtance is 71. *known; to find the diſtance of an object, that its diameter may be to the diameter of its image, in the given ratio of a to* 1.

This is eaſily reſolved by Cor. 1. Prop. XX. For ſince $d : f :: a : 1$; therefore $\frac{d}{a} = f = \frac{pdrR}{d \times \overline{r + R} - prR}$, and $d \times \overline{r + R} - prR = aprR$; there-

Fig. 71. therefore $d \times \overline{r + R} = aprR + prR = \overline{a + 1} \times prR$, whence $d = \frac{a + 1}{r + R} \times prR$. Let t = principal focal diſtance; then (Cor. 1. Prop. XIX.) $t = \frac{prR}{r + R}$, and therefore $d = t \times \overline{a + 1}$.

Cor. *In like manner, if all the quantities,* f, p, d, r, R, *except one, be given; that one may be found by reſolving the equation* $f = \frac{pdrR}{d \times \overline{r + R} - prR}$.

PROP. XXXVI.

The image made by a lens, becomes viſible, by placing the eye to receive the diverging rays going from the image, and at a proper diſtance for diſtinct viſion.

To ſee the image, one muſt behold it as one would do any other viſible object. And conſequently in order to ſee it, the eye muſt be placed in the diverging rays, ſo that they may enter the eye, and thereby make the image viſible. But it cannot be ſeen in any other ſituation, than in the diverging rays; for if theſe miſs the eye there can be no viſion. For as all the points of any viſible body are radiant points, ſo all the points of an image are radiant points, each in its proper pencil of rays iſſuing therefrom. All the difference is, the image can only be ſeen, when the eye is placed in thoſe pencils; whereas a luminous body, that ſend rays every way, may be ſeen in any ſituation.

Cor. 1. *The image made in the air by the lens, may not only be ſeen by the eye placed in the diverging rays; but it may be ſeen upon a paper, placed in the focus of the glaſs.*

Cor. 2. *If* R *be an object*, F *the focus*; *then to an eye at* O, *beyond the glaſs*; *the object* R *ſhall appear at* F; *and therefore it is magnified.* Fig. 92.

For an object appears to be ſituated in the place, from which the rays come laſt to the eye, which is at F.

Cor. 3. *Concave glaſſes, make objects appear nearer, ſmaller and fainter than they are.* 89.

For by the ſpreading of the rays, by the refraction of a concave, ſome of them miſs the eye; which would fall upon it if the glaſs was not there.

Cor. 4. *In a convex lens if the eye be placed between the glaſs and the real image*; *the image will appear erect, but confuſed, and nearer the eye.* 88.

For the rays do not croſs one another before they come to the eye. As if the eye was between D and *q*, there is no croſſing but at *q*, and the eye takes them before they come there. It appears nearer the eye, becauſe it is confuſed; for experience has taught us, that objects very near the eye, always appear confuſed; and therefore thro' cuſtom we judge theſe objects always to be near the eye, which appear confuſed.

Cor. 5. *Likewiſe if the eye be placed either between* D *and q*, *or beyond q*, *and you look with both eyes*; *the object will appear double.*

For in ſuch a poſition, the axis of the eyes cannot both of them be directed to the ſame point of the image, to make it appear ſingle.

Cor. 6. *If a convex lens be fixt in the window ſhut, and the room made dark, the pictures of external objects will be painted upon a paper held in the focus of the glaſs.*

For by Cor. 2. the images will be found there, and if the objects are very luminous, and the room dark, they will appear bright, and in their proper colours, but inverted.

PROP. XXXVII.

94. *If an object* PR *be placed in the principal focus* Q, *of a lens* AB; *its apparent magnitude at any place whatever as* O, *beyond the lens, will be invariably the ſame; and equal to the apparent magnitude when ſeen from* D, *the center of the lens, with the naked eye.*

For (by Cor. 10. Prop. XX.) all the rays of the pencil PAD, will emerge parallel to one another, and to the axis PD; and therefore the angle DOA will be equal to QDP; but (Cor. 8. Prop. XXXII.) DOA is the apparent magnitude of QP, ſeen by refraction at O, and QDP is the magnitude as ſeen from D. Therefore $< QDP = < O = < L = < I$, (which meaſures the apparent magnitude of PQ,) whether the eye be placed at O, L, or I, &c.

Cor. 1. *Hence the apparent magnitude of a body placed in the principal focus will always continue the ſame, however the eye is moved backward or forward from the lens.*

Cor. 2. *The nearer the eye is to the lens, the more of the object appears; and the further off, the leſs of it is ſeen.*

For the eye at O can ſee no more of the object than P; but at L, ſome parts above P may be ſeen by rays coming from thence thro' A to L. But if the eye be removed back from O, the ray AO will then miſs the lens, and the point P will not be viſible.

Cor. 3. *If the object be nearer than the principal focus, its apparent magnitude grows leſs in going from* *the glaſs. If the object be further, the apparent magnitude increaſes, in going from it.* Fig.

When the object is nearer than the principal focus, as at R (fig. 92.); then the focus F, and image there, will be on the ſame ſide of the glaſs; therefore the eye at O going further from the glaſs, goes alſo further from the image at F, and upon that account, it will appear leſs and leſs, but by ſlow degrees.

But when the object is beyond the principal focus as at Q (fig. 90), then the focus *q* will be on the contrary ſide of the glaſs; that is, it will be on the ſame ſide with the eye. Then let the eye be placed beyond *q* to view the image there; and as it approaches *q*, the apparent magnitude of the image will grow bigger and bigger (being as the angle it appears under), and at *q*, it becomes infinite, and infinitely confuſed, and begins to be inverted again. Then proceeding ſtill from *q* to D it decreaſes again from infinity to a given magnitude. So that on either ſide of *q*, its apparent magnitude will be as the diſtance from *q*, therefore it increaſes going from D till it be infinite at *q*.

SCHOLIUM.

There are other phœnomena of glaſs lenſes, reſolvable the ſame way, ſuch as the following.

1. If the eye be fixt in the principal focus, the apparent magnitude of an object will be invariably the ſame, wherever the object is placed beyond the glaſs.

2. If the eye is fixed at a leſs diſtance than the principal focus; the apparent magnitude of an object is diminiſhed, as it is further removed from the glaſs, but by ſlow degrees.

3. If the eye be fixed at a greater diſtance than the principal focus; the apparent magnitude of an object is increaſed, as it is removed from the glaſs; till

Fig. till it comes to the conjugate focus, in reſpect to the eye; and then it becomes infinite, and infinitely confuſed, and begins to be inverted; and going further off, it is diminiſhed again.

4. If the eye and object be fixt, and a convex lens be moved from the object to the eye; the apparent magnitude of the object increaſes to the middle, and then decreaſes to the eye. When it comes into ſuch a poſition, that the eye and object are conjugate foci, the object is infinitely confuſed.

5. If the eye and object be fixt, and a concave lens be moved from either of them to the other; the apparent magnitude of the object will decreaſe to the middle, and then increaſe again.

The apparent diſtance is reciprocally as the apparent magnitude; and in general we judge the apparent diſtance of an object to be the very ſame, as we ſuppoſe the diſtance of a real object to be, from which the rays come to our eyes with the ſame degree of confuſion.

The apparent magnitude, as has been obſerved before, is in all caſes, reciprocally as the diſtance of the eye from the image, or directly as the angle it ſubtends; and that whether the rays diverge or converge, or whether the image be before or behind the eye. But the image hardly ever appears diſtinct in any of theſe caſes.

PROP. XXXVIII.

88. *The brightneſs of the image pr, formed by the lens* AB *from the luminous object* PR; *will be as* $\frac{AB^2}{Dq^2}$, *the area of the lens directly, and the ſquare of the diſtance of the image reciprocally.*

For the brightneſs of the object being given, the quantity of light flowing from it, and falling on the lens, and tranſmitted to the image, will be directly as the magnitude of the object, and that of the lens, and reciprocally as the ſquare of the diſtance. Therefore the light falling on the picture rp, is as $\frac{PR^2 \times AB^2}{QD^2}$. But the denſity of the rays, or the brightneſs of the image, is as the quantity of light directly, and the area reciprocally, that is, as $\frac{PR^2 \times AB^2}{QD^2 \times rp^2}$. But by ſimilar triangles (Prop. XXXI) $\frac{PR^2}{pr^2} = \frac{QD^2}{qD^2}$; therefore the brightneſs is as $\frac{AB^2 \times QD^2}{QD^2 \times qD^2} = \frac{AB^2}{qD^2}$. Fig. 88.

Cor. 1. *If the brightneſs of the object be increaſed in any proportion, which call* B; *then the brightneſs of the image will be as* $\frac{AB^2}{Dq^2}$B.

Cor. 2. *In very remote objects, if f be the principal focal diſtance, then the brightneſs of the image will be as* $\frac{AB^2}{ff}$ B.

For then $Dq = f$.

Cor. 3. *The brightneſs of the image increaſes with the area of the glaſs; but the diſtinctneſs of it decreaſes.*

For it is only the rays very near the vertex, that are refracted to a ſingle point, in the picture; thoſe that are farther off deviate more and more, and ſo make the picture confuſed. All this is plain from Prop. X. and XI, where you find different proportions for the focus, when the arch is large and when ſmall, in a ſingle refraction; and the refraction at two ſurfaces increaſes the error.

PROP.

Fig.

PROP. XXXIX.

95. *If a convex lens* AB *be held directly to the rays of the ſun; and a combuſtible body be held in the principal focus* P, *the heat of the rays will ſet it on fire.*

For all the rays of the ſun which fall upon the lens, are refracted into the focus, making there the image of the ſun. And all theſe rays being brought into ſo ſmall a ſpace, muſt have an intenſe heat; and the more ſo, the more rays are brought thither, or the broader the lens is. And by increaſing the breadth of the lens, the denſity of the rays, and the heat thereof may be made ſo great, as to burn or melt all ſorts of bodies placed there; as is evident by experiments.

Cor. 1. *The heat in the focus of the lens, is to the ſun's direct heat; as the area of the glaſs, to the area of the image in the focus.*

For the denſity or heat is reciprocally as the ſpace it is contained in.

Cor. 2. *The degrees of heat in the foci of different lenſes; are as the ſquares of the diameters directly, and the ſquares of the focal diſtances reciprocally.*

For the heat is as the brightneſs of the ſun's image, that is (by Prop. XXXVIII.) as $\frac{AB^2}{DP^2}$.

SCHOLIUM.

Upon account of the ſun's rays meeting in that place, and burning bodies therein, it is called the *focus* or *burning point*; and the lens itſelf a *burning glaſs*.

When the ſun's rays cannot be collected into a focus ſmall enough to burn briſkly, as in large glaſſes, or thoſe of a long focus; they may be made to paſs thro' a ſmaller lens, held in the converging rays, which will contract them, and bring them into a leſs ſpace in the focus. Fig. 95.

PROP. XL.

If AB be a plane convex lens, expoſed with the plane ſide to the parallel homogeneous rays NA, CV. *And if* P *be the principal focus, or that of rays falling near* V. *and* F *the focus of the ray* AF. *m*, *n the ſines of incidence and refraction out of a rare into a denſe medium*, $x = \text{DV}$, *the verſed ſine of the arch* AV. *Then the aberration* $\text{PF} = \frac{mm}{n \times \overline{m-n}} x$. 96.

From the center C draw CN, CM perp. to the rays NA, MP; and put $\text{CV} = a$, $\text{DV} = x$, $\text{CP} = z$. Then $\text{AD}^2 = 2ax - xx$, $\text{DP}^2 = \overline{z-a+x}^2 = zz + 2xz - 2az + xx - 2ax + aa$, and $\text{AP}^2 = (\text{AD}^2 + \text{DP}^2 =)\ zz + 2xz - 2az + aa$. By ſimilar triangles, $\text{AP}^2\ (zz + 2xz - 2az + aa.) : \text{CP}^2\ (zz) :: \text{AD}^2$ or $\text{CN}^2\ (nn) : \text{CM}^2\ (mm)$, whence $nnzz = mmzz + 2mmxz - 2mmaz + mmaa$, and by reduction, $z = \frac{mma - mmx}{mm - nn} + \frac{m\sqrt{nnaa - 2mmax + mmxx}}{mm - nn}$, and putting $dd = mm - nn$, and extracting the root, $z = \frac{mm}{dd} \times \overline{a-x} + \overline{\frac{m}{dd} \times na - \frac{mm}{n} x - \frac{ddmmxx}{2n^3a}}$ &c. $= \frac{mma + mna}{dd} - \frac{mmn + m^3}{ndd} x - \frac{m^3}{2n^3a} xx$ &c. = (becauſe $dd =$

Fig. 96. $\overline{m+n} \times \overline{m-n}) \frac{ma}{m-n} - \frac{mm}{n \times \overline{m-n}} x - \frac{m^3}{2n^3a} xx$ &c. $= \mathrm{CP}$. But when $x = 0$, then $\mathrm{CF} = \frac{ma}{m-n}$. Therefore CF — CP or $\mathrm{PF} = \frac{mm}{n.\overline{m-n}} x + \frac{m^3}{2n^3a} xx$ &c. $= \frac{mm}{n.\overline{m-n}} x$, nearly. Here the rays being perp. to the plain ſurface AB, ſuffer no refraction; and therefore it is the ſame thing, as if the rays were only refracted at the ſpherical ſurface AV.

Cor. 1. *Putting* $\mathrm{AD} = y$, *the aberration by the figure of the lens, that is,* $\mathrm{PF} = \frac{mmyy}{2an.\overline{m-n}}$.

For $yy = 2ax - xx = 2ax$ nearly, and $x = \frac{yy}{2a}$.

Cor. 2. *The aberration by the ſphericalneſs of the figure, is as the verſed ſine, or as the ſquare of the cord, of half the arch* AV.

Cor. 3. *The aberration in length* PF, *of a plano convex lens, is* $4\frac{1}{2}$ *times its thickneſs, nearly.*

For $m = 3$, $n = 2$, and $\frac{mm}{n.\overline{m-n}} x = \frac{9}{2} x = 4\frac{1}{2}x$.

Cor. 4. *The lateral aberration* FG *correſponding to* PF, *is* $= \frac{mm}{nn} \times \frac{y^3}{2aa}$.

For $\mathrm{CF} = \frac{ma}{m-n}$, and $\mathrm{VF} = \frac{ma}{m-n} - a = \frac{na}{m-n}$. And by ſimilar triangles PD or FV $\left(\frac{na}{m-n}\right)$: AD (y) : : PF $\left(\frac{mm}{2an.\overline{m-n}} yy\right)$: $\mathrm{FG} = \frac{mm}{nn} \times \frac{y^3}{2aa}$.

Cor. 5. *The aberration* PF $= \frac{mm}{\overline{m-n}^2} \times \frac{AD^2}{2VF}$, Fig. 96.
and the correſpondent lateral aberration FG $= \frac{mm}{\overline{m-n}^2} \times \frac{AD^3}{2VF^2}$.

For (Prop. XIII.) the rays going into a rarer medium, $n : m :: VF : CF$. And $n : \overline{m-n} :: VF : CV$; whence $n \times CV$ (or na) $= \overline{m-n} \times VF$. Therefore PF $= \frac{mm \times yy}{2an \times \overline{m-n}} = \frac{mm \times yy}{\overline{m-n} \times 2VF \times \overline{m-n}}$.

And FG $= \frac{mm}{nn} \times \frac{y^3}{2aa} = \frac{mm \times y^3}{2 \times \overline{m-n}^2 \times VF^2}$.

Cor. 6. *In a double convex lens, of the ſame focal diſtance; the aberrations will be nearly the ſame as in the plano convex.*

For the rays AP, VF will be nearly of the ſame length as before; and therefore their difference, or the aberration will be the ſame.

Cor. 7. *If the rays don't come parallel, but diverging, the aberration will be ſomething greater; and converging ſomething leſs; but may be taken for the ſame.*

Cor. 8. *If parallel rays (or any others nearly) fall upon the plane ſide of a plano convex lens, the diameter of the leaſt circle of aberration, is $\frac{1}{4}$ the aberration made at the principal focus; or* $2QI = \frac{1}{2} FG = \frac{mm}{nn} \times \frac{y^3}{4aa} = \frac{mm}{\overline{m-n}^2} \times \frac{AD^3}{4VF^2}$.

For this is the leaſt, when $VE = \frac{1}{2} VB$, and EV cuts AG in Q. And is demonſtrated exactly the ſame way as Prop. XXIV. B. II. And hence

Cor. 9. *The circle whoſe diameter is 2QI, contains all the rays refracted by the lens, between A and B.*

Fig. 96. For EQ cuts AP in Q, at the greateſt diſtance from the axis VF.

Cor. 10. *Hence if the principal focal diſtance is given; the lateral aberrations are as the cubes of the breadths of the lens. And if the breadth of the lens be given, the lateral aberration is reciprocally as the ſquare of the focal diſtance.*

Cor. 11. *If the focal diſtances and apertures of a reflecting concave, and a convex lens, be the ſame. The diameter of the circle of aberration cauſed by their figures, will be above 30 times leſs in the reflector than in the refractor.*

For (by Cor. 1. Prop. XXIV. B. II. and Cor. 8, of this); theſe diameters are $\frac{B^3}{4RR}$, (where f the focal diſtance $= \frac{1}{2}R$), and $\frac{mm}{nn} \times \frac{y^3}{4aa}$ (where $f = 2a$), and $B = y$. And theſe are as $\frac{B^3}{16ff}$ and $\frac{mm}{nn} \times \frac{y^3}{ff}$; that is, as $\frac{1}{16}$ to $\frac{mm}{nn}$ or as 1 to $\frac{16mm}{nn}$, or as 1 to $\frac{16 \times 9}{4}$ or 36.

PROP. XLI.

97. *Let the ſine of incidence, going out of a denſe medium be n, the ſines of refraction, (into a rare medium) of the leaſt, the mean, and the moſt refrangible rays be p, m, q. Then if a plano convex lens be expoſed with the plain ſide to the ſun; the diameter of the circle of aberration* IK *(by the different refrangibility), is to the diameter of the lens* AB; *as* $q - p$, *to* $q + p - 2n$.

Becauſe all the rays fall perp. on AB, they are only refracted by the ſurface AVB. Draw the radius

Fig. 97. dius CAL, and the axis CVF, and RAG, SBH parallel to CV. And let AIf and BKf be the leaſt refrangible rays, and AFK, BFI, the moſt refrangible. Thro' I, K draw LH which will be perp. to Cf. Now ſince the arch AV is ſuppoſed to be very ſmall, the angle ACV is very ſmall, and conſequently the angles LAG, LAI, LAK are likewiſe very ſmall; and conſequently the lines LG, LI, LK are nearly as the ſines of theſe angles; therefore if LG $= n$, then will LI $= p$, LK $= q$: For LAG or RAC is the angle of incidence. Hence GI = (LI — LG =) $p - n$, IK = (LK — LI =) $q - p$. And IO $= \frac{1}{2}$IK $= \frac{q-p}{2}$. Therefore GO or AD = (GI + IO =) $p - n + \frac{q-p}{2} = \frac{p+q-2n}{2}$; and 2AD $= p + q - 2n$. Therefore 2AD or AB : IK :: $p + q - 2n : q - p$. And all the rays between A and B, paſs thro' IK.

Cor. 1. *Diameter of the circle of aberration* IK : *diameter of the lens* AB :: $\frac{q-p}{2} : m - n$.

For $p + q = 2m$, and IK : AB :: $q - p : 2m - 2n$: $\frac{q-p}{2} : m - n$.

Cor. 2. *The aberration* Ff : *focal diſtance* fD :: $q - p : m - n$.

For by ſimilar triangles, fO : OI :: fD : AD; and 2fO or fF : 2OI or IK :: fD : AD or $\frac{1}{2}$AB. And fF : fD :: IK : $\frac{1}{2}$AB :: $\frac{q-p}{2} : \frac{m-n}{2} :: q - p : m - n$.

Cor. 3. *In a plano convex glaſs the diameter of the circle of aberrations, or that which cuts all the rays, is the 55th part of the diameter of the glaſs. And*

Fig. *the aberration in length, is the 27th or 28th part of*
97. *the focal diſtance.*

For in glaſs $n = 50$, $p = 77$, $q = 78$. And AB : IK : : $\overline{m-n}$ $(27\frac{1}{2})$: $\frac{\overline{q-p}}{2}$ $(\frac{1}{2})$: : 55 : 1. And $fD : Ff : : \overline{m-n}\,(27\frac{1}{2}) : \overline{q-p}\,(1)$.

Cor. 4. *The diameter of the circle of aberrations, in which all the rays are contained, that affect the ſenſe, is hardly the 200th part of the diameter of the glaſs.*

The foregoing computation takes in all the rays, but ſince they are exceeding thin toward the circumference of the circle of aberrations, and exceeding denſe near the center; not above a quarter of that makes any ſenſible impreſſion. And therefore 4 × 55 or 220 to 1 will now be the ratio of AB to KI.

Cor. 5. *In different glaſſes, the aberrations are as the diameters of the glaſſes. And the angles of aberration are as the breadth of the glaſſes directly, and the focal diſtance, inverſely.*

96. Cor. 6. *Hence if b be the breadth of the lens, f the*
97. *focal diſtance; the aberration by the figure of the lens, is to the aberration by the different refrangibility; as* $\frac{mm}{\overline{m-n}} \times \frac{bb}{16ff}$ to $q - p$.

For Cor. 8. Prop. XL.) the diameter by the figure is $\frac{mm}{\overline{m-n}^2} \times \frac{AB^3}{8 \times 4VF^2} = \frac{mm}{\overline{m-n}^2} \times \frac{b^3}{32ff}$. And the diameter by the aberration (by Cor. 1. of this) is $\frac{\overline{q-p}}{2 \times \overline{m-n}} \times b$. Divide both by $\frac{b}{2 \times \overline{m-n}}$: and you have the ratio as above.

Cor. 7. *In the glaſſes of teleſcopes, the error cauſed by the figure of the glaſſes, is quite inconſiderable, and may be reckoned as nothing, in reſpect of the error by the different refrangibility.* Fig. 96. 97.

For in glaſs $n = 50$, $p = 77$, $m = 77\frac{1}{2}$, $q = 78$, and the error by the figure, is to the error by the different refrangibility, as $\frac{13.65bb}{ff}$ to 1. For $q-p = 1$, and $\frac{mm}{m-n} \times \frac{bb}{16ff} = \frac{77\frac{1}{2} \times 77\frac{1}{2}}{27\frac{1}{2}} \times \frac{bb}{16ff} = \frac{31 \times 77\frac{1}{2}}{11} \times \frac{bb}{16ff} = \frac{31 \times 155}{11 \times 32} \times \frac{bb}{ff} = 13.65 \times \frac{bb}{ff}$.

Ex. 1.

Suppoſe $b = 4$ inches, $f = 100$ feet $= 1200$ inches, then $\frac{13.65bb}{ff} = \frac{13.65 \times 16}{1440000} = \frac{218.4}{1440000}$, and the error by the figure is to the error by refraction, as 218.4 to 1440000, or as 1 to 6593. And if the rays that affect the ſenſe be only a quarter of this, (as in Cor. 4.), the ratio becomes as 1 to 1648.

Ex. 2.

Suppoſe $b = 2$ inches, $f = 240$ inches; and $\frac{13.65bb}{ff} = \frac{13.65 \times 4}{240 \times 240} = \frac{13.65}{120 \times 120} = \frac{13.65}{14400} = \frac{1}{1055}$; and theſe errors as 1055 to 1. And if we reckon only the mean rays; it will be no leſs than 263 to 1; a trifling quantity for the figure of the glaſs.

From hence it appears, that the leſſer the focal diſtance of the glaſſes are, the more conſiderable is the error proceeding from the figure of the glaſs. And therefore in microſcopes, the aberration ariſing

Fig. from the ſpherical figure of the glaſſes, may have 96. a ſenſible effect; by reaſon of the ſmallneſs of the 97. ſpheres, and the large apertures required therein.

Cor. 8. *If the breadth of a glaſs lens be* $\frac{2}{3}$ *the diameter of the ſphere; the aberration cauſed by the figure, is nearly equal to the aberration cauſed by the different refrangibility of the rays.*

For by Cor. 6. the aberration by the figure is to the aberration by the different refrangibility; as $\frac{mm}{m-n} \times \frac{bb}{16ff}$ to $q-p$. Put them equal, then $\frac{mm}{m-n} \times \frac{bb}{16ff} = q-p$, or in numbers, $\frac{9}{1} \times \frac{bb}{16ff} = 1$. And $9bb = 16ff$. And extracting the root, $3b = 4f = 4 \times$ radius in a double convex $=$ 2 diameters, and $b = \frac{2}{3}$ the diameter.

Cor. 9. *The aberration in any lens by the different refrangibility, will be nearly the ſame, as in a plano convex of the ſame focus.*

For in a double convex of equal radii, the refraction at each ſurface will be but half ſo much as at the convex ſurface of the plano convex, which has double the curvature.

SCHOLIUM.

Altho' glaſſes formed into the figures of the conic ſections or any other curves, could refract the rays accurately to one point. Yet ſince no method has been found out to give them theſe figures, by grinding them into their proper forms; and a glaſs may be very truly ground into the form of a ſphere. Therefore Opticians are obliged to make uſe of ſpherical glaſſes upon all occaſions. And if we had glaſſes in the form of any curve, we chuſe, yet none of them can, in the leaſt, alter the different Fig. refrangibility of the rays. Neither can any curve 96. refract the rays iſſuing from the collateral parts of an 97. object, to a ſingle point. And therefore ſince all the errors of glaſſes are to be attributed ſolely to the different refrangibility of the rays. Therefore ſpherical ſurfaces may be of the ſame uſe as the beſt curves; and being ſo eaſily made, they are more commodious for dioptrical uſes, than any other curves whatever.

BOOK IV.

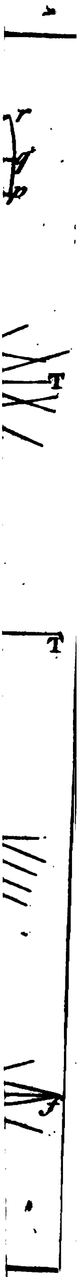
The Combination of GLASSES; *and the Conſtruction of* TELESCOPES *and* MICROSCOPES, *and other Optical Inſtruments.*

PROB. I.

Fig. *To explain the Nature and Conſtruction of the Eye.*

AS the ſenſe of ſeeing is the nobleſt ſenſe belonging to an animal, without which he would live in perpetual darkneſs, unable to perform the neceſſary exerciſes of life; and ſince all that has been demonſtrated of glaſſes, and will hereafter be demonſtrated of all optical machines, tend only to this ſingle end, to help and improve this ſenſe of ſeeing. And ſince the eye is the organ, or inſtrument by which viſion is performed; it is by its nature an optical inſtrument, and the foundation of all others. And therefore it cannot be amiſs to give a ſhort deſcription thereof.

98. ABCE is the eye, its figure ſpherical; by reaſon of which, it is eaſily moved any way in its ſocket. The fore part at A is more convex than the reſt. It is contained in three membranes, the outermoſt is the *Sclerotica*; the ſecond the *Tunica choroides*; the fore part of it is called the *Iris*, which conſiſts of many fibres like ſo many radii. The third or inner-



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innermoſt is called the *Retina*, which is nothing Fig. 98.
but the optic nerve, ſpread over the bottom of the eye.

In theſe are contained the three humours of the eye; the firſt is HAI, called the *Aqueous* humour, which is a thin liquor like water. The ſecond is FG the *Chriſtalline*, in the form of a lens, more convex behind towards C. Adjoining to this is the third KL, called the *Vitreous* humour.

The chriſtalline is more denſe than the vitreous, and the vitreous more denſe than the aqueous humor, and all together make a compound lens, which refracts the rays of light, iſſuing from an object PR, to the bottom of the eye, and there paints its image *pr* in the focus upon the retina, inverted.

The figure of the aqueous humour is a meniſcus, and ſo is that of the vitreous. The fore part of the ſclerotica is called the *Cornea*, and that part adjoining to it, is called the *White* of the eye. Within the cornea is a coat called the *Uvea*; in the middle of this is a hole O called the *Pupil*, to let in the rays of light. And the pupil is contracted or dilated by ſeveral muſcular fibres in the uvea or iris, according as more or leſs light falls on the eye, and that by an involuntary motion.

D is the optic nerve going from the retina to the common ſenſorium in the brain, where theſe images are perceived. This is not in the middle of the eye, but lies nearer the ſide E, which is towards the noſe.

Round the edge of the chriſtalline FG, is a ring of fibres, by help of which the diſtance AC is made longer or ſhorter, in order to bring the image *pr* upon the retina, for diſtinct viſion. And perhaps theſe fibres alſo make the chriſtalline more or leſs convex at the ſame time. This ring of fibres is called the *Ligamentum ciliare*; and its back part

Fig. 98. is black, to ſtifle the rays that are reflected upon it.

The eye is moved in the head by ſeveral muſcles inſerted in the ſclerotica, ſo that it may be quickly directed to any object.

If the image of an object does not fall upon the retina at *pr*, the viſion will be confuſed. If it fall ſhort, or nearer FG; then a concave lens that makes the rays more diverging will bring them to the retina. This is the caſe of purblind or ſhort-ſighted people, who are forced to look very near an object; or elſe they muſt uſe concave ſpectacles.

If the rays do not unite till they get beyond the retina, as in moſt old people; then a convex lens of a due focus, held between the eye and object, will make them converge, and fall upon the retina. Therefore long-ſighted people muſt uſe convex ſpectacles.

Altho' the image is painted inverted in the eye, yet we judge it erect; becauſe we always find it ſo by cuſtom and experience.

The diameters of images at the bottom of the eye, as *pr*, are proportional to the angles which the objects ſubtend at the eye, as POR; the ſame as in a lens; and are reciprocally as the diſtances of the ſame object viewed in different places.

The eye is in reality no more than a camera obſcura, for the rays of light flowing from all the points of an object, thro' the pupil of the eye, do by the refraction of the humours of the eye, paint the image thereof in the bottom of the eye. Juſt ſo it is in the camera obſcura, where all the rays refracted by a lens in the window ſhut, or paſſing thro' a ſmall hole in it, paint the image on the oppoſite wall. Some properties of the eye are theſe.

1. The eye can only ſee a very ſmall part of an object diſtinctly at once. For the collateral parts

of an object are not repreſented diſtinctly in the eye; and therefore the eye is forced to turn itſelf ſucceſſively to the ſeveral parts of the object it wants to view, that they may fall near the axis of the eye, where alone diſtinct viſion is performed. Fig. 98.

2. When any point of an object is ſeen diſtinctly with both eyes, the axes of both eyes are directed to that point and meet there. And then the object appears ſingle tho' lookt at with both eyes. For the optic nerves are ſo framed, that the correſpondent parts in both eyes, lead to the ſame place in the brain, and give but one ſenſation; and the image will be twice as bright with both eyes as with one. But if the axes of both eyes are not directed to the object, that object will appear double, as the pictures in the two eyes do not fall upon correſpondent or ſimilar parts of the retina.

3. The beſt eye can hardly diſtinguiſh a particle of matter that ſubtends at the eye, an angle leſs than half a minute. And very few can diſtinguiſh it when it ſubtends a minute. If the diſtance of two ſtars in the heavens be not greater than this, they will appear as one.

4. Young people can ſee nearer hand than old people; for as men grow old their eyes grow flatter; and to collect the rays of light at the retina, the object muſt be placed further off. And this is the reaſon that ſhort-ſighted people as they grow older ſee at a greater diſtance.

5. Tho' men may ſee diſtinctly at different diſtances, by altering the poſition and figure of the chriſtalline, yet they can only ſee diſtinctly within certain limits, and nearer than that, objects appear confuſed. But theſe limits are not the ſame in different people. A good eye can ſee diſtinctly, when the rays fall parallel upon it; and then the principal focus is at the bottom of the eye.

Fig. 98. 6. If a good eye views an object at the leaſt diſtance it can be ſeen diſtinctly, and then at twice that diſtance, and then at an infinite diſtance; there is about the ſame alteration made in the figure of the eye between the two laſt caſes, as there is between the two firſt.

7. If a ſhort-ſighted perſon can ſee an object diſtinctly at two diſtances, whereof one is double the other. As great an alteration muſt be made in the figure of their eyes to ſee it at an infinite diſtance, as there was made for the firſt two diſtances; or as great an alteration as is made in a good eye, to ſee diſtinctly from the greater diſtance to infinity. And therefore a ſhort-ſighted perſon can ſee diſtinctly at all diſtances with a ſingle concave glaſs, of a proper figure. And hence the cauſe of ſhort-ſightedneſs is not a want of power to change the figure of the eye; but that change is not great enough to accommodate it to all diſtances, taken from a point ſo near the eye.

8. A man can judge of a ſmall diſtance with a ſingle eye, by frequently obſerving how much variation is made in the eye to make the object diſtinct; and from this a habit of judging is acquired. But this cannot be done at great diſtances, becauſe tho' the diſtance be varied, the change in the eye becomes then inſenſible.

9. But a man can judge of greater diſtances with both eyes, than he can with one eye. For the eyes being at a diſtance from one another, as long as that diſtance has a ſenſible proportion to the diſtance of the object, one gets a habit of judging, by the poſition of the two axes of the eyes, which are always directed to that point. For different diſtances require different poſitions, of the axes, which depends on the motions of the eyes which we feel. But in very great diſtances, no judgment can be made from the motion of the

eyes, or their internal parts. Therefore we can only gueſs at the diſtance from the magnitude, colour, and the poſition of interjacent bodies. Fig. 98.

10. Whatever light falls upon that part of the retina, where the optic nerve D, ſprings, makes no impreſſion; and therefore if the picture of an object falls thereon, it is not perceived, and ſo that object is inviſible. This will appear by placing a ſmall bright object before you, and looking at it with one eye, and moving your eye laterally towards the contrary ſide (towards the left, if it be the right eye), the object will diſappear, and ſeem to be loſt; and moving it ſtill further, it will appear again. Now this place is not at the bottom of the eye, but nearer the noſe in both of them; ſo that no rays either parallel or diverging, that come from any object, can fall upon that place in both the eyes; ſo that any object we direct the eyes to, will always be viſible, at leaſt to one eye. But the ſame bright object may be made to diſappear to both eyes, by directing the axes of both eyes to a point a little beyond the noſe, to be found by trials.

11. Dimneſs of ſight generally attend old people, and this may ariſe from two cauſes, 1. by the eyes growing flat, and not uniting the rays at the retina, which cauſes indiſtinctneſs of viſion; or, 2. by the opacity of the humours of the eye, which in time loſe their tranſparency in ſome degree; from whence it follows that a great deal of the light is ſtopt and loſt, that enters the eye; and every object appears faint and dim.

12. As the rays of light flowing from an object, and painting its image upon the retina, is the immediate cauſe of ſeeing. So where there is no light, there can be no viſion; conſequently without light, the eye becomes a machine utterly uſeleſs;

Fig. 98. uſeleſs; as it can give us no manner of information of the exiſtence of bodies at a diſtance from us.

PROB. II.

To find the places and poſitions of the images formed by ſeveral lenſes, or reflectors.

There may be an infinite number of caſes, of this problem, according to the number of lenſes and their diſtances from one another; but any of them may be ſolved in general thus.

Draw a right line thro' the centers of all the lenſes, for their common axis. Then having the poſition of the object, find the focus of the firſt glaſs, or the place of the firſt image. This being had, and the place of the ſecond lens; find the other focus, or the conjugate focus of the ſecond lens, or the place of the ſecond image. This being found, and the place of the third lens; find its conjugate focus, or the place of the third image, and ſo on from one to another, as long as there are any lenſes, till you get the laſt image.

The places of the images being known, their poſition as to their being erect or inverted may be found thus. Take a point in the object above the axis, from which draw a line thro' the center of the firſt glaſs, and continue it to the focus, there you will have the firſt image of that point, and if it is above the axis it is erect; but if below, 'tis inverted. From that point draw a line thro' the center of the ſecond glaſs, to the ſecond image, and that gives the ſecond image of that point, which image is erect or inverted, according as that point is above or below the axis. After the ſame manner purſue the ray thro' the third glaſs, and then thro' the fourth glaſs, and ſo on to the laſt, where if the laſt image of the ſaid point be above the axis, the image is erect; if below, it is Fig. inverted. If there be any reflectors, purſue the ray, after the ſame manner, to the reſpective foci. The ſhorteſt way to find the conjugate foci, of all the glaſſes, ſucceſſively, is to get the principal foci firſt; and then theſe will be had by Prop. XXI. Book III.

Ex. 1.

Let AB be a reflecting concave, PQ an object, 99. draw the axis QD, and find the focus *q*. From P the higheſt point of the object, draw PD, and the reflected ray D*p*, then *p* is the image of P, and *pq* the image of PQ; and ſince *qp* is below the axis, the image is inverted.

Ex. 2.

Let A, B, C, be three lenſes, PQ an object. 100. Draw QH the common axis of all the lenſes; in the axis QH find the conjugate foci Q, F, of the lens A; and F, G, the conjugate foci of the lens B; and laſtly G, H the conjugate foci of the lens C. Take the point of the object P above the axis, and thro' A the center of the firſt lens, draw PA, and continue it to L at the focus F; then L is the image of P. Again, draw LBM thro' the center B of the ſecond lens, to the focus G; then M is the image of L, or the ſecond image of P. Laſtly, draw MCN thro' the center C of the third lens, to the focus at H; and N is the image of M, or the third image of P. And becauſe HN lies below the axis, or contrary to the poſition of PQ, the laſt image is inverted.

Ex. 3.

Let A, B, C, D, be four lenſes, PQ an object. 101. Draw QK the common axis of all the lenſes. In this axis find the conjugate foci Q, F, of the firſt lens; F and G, the conjugate foci of the ſecond

Fig. 101. lens; G and H, the conjugate foci of the third lens; laſtly, H and I the conjugate foci of the fourth lens, which is a concave. Take a point of the object P, above the axis, and draw PAL thro' the center of the firſt lens to the focus F; then L is the image of P. Draw LB to the center of the ſecond lens, cutting the perpendicular GM, at the focus G, in M; then M is the image of L, or the ſecond image of P. Here the rays, which by the glaſs A, converged to L; by the glaſs B are made to converge to M. Again, draw MCN thro' the center of the third glaſs, to cut the perp. HN at the focus H, in N; and N is the image of M, or the third image of P. Laſtly, draw ND to the center of the fourth glaſs, cutting the perp. IO, at the focus I, in O; and O is the image of N, or the fourth image of P. Here the rays diverging from N, and falling on the concave glaſs D, are made to diverge from O; juſt as the rays diverging from H, are made to diverge from I. And ſince IO is on the ſame ſide of the axis, as the object PQ, it will appear to be erect.

And thus the rays proceeding from an object, may be traced thro' any number of lenſes, to the place of the laſt image.

Cor. 1. *Hence, if part of the rays are ſtopt by the way, what remains will belong to the ſame focuſes as before; and the ſame images will be made by theſe partial pencils of rays that remain.*

For in the oblique pencils, a great many rays miſs the following glaſſes, and are loſt; and the more ſo, the more oblique they are. But that does not affect the reſt, which will ſtill go to their own focus.

Cor. 2. *Hence the proportion of the laſt image to the object is eaſily found. And that is, as* IO : PQ : :

ſo AF × BG × CH × DI : *to* QA × BF × CG × DH. Fig. 101.

For PQ to IO is compounded of the ratio of PQ to FL, FL to MG, MG to HN, and HN to IO; that is, of QA to AF, BF to BG, CG to CH, and DH to DI; or of QA × BF × CG × DH to AF × BG × CH × DI.

Cor. 3. *Likewiſe the apparent magnitude of the laſt image* IO *ſeen from any point* K : *is to the apparent magnitude of the object* PQ *ſeen from* A, *with the naked eye* : : *as* DI × CH × BG × AF : *to* KI × DH × CG × BF.

For the apparent magnitude of IO, to the apparent magnitude of PQ, is as the angle IKO to the angle QAP. But IKO to QAP is compounded of the ratios of IKO to IDO, and IDO or HDN to HCN, and HCN or GCM to GBM, and GBM or FBL to FAL or QAP; that is, of the ratios of DI to KI, and CH to DH, and BG to CG, and AF to BF; or as DI × CH × BG × AF to KI × DH × CG × BF.

Cor. 4. *Therefore where there are but three lenſes*, A, B, C; *and the eye at* C. *The apparent magnitude of the laſt image* GM : *is to the apparent magnitude of the object* PQ *ſeen from* A : : *as* BG × AF : *to* CG × BF.

PROB. III.

To find the place of the eye, where the viſible area of the object is the greateſt, when ſeen thro' ſeveral glaſſes. 102.

This will be in the place where the axes of all the pencils meet. Suppoſe A the centre of the firſt glaſs, to be a radiant point, and that the rays iſſuing from A, are refracted ſucceſſively by all the

Fig. 102. glaſſes BC, EF, &c. Then find the focus G made by refraction at the laſt glaſs EF; and G is the place of the eye. And this is found after the manner of the laſt Prob. by finding the conjugate foci A, D, for the glaſs BC; and then the conjugate foci, D, G, for the glaſs EF, and ſo on to the laſt, which is done either by Prop. XXI. or Cor. 2. Prop. XX. of Book III.

For ſince the glaſs A is the common baſe of all the pencils of rays that come from the object; the axis of every pencil, LA, OA, NA, will paſs ſtreight thro' the center A of the glaſs, and LAC, OAD, NAB, &c. will be right lines. Therefore A is the radiant point from which all theſe axes proceed. Therefore it follows of courſe, that all theſe axes or middle rays interſect again in the ſeveral foci D, G &c. And conſequently if the eye be placed at the laſt focus G, it will receive all theſe middle rays, and beſides as many of the collateral rays of every pencil, as the breadth of the pupil can admit. Therefore the viſible area of the object will be greateſt, when the eye is in the focus G. For if the eye be ſuppoſed at any other place, the eye will take in leſs, becauſe the ſection of all the rays becomes greater; (for it is the very leaſt at D), and fewer of them will fall on the pupil.

Cor. 1. *If the laſt glaſs* EF *be a concave, the eye muſt be put cloſe to the glaſs.*

For its focus G will fall between the glaſs EF and the point D; and the rays coming from it thro' the glaſs will diverge, and the ſection of the rays, on the outſide of the glaſs, will be the leaſt neareſt the glaſs.

Cor. 2. *The focus of the axes, of all the pencils, is a quite different thing from the focus of any one pencil.*

For the former has its radiant point in the centre Fig.
of the firſt glaſs; and the latter, in the object itſelf. 102.

Cor. 3. *For the ſame reaſon, if* A *be a concave reflector; and* A, D, *conjugate foci to* BC; D *will be the interſection of all the axes of the ſeveral pencils, or the place of the eye.*

PROB. IV.

Having given AF, *the leaſt diſtance that a long-* 103.
ſighted perſon can ſee diſtinctly; or the greateſt diſtance 104.
a ſhort-ſighted perſon can ſee diſtinctly; and AQ *the diſtance he wants to ſee diſtinctly. To find the radius of a glaſs fit for that purpoſe.*

Let AC be the radius of a double convex, or double concave; take $AC = \frac{AF \times AQ}{AF - AQ}$, for the radius of a convex lens, for the long-ſighted perſon; or for the radius of the concave, for the ſhort-ſighted perſon.

For if C be the principal focus of the double convex, or double concave lens; or which is the ſame thing, CA = the radius of the ſphere (by Art. 1. Schol. Prop. XIX. B. III.) Then (Prop. XXII. ib.) $QF : QA :: AF : CA = \frac{AF \times AQ}{QF} = \frac{AF \times AQ}{AF - AQ}$. Then the rays that come from Q, will enter the eye of the long-ſighted perſon, as if they came from F, and will therefore make the object at Q appear diſtinctly (fig. 103.) and the rays proceeding from Q (fig. 104.), will enter the eye of the ſhort-ſighted perſon, as if they came from F; and will therefore make the object at Q appear diſtinctly to him, when AC the radius of concavity $= \frac{AF \times AQ}{AF - AQ}$, or $\frac{AF \times AQ}{AQ - AF}$.

Fig. 103. Cor. 1. *A long-ſighted perſon that cannot ſee diſtinctly nearer than* F; *cannot ſee diſtinctly nearer than* Q, *by help of the glaſs* BD. *But from* Q *to* C, *or further, he can ſee diſtinctly.*

For as F approaches the glaſs, Q approaches it (Cor. 2. Prop. XXXV. B. III.); but nearer than F, the object is confuſed; therefore nearer than Q, the image is confuſed. But beyond F it is diſtinct, therefore beyond Q it is diſtinct.

Cor. 2. *If the long-ſighted perſon can only ſee objects diſtinctly, which are very remote; then he muſt have a convex lens whoſe focal diſtance is* AQ, *to ſee objects diſtinctly at the diſtance* AQ.

For in this caſe the radius is $\frac{AF \times AQ}{AF} = AQ$, becauſe AF is infinitely greater than AQ.

104. Cor. 3. *A ſhort-ſighted perſon that cannot ſee diſtinctly further than* F; *cannot ſee diſtinctly further than* Q, *by help of the concave glaſs* BD. *But can ſee diſtinctly from* Q *to* F, *or nearer.*

For as F and Q both recede from the glaſs; and beyond F, the object is confuſed, ſo is the image beyond Q; but on this ſide of Q it is diſtinct.

Cor. 4. *If* AF *be the greateſt diſtance, a ſhort-ſighted perſon can ſee diſtinctly; then by help of a concave, whoſe radius or focal diſtance is* AF, *he will be able to ſee all remote objects, to infinity.*

For in this caſe AQ is infinite, and the rays from Q will be parallel; and conſequently F is the focus of theſe parallel rays, and FA the radius of the glaſs. And in all remote objects, the rays are nearly parallel.

Cor. 5. *From theſe rules a man may be fitted with the moſt proper ſpectacles, tho' he lives at a diſtance*

from the maker; by ſending him the focal diſtances. Fig.
But if glaſſes are at hand, he may be better fitted by trial; taking notice to chuſe thoſe that are the leaſt convex or concave that will fit him. Theſe will prevent a perſon from growing more long-ſighted or ſhort-ſighted, for ſome time.

SCHOLIUM.

When glaſſes are put into frames for ſpectacles,
theſe frames ought not to be ſtreight, ſo as both
eyes may lie in the ſame plane; but they ought to
be ſo bent in the middle, that the axes of both
glaſſes may be directed to one point, at ſuch a diſ-
tance as you generally look with ſpectacles. By
this means the rays will fall perp. upon both glaſſes,
and make the object appear diſtinct. But if they
fall obliquely upon the glaſſes, it will cauſe a con-
fuſed appearance in the objects. Therefore the
ſhape of the frame ought to be as repreſented in
fig. 105, where AB is the plane where one glaſs is 105.
fixt, and CD the other.

Spectacles are the moſt uſeful optical machine in the world. Few people that grow into years, but have occaſion for them, to help their defective eyes, which without them would be uſeleſs, in a great many occaſions in life.

PROB. V.

To make a concave ſpeculum HD *of a given thickneſs* BD, *and whoſe inner radius* AB *is given; that ſhall reflect parallel rays* RH, *to the ſame focus* F, *from both ſurfaces* BH, DI. 106.

It is plain, that the ray which is reflected from the hindmoſt ſurface, will be twice refracted. For RH is firſt refracted at H to the focus *f*, and then

Fig. 106. reflected at I to the focus G, and laſtly refracted at L to the focus F.

Let AB $= a$, BD $= t$, DG $= x$, radius CD $= z$; m, n the ſines of incidence and refraction. Then to find f, the focus of the refracted ray HI; we ſhall have (by Cor. 1. Prop XIII. B. III.) $m - n : m :: \text{AB } (a) : \text{B}f = \frac{ma}{m-n}$; put $q = \frac{m}{m-n}$. Then to find the reflected ray IG, (by Prop. V. B. II.) Cf $(qa + t - z)$: CG $(z - x)$: : Df $(qa + t)$: DG (x). Whence $qax + tx - zx = qaz - qax + tz - tx$; and when reduced $x = \frac{qaz + tz}{2qa + 2t - z} =$ DG, and BG $= x - t = \frac{qaz + 2tz - 2qat - 2tt}{2qa + 2t - z}$.

But (Prop. VI. B. II.) BF $= \frac{1}{2}a$. And ſuppoſing the refracted ray ILF to go back again, and be refracted along LI, whoſe focus is G; then (Cor. 1. Prop. XI. B. III.) $m \times \text{BF} + n \times \text{AF} : m \times \text{BF} ::$ AB : BG. That is $\frac{1}{2}ma + \frac{1}{2}na : \frac{1}{2}ma :: a : x - t$, or $m + n : m :: a : \frac{qaz + 2tz - 2qat - 2tt}{2qa + 2t - z}$, and $\overline{ma} \times \overline{2qa + 2t - z} = \overline{m + n} \times \overline{qaz + 2tz - 2qat - 2tt}$. And putting $s = \frac{m}{m + n}$, we ſhall have $qaz + 2tz + saz = 2sqaa + 2sat + 2qat + 2tt$, and $z = \frac{2sqaa + 2sat + 2qat + 2tt}{qa + sa + 2t}$, and if m, and n be 3 and 2, then $z = \frac{9aa + 18at + 5tt}{9a + 5t}$.

Cor. 1. CD *or* $z = a + \frac{13}{9}t - \frac{tt}{4a}$ *very near*; *as will appear by diviſion*; *and therefore if* AD *or* $a + t$ *be ſubtracted, there will remain* AC $= \frac{4}{9}t$, *nearly.*

Cor. 2. *Hence alſo if z be given and a required,* Fig.
it may be found by reſolving the equation $9az + 5tz$ 106.
$= 9aa + 18at + 5tt$. *Or if z and a be given, t may be found from the ſame equation.*

Cor. 3. *If ſuch a ſpeculum could be exactly ground, it would be better than any reflecting metal, not only becauſe it would reflect more light, but it would preſerve the poliſh longer. But the difficulty of executing them truly, has prevented the uſe of them.*

SCHOLIUM.

If inſtead of 3 and 2, 31 and 20 be put for *m* and *n*, as being more exact; we ſhall find *z* or CD $= \frac{2qs}{q+s}a + \frac{2qq+2ss}{\overline{q+s}^{2}}t$, nearly. And if $a+t$ be ſubtracted, then AC $= \frac{2sq-q+s}{q+s}a + \overline{\frac{q-s}{q+s}}^{2}\,t$, nearly. According to this computation, we ſhall have AC $= .416t$, which is not much different from the former, which was $\frac{4}{9}t$ or $.444t$.

The principal difficulty is to find the vertex of this reflector after it is made, or the point where it is the thinneſt. To do this, cover the ſpeculum with a paper, having a hole in it of the bigneſs the ſpeculum is to be when finiſhed, or when put into a teleſcope. Then it muſt be placed at a good diſtance from a candle, and the image received on a bit of paper juſt big enough to receive it. And moving the paper cover to all places of the ſurface, till you find ſuch a poſition for it, as makes the image the moſt diſtinct poſſible, there fix it; and the middle of the hole is the vertex. For tho' the two ſurfaces be ground very truly to their proper radii, yet if the centers are not in a right line, paſſing thro' the middle of the reflector, the two images

Fig. 106. images cannot coincide, and they will confuſe one another.

If a concave reflector be made after this manner, it will burn bodies in its focus more fiercely than any other ſort; for both the reflected and refracted rays meeting there, conſpire to make the effect greater.

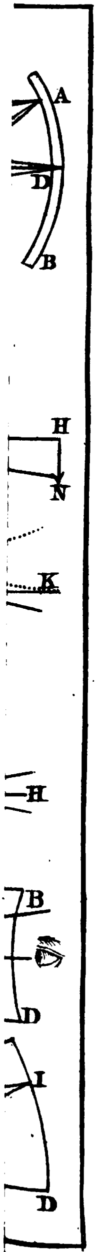
PROB. VI.

107. *To make a teleſcope with a convex and a concave glaſs, to ſhew objects erect; called,* Galileo's *Teleſcope.*

A Teleſcope is an optical inſtrument made by the combination of glaſſes, for ſeeing objects at a great diſtance. The glaſs next the object, is called the *object glaſs*; and that next the eye, is the *eye glaſs*. The *aperture* is the diameter of the hole through which the rays paſs. And by reaſon of the great diſtance of the object, all rays of the ſame pencil, incident on the object glaſs, muſt be eſteemed to be parallel; for the eye can diſcover no manner of difference between theſe and parallel rays.

To make this teleſcope, fix the convex lens AB in the tube AEFB, and the concave lens CD in the tube CGHD, the concave lens muſt be of a leſs focal diſtance than AB, and the tube GD muſt be made to ſlide within the tube AF; and the glaſſes muſt be ſo placed, that the axes of the tubes and glaſſes may coincide; then ſliding the tube GD within the tube AF, till the diſtance of the glaſſes AC be equal to the difference of their principal focal diſtances, or that both their principal foci may coincide in the ſame point P; and then the teleſcope is fit for uſe.

The teleſcope being fitted thus, direct it with the axis RP ſtreight upon the object, and apply the



pa.170.

NIV

the eye cloſe to CD, and you'll ſee the object magnified, and erect. Fig. 107.

For ſince the rays proceeding from the object tend to the focus P; and before they arrive at it, are refracted by the concave CD; therefore they do not croſs one another before they come at the eye, and conſequently the image is erect. Or thus, ſince the conjugate focus to P (in the lens CD) is ſomewhere on the other ſide of CD, therefore if a line be drawn from Q, the lower part of the image at P (which is there inverted), thro' the middle of the lens CD, it will come to that conjugate focus above the axis, and therefore the image is erect.

The properties of this teleſcope.

1. The focal diſtance of AB muſt be greater than that of CD; or elſe it will not magnify an object. If AB be leſs, it will diminiſh it.

2. The magnifying power is equal to the quotient ariſing by dividing the focal diſtance of the object glaſs, by the focal diſtance of the eye glaſs $= \frac{AP}{CP}$.

3. The rays of any pencil coming out of the glaſs CD to the eye, are nearly parallel. If this does not ſuit all eyes, the tube GD muſt be put in, or drawn out a little, till the object appears diſtinct.

4. The viſible area of the object is the greater, the nearer the eye is to the glaſs; and depends on the breadth of the pupil of the eye, and of the object glaſs. Therefore the view will be ſmall.

5. In order to ſee more of the object, the eye muſt be moved laterally, or up and down, the teleſcope remaining fixt. By this means one may ſee ſucceſſively all the parts, that the teleſcope can take in at once, and that depends upon the breadth of the eye glaſs.

6. This

Fig. 107. 6. This ſort of teleſcopes muſt not be made long; for then the viſible area is ſo little, that they become uſeleſs.

For if you ſhut your eye for a while that the pupil may dilate, as it will in the dark; and then ſuddenly opening it, and looking ſteadily at an object thro' the teleſcope, you'll ſee it largeſt at firſt, and afterwards it gradually leſſens, as the pupil contracts by the light. Alſo if you cover the object glaſs with a paper with a hole in it; leſs of the object will appear than before; and ſo much leſs as the hole is leſs. And therefore the viſible area depends both upon the breadth of the pupil, and of the object glaſs.

Alſo if a paper be put over the eye glaſs, having a ſmall hole in it; leſs of the object will appear, by moving to and fro, ſideways; than did before, thro' the whole eye glaſs. And if the hole be leſs than the pupil, the direct view will be leſs.

In a long teleſcope of this ſort, the viſible area will be very ſmall, by the narrowneſs of the object glaſs. And if the object glaſs be made broader, the object will be coloured, and conſequenly confuſed.

This teleſcope is the ſame as is generally called a *common perſpective glaſs*.

PROB. VII.

108. *To make a teleſcope with two convex glaſſes, to ſhew objects inverted; called the* Aſtronomical Teleſcope.

Let A, B be two convex lenſes, HC, DE two tubes ſliding within one another. Fix the lens A with the longer focus, in the wider tube HC; and the lens B with the ſhorter focus, in the ſmaller tube DE; and let the axes of the glaſſes coincide with the axis of both tubes. Then ſliding the tube DE, within the other, till the diſtance of the lenſes be equal to the ſum of their principal focal diſtances; or that their principal foci may coincide in the ſame point P; and near P there muſt be a ſtop, with a hole to circumſcribe the image, and to exclude all ſtraggling rays. Fig. 108.

And then the teleſcope is fit for uſing, which is done by directing it ſtreight to the diſtant object; and then applying the eye beyond B, the diſtance of its focus; the object will appear magnified and inverted.

For the rays from the object R, croſſing at A, will form the image at the focus P, inverted; which viewed thro' the lens D, will therefore appear inverted. And P being the principal focus of B, the emerging rays will fall parallel upon the eye, and cauſe diſtinct viſion.

The properties of this teleſcope.

1. The magnifying power is as the ratio of the focal diſtance of the object glaſs A, to that of the eye glaſs B.

2. The rays emerging out of the eye glaſs to the eye, muſt be nearly parallel, for a good eye. If this happens not to ſuit any other eye; then the tube DE muſt be thruſt in or pulled out, till the object appears diſtinct.

3. The apparent magnitude of the object is the ſame wherever the eye is placed, by Prop. 37. B. 3. But the viſible area is the greateſt, when the diſtance BO of the eye is nearly equal to BP, the focal diſtance of the eye glaſs. By Prob. III.

4. The object in this teleſcope is always inverted; and therefore it is only applied to aſtronomical uſes, to look at the celeſtial bodies. For it ſignifies little whether they appear erect or invert-

ed,

Fig. ed, in theſe caſes. And they are commonly uſed
108. in the night.

5. The viſual angle depends on the breadth of the eye glaſs. For it is equal to the angle which the eye glaſs ſubtends at the object glaſs.

PROB. VIII.

109. *To make a teleſcope with three convex glaſſes to ſhew objects erect.*

Combine the two glaſſes A, B, ſo that FA may be the principal focal diſtance of A; and FB twice the principal focal diſtance of B, and BP the ſame; let theſe be fixt in one tube. Then place the lens C, in another tube ſliding within the firſt, that PC may be its principal focal diſtance, which may be done by ſliding the inner tube back or forward; and then the teleſcope is finiſhed. The eye muſt be placed at the diſtance of 2PC behind C.

For parallel rays proceeding from the object R, and falling on the lens A, are refracted to the focus F, and there give the image inverted. And (Prop. XXI.) F, P are conjugate foci, and (Cor. Prop. XXIII.) they are at the leaſt diſtance poſſible. And ſince the image at F is inverted, that at P will be erect. And ſince PC is the focal diſtance of C, the rays falling on it, will emerge parallel, upon the eye; which therefore will ſee the object erect.

This teleſcope is of little uſe, by reaſon it takes in ſo little of the object; and makes the viſible area ſo ſmall.

SCHOLIUM.

A teleſcope may be made with two convex glaſſes to ſee objects erect. And that is by combining the two lenſes A, B, as before, which will give the image at P erect, which image may be ſeen by an eye placed ſix or eight inches behind it. Fig. 109. This teleſcope is the ſame as the former, when the glaſs C is taken away; but it takes not in ſo much of the object as the other, and therefore is of no uſe for teleſcopes.

A teleſcope may alſo be made with two convex lenſes, and a concave, which will ſhew objects inverted. Here the concave is the middlemoſt, being put inſtead of B. And they are placed thus, the foci of A and B muſt not coincide as in Prob. V; but they muſt be a little further aſunder, the focus of B being further from A than its focus is; and ſo far, that the conjugate focus (of B) may be at P; then if the eye glaſs C be placed, that its focus may be at P, then looking thro' the teleſcope the object will appear diſtinct, and inverted. This teleſcope, has full as great magnifying power as any refracting teleſcope; but becauſe the object is inverted, it is not uſed. The conſtruction of this teleſcope is the ſame as the ſolar teleſcope in Prob. XIII. following, fig. 114, having a concave eye glaſs L. For if *i* be its focus, and *b* the focus of AB; and *b*, K conjugate foci of L. Then K is the focus of a convex eye glaſs for this teleſcope.

PROB. IX.

To make a teleſcope with four convex glaſſes to ſhew objects erect.

110. Fix the lens A of a great focal diſtance in a wide tube, and fix the three ſmall lenſes or eye glaſſes B, C, D in another tube, to ſlide within the former. The lenſes B, C, D, muſt be all of one focal diſtance, and muſt be placed equidiſtant in the tube, ſo that BC, and CD, may be twice the focal diſtance. Then if CP = PD, P will be the principal focus of C and D. A ſtop muſt be

Fig. placed at P or F, to ſtop the ſtraggling rays.
110. Then ſlide the tube with the three eye glaſſes, within the other, till the foci of A and B coincide in F. Then the teleſcope is made, then directing it to the object R, and placing the eye in the other principal focus of D; and the object will appear to it magnified and erect.

For parallel rays coming from the far diſtant object R, will be refracted by the object glaſs A to its focus F, and there make the image inverted. Then ſince F is the principal focus of B, the rays diverging from the image at F, and falling on the lens B will emerge parallel, and fall parallel on the glaſs C, and therefore will be refracted by it, to its principal focus P, and there form the image erect. Then the rays flowing from P the principal focus of D, will emerge a third time parallel, in falling on the eye; which (by Prob. III.) muſt be placed at the principal focus; and then the eye will diſtinctly ſee the image P of the object R, erect and magnified. Here then the firſt image is at F inverted, and the ſecond at P erect, from whence rays flow to the eye.

If BC be leſs than twice the focal diſtance of B or C, then ſome of the rays will miſs the eye glaſs D. And if BC be greater than twice the focal diſtance, ſome of the rays will miſs the glaſs C. And in either caſe the proſpect will be diminiſhed.

This teleſcope with four glaſſes is in effect two teleſcopes, of two glaſſes each, as in Prob. VII. The firſt teleſcope is AB, where the rays emerge parallel from B. The ſecond is CD, where the rays fall parallel on C, and emerge parallel from D. And ſince C and D are equal, they magnify nothing, and therefore the image at P is equal to the image at F. All that the glaſſes C, and D do, is to invert the image at F, or make the image of R erect. This is the common teleſcope uſed in the day time to look at diſtant objects. Fig. 110.

If a ſtop be placed at P, with a hole to contain the image; it will cut off the reſt of the rays which come irregularly, and would diſturb the diſtinctneſs of the image; and will take away the colours which ariſe by the different refrangibility of the rays, which would make the image confuſed. But the image will no more be coloured by four glaſſes than by two; for the glaſs C corrects and takes away the colours which the glaſs B produced.

The properties of this teleſcope.

1. The magnifying power, is in the proportion of the focal diſtance of the object glaſs, to the focal diſtance of the eye glaſs. This will be ſhewn afterwards.

2. It ſhews the object erect, but not ſo bright as in the teleſcope with two convex lenſes, becauſe the other two reflect a deal of the light which is loſt. In other reſpects it has the ſame properties as the other teleſcope.

3. The viſible area depends on the breadth of the eye glaſs, for as the eye glaſs is bigger or leſſer, the angle it ſubtends at A is greater or leſſer, or its oppoſite angle, which is the angle the part of the object ſubtends at A, which determines the viſible area. And this teleſcope takes in more than that in Prop. VI, with a concave eye glaſs.

4. The brightneſs of an object ſeen thro' a given teleſcope is ſo much the greater as the aperture of the object glaſs is greater. For ſuppoſe the object glaſs covered with paper, with a ſmall hole in the middle; the magnitude of the image in the focus will not be altered, nor conſequently that upon the retina of the eye. But the hole in the paper being ſmaller than the object glaſs, fewer rays of every pencil will come to the focus, and the image will be more faint.

Fig. 110. 5. If the aperture and focal diſtance of the object glaſs remains the ſame; an object appears brighter or fainter, according to the greater or leſſer focal diſtance of the eye glaſs; that is according to the leſſer or greater magnifying power. For the ſame quantity of light ſpread over a ſmaller or greater image, makes it brighter or duller.

6. It is found by experience, that an object glaſs of 55 feet focal diſtance will bear an aperture of 4 inches, and an eye glaſs, whoſe focal diſtance is 4.4 inches. And for other object glaſſes, the apertures thereof, and the focal diſtances of the eye glaſſes, muſt be as the ſquare roots of the focal diſtances of the object glaſſes. But to view objects by day light, requires eye glaſſes whoſe focal diſtances are longer; and by night ſhorter.

SCHOLIUM.

It may be thought, that in a teleſcope of any length, an object may be magnified in any proportion, by making the focal diſtances of the object glaſs and eye glaſs in that ratio. But there are two cauſes which prevent this. 1. Whilſt the object glaſs remains the ſame, the more the object is to be magnified, the more convex the eye glaſs muſt be, and then the more obſcure and dark the object will appear. 2. By this means the object will be repreſented the more confuſedly; and if one attempts to remedy this by increaſing the aperture, the confuſion will be more increaſed, and the objects will at laſt appear coloured. So that increaſing the aperture of the object glaſs, makes the object appear brighter, but at the ſame time it is confuſed and indiſtinct.

Since the pupil dilates itſelf very much in the dark, therefore there is not occaſion for ſo much light in looking at the moon or ſtars.

There are teleſcopes made with five or ſix or more glaſſes, which by refracting the rays gradually to a focus, will bear a larger aperture; but then the object is more faint, by the loſs of ſo many rays, by the reflection from ſo many glaſſes. Fig. 110.

Sometimes the eye glaſs D is of a different focus from B and C. But the diſtance BC muſt ſtill be the ſum of the focal diſtances of C and D. Thus ſhorter eye glaſſes are uſed in the night, to increaſe the brightneſs of the object.

PROB. X.

To find the magnifying power of a teleſcope.

Let the far diſtant object appear from A under the angle RAS, or its oppoſite GAF, drawn thro' the center of the object glaſs A. And let B be eye glaſs, FG the image in the focus of the glaſſes A and B. Then thro' the extremes F, G, of the image, draw the lines FBL, GBN, thro' the center of the lens B, and BL, BN will be the refracted rays (by Prop. XVII. B. III.) Then the object is ſeen thro' the teleſcope under the angle LBN or FBG, and it is ſeen from A or from B under the angle RAS or GAF. Therefore the apparent angle thro' the teleſcope, is to the apparent angle to the naked eye; as angle GBF, to the angle GAF; that is as AF to BF; that is, as the focal diſtance of the object glaſs, is to the focal diſtance of the eye glaſs. And if there be four convex glaſſes; if the glaſſes B, C, D, are all equal (fig. 110), the image at P is equal to the image at F, and the magnifying power continues the ſame as before. But if they be different glaſſes, the image at F will be to the image at P, as the focal diſtance of B, to the focal diſtance of D. So that at laſt, the apparent angle thro' the teleſcope, will be to the 111. 112.

Fig. apparent angle to the naked eye; as the focal diſ-
111. tance of the object glaſs, to the focal diſtance of
112. the laſt eye glaſs. See alſo the Corollaries of
Prob. II.

Cor. 1. *The magnifying power by a refracting teleſcope, is equal to the focal diſtance of the object glaſs divided by the focal diſtance of the eye glaſs.*

111. Cor. 2. *In a teleſcope with convex glaſſes; the viſual angle or linear view, is as the diameter of the eye glaſs, divided by the focal diſtance of the object glaſs. And the viſible area as the ſquare of that quantity.*

This depends upon the breadth of the eye glaſs, therefore drawing AD to the edge of it, BAD will be half RAS or half the viſual angle; and that angle BAD or its double is as $\frac{BD}{BA}$; and the viſible area as $\frac{BD^2}{BA^2}$. In the teleſcope with a concave glaſs (fig. 112.) it depends on the breadth of the pupil.

Cor. 3. *As the diſtance between the object glaſs and eye glaſs, to the breadth of the eye glaſs; ſo is radius, to the tangent of the angle which the teleſcope takes in.*

For all the teleſcope can take in, is contained between two lines drawn from the edges of the eye glaſs, through the center of the object glaſs. The contained angle is what the teleſcope can take in.

PROB.

PROP. XI.

To find the degree of indiſtinctneſs, or confuſed appearance of an object, ſeen thro' a teleſcope.

Since no lens can be made ſo perfect, as to refract all the rays iſſuing from one point of an object, to a ſingle point in the picture, but there will always be a deviation or aberration more or leſs, according as the lens is broader or narrower. And by reaſon of this aberration, the rays proceeding from one point will be ſpread over a certain ſpace of the image, and cauſe that confuſion or indiſtinctneſs obſerved in teleſcopes; which ſpace is the circle of aberration in the focus of the object glaſs. And this confuſion will be ſo much the greater, as that circle is larger, becauſe there are ſo many more different points of that ſpace, which repreſent but one point of the object; and therefore that confuſion or indiſtinctneſs is, *ceteris paribus*, as the circle of aberration in the focus of the object glaſs; that is, by (Cor. 5. Prop. XLI. B. III.) as the area of the object glaſs.

Now the confuſion of the image in the bottom of the eye (which is that we ſeek) is as the apparent magnitude of this circle of aberrations. And the apparent diameter of aberration ſeen thro' the eye glaſs, is as the angle made by the extreme rays in falling upon the eye, or (which is equal to it, by Cor. 2. Prop. XVII. B. III.) as the angle ſubtended by that diameter at the center of the eye glaſs, that is, as the diameter itſelf directly, and the focal diſtance of the eye glaſs reciprocally. Therefore the apparent magnitude of the circle of aberrations, is as the ſquare of that diameter directly, and the ſquare of the focal diſtance of the eye glaſs reciprocally; or as the ſquare of the aper-

Fig. ~~ture of the object glaſs directly, and the~~ ſquare of the focal diſtance of the eye glaſs reciprocally. And therefore the confuſion or indiſtinctneſs is in that ratio.

Cor. 1. *~~The indiſtinctneſs of an~~ object ſeen thro' a refracting teleſcope, is as the ſquare of the aperture of the object glaſs, divided by the ſquare of the focal diſtance of the eye glaſs.*

Cor. 2. *An object appears equally diſtinct in ſeveral refracting teleſcopes, when the diameters of the apertures of the object glaſſes, are in a given ratio to the focal diſtances of the eye glaſſes.*

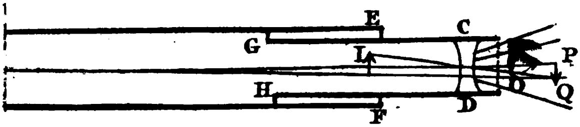
PROB. XII.

To find the apparent brightneſs of an object, ſeen thro' a refracting teleſcope.

If the area of the image upon the retina be the ſame, the brightneſs will be as the quantity of light upon it, that is, as the area of the aperture of the object glaſs. And if the aperture or quantity of light be the ſame, the brightneſs will be reciprocally as the area of the image, that is, reciprocally as the ſquare of the magnifying power. And therefore in general the brightneſs is as the area of the object glaſs directly, and the ſquare of the magnifying power reciprocally.

Otherwiſe thus.

113. Let two pencils of rays falling on the object glaſs CD, be refracted to the foci F, *f*; and paſs thro' the eye glaſs B*b*; and be refracted in the directions BL, *b*l. Let BL = FB, then L is the place of the eye, being the place where the ſeveral



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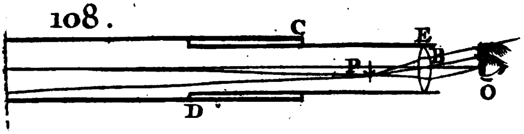

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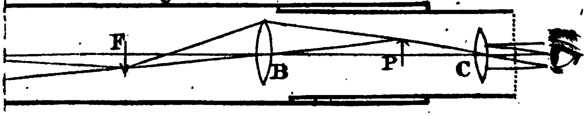

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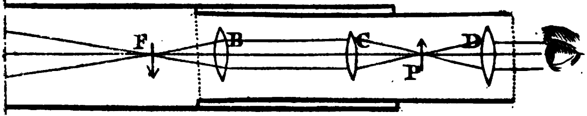

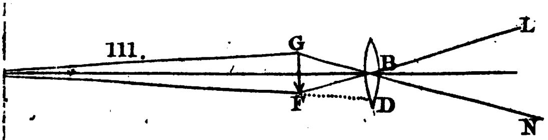

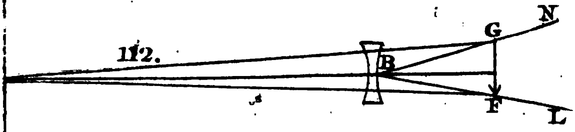

Pl. IX. *pa.182.*

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ral pencils (each conſiſting of parallel rays) converge to, and interſect, after refraction at B*b*. Fig. 113.

Let *m* = magnifying power, *p* = diameter of the pupil. Then by ſimilar triangles, AF : FB :: CD : B*b* or LI the breadth of any pencil of rays at B, or at L where the eye is; becauſe beyond B, the rays of each pencil are parallel. Therefore the breadth $\text{LI} = \frac{\text{FB}\times\text{CD}}{\text{AF}} = \frac{\text{CD}}{m}$.

Let the degree of common light be = 1, the area of the picture in the naked eye = *a*, then thro' the teleſcope it will be = *mma*. Then ſince the brightneſs is as the area of the aperture divided by the area of the picture, the brightneſs of the object by the naked eye will be $\frac{1\times pp}{a}$; and by the teleſcope, $\frac{\text{CD}^2}{mma}$. Therefore common light : to teleſcopic light :: $\frac{pp}{a} : \frac{\text{CD}^2}{mma} :: 1 : \frac{\text{CD}^2}{mmpp}$. And ſince the common light is 1, the teleſcopic light will be $\frac{\text{CD}^2}{mmpp}$.

Now ſuppoſe LI to be juſt equal to *p*. Then $pp = \frac{\text{CD}^2}{mm}$, and in this caſe the teleſcopic light becomes 1, and is therefore equal to common light.

But diminiſhing CD, LI is alſo diminiſhed, and therefore in this caſe, leſs light enters the pupil; and therefore in all caſes where LI is leſs than *p*, the brightneſs = $\frac{\text{CD}^2}{mmpp}$.

But if CD be increaſed never ſo much, the brightneſs is not increaſed at all by the ſide rays, becauſe they will all miſs the pupil. For as CD increaſes in breadth, B*b* and LI alſo increaſe in breadth, and will therefore, in this caſe, be bigger

Fig. 113. than the pupil. And hence the denſity of the light is the ſame as before. For there is a greater quantity of light on a greater ſpace CD, or on a greater ſpace B*b* or LI; and the quantity of light being as the ſpace, the denſity is not increaſed.

By ſome experiments, I have found $\frac{1}{6}$ part of the light loſt in paſſing thro' two glaſſes; and near $\frac{1}{3}$, in paſſing thro' four, and more ſtill in paſſing thro' five or ſix. This happens by the reflexion of the two ſurfaces of every lens, which reflect a part of the light, that is loſt; and ſo much is the brightneſs decreaſed.

Therefore in general the brightneſs is increaſed, by increaſing the object glaſs, till LI becomes equal to the breadth of the pupil; but no longer.

Cor. 1. *If an object is ſeen thro' a refracting teleſcope, and the rays do not fill the pupil; its apparent brightneſs is as the ſquare of the aperture of the object glaſs, multiplied by the ſquare of the focal diſtance of the eye glaſs, and divided by the ſquare of the focal diſtance of the object glaſs.*

This follows from the firſt method. For the magnifying power is as $\frac{\text{AF}}{\text{BF}}$; and therefore the brightneſs is as CD^2, divided by $\frac{\text{AF}^2}{\text{BF}^2}$, or as $\frac{\text{CD}^2 \times \text{BF}^2}{\text{AF}^2}$. Or by the ſecond method, the brightneſs is as $\frac{\text{CD}^2}{mmpp}$, p being given.

Cor. 2. *According to this; an object appears equally bright, thro' ſeveral teleſcopes, when the apertures are as the magnifying powers.*

Cor. 3. *The ſame thing ſuppoſed; in different teleſcopes, an object will appear equally bright and diſtinct,*

tinct, when the apertures (of the object glaſſes), and focal diſtances of the eye-glaſſes, are each of them as the ſquare roots of the focal diſtances of the object glaſſes. And then the magnifying powers will alſo be as the ſquare roots of the focal diſtances of the object glaſſes.

Fig. 113.

For put F, f for the focal diſtances of the object glaſs and eye glaſs; and that the brightneſs and diſtinctneſs may be in the ſame ratio, (by Cor. 2. of the laſt Prop. and Cor. 2. of this), we ſhall have the aperture $\propto f \propto \frac{F}{f}$, whence $F \propto ff$, and $f \propto \sqrt{F}$, conſequently the aperture $\propto \sqrt{F}$. And laſtly, the magnifying power $\propto \left(\frac{F}{f} \propto\right) \frac{ff}{f}$ or as f, that is as $\sqrt{F}$.

Cor. 4. *In ſimilar teleſcopes; the diſtinctneſs, viſual angle and magnifying power are the ſame; and the brightneſs as the ſquare of the length.*

For let D be the diameter of the aperture; F, f the focal diſtances of the object glaſs and eye glaſs; then the diſtinctneſs is $\frac{D}{f}$, which is a given ratio. And the magnifying power is $\frac{F}{f}$, which is alſo a given ratio. And the viſual angle is as the diameter of the eye glaſs divided by F, which is a given ratio. And laſtly, the brightneſs is as $\frac{CD^2}{mm}$ or $\frac{DD}{mm}$, and m or the magnifying power is given; and the aperture CD is as the length of the teleſcope.

Cor. 5. *Hence, no teleſcope can ſhew an object ſo bright as it is to the naked eye. The neareſt it can approach to it is $\frac{5}{6}$ the light for two glaſſes, and $\frac{2}{3}$ for four glaſſes.*

Fig. 113. For when all the rays enter the eye, the teleſcopic light, or what paſſes thro' the teleſcope into the eye, is $\left(\frac{CD^2}{mmpp} =\right) 1$, or the ſame as common light. Of which $\frac{1}{6}$ or $\frac{1}{7}$ is to be deducted for reflected rays, which are loſt.

Cor. 6. *According to what has been delivered before (after Prob. IX. and in Cor. 3 of this), the focal diſtances and apertures muſt be made according to the following table.*

foc. obj. glaſs	aper. obj. glaſs	foc. eye glaſs	mag. power	foc. obj. glaſs	aper. obj. glaſs	foc. eye glaſs	mag. power
feet	inch	inch		feet	inch	inch	
1	.54	.60	20	35	3.20	3.52	119
2	.76	.83	28	40	3.41	3.75	128
3	.94	1.03	34	45	3.62	3.98	135
4	1.08	1.19	40	50	3.82	4.20	143
5	1.22	1.34	44	55	4.00	4.40	150
6	1.32	1.45	49	60	4.18	4.60	156
7	1.43	1.57	52	70	4.52	4.97	169
8	1.53	1.68	56	80	4.83	5.31	181
9	1.62	1.78	60	90	5.13	5.64	192
10	1.71	1.88	64	100	5.40	5.94	202
13	1.95	2.14	73	120	5.91	6.50	222
15	2.09	2.30	78	140	6.39	7.03	239
20	2.42	2.66	90	160	6.83	7.51	255
25	2.70	2.97	101	180	7.25	7.97	271
30	2.96	3.25	110	200	7.64	8.40	216

SCHOLIUM.

It follows from Cor. 3, that if a parabola be deſcribed, and in the axis thereof, taking the abſciſſa's equal to the focal diſtances of the object glaſſes; then the correſpondent ordinates will be proportional to the apertures, or the focal diſtances of the eye glaſſes, or the magnifying powers. For theſe are as the ſquare roots of the abſciſſæ, that is, as the ſquare roots of the focal diſtances of the object glaſſes. And inſtead of the focal diſtances of the object glaſſes, one may ſubſtitute the length of the teleſcope, without ſenſible error. Fig. 113.

In the inveſtigation of the foregoing Problems, the indiſtinctneſs ariſing from the aberrations cauſed by the eye glaſſes are ſet aſide as being inconſiderable. For in ſo ſhort a diſtance as from the eye glaſs to the eye, the heterogeneous rays cannot ſeparate ſo as to become ſenſible, and therefore the errors may be lookt upon as nothing. And the refractions at ſo ſmall a diſtance will be ſufficiently true and regular.

To try the goodneſs of an object glaſs; proceed according to the ſecond method laid down in Prop. XVI. B. III. And if all the holes unite in one, the glaſs is good, otherwiſe not. But there is no better way for trying the goodneſs of an object glaſs than putting it in a tube, and trying it with ſeveral ſmall eye glaſſes, by looking at ſeveral diſtant objects, and particularly at the title page of a book. For that glaſs which repreſents objects the moſt bright and diſtinct, and bears the greateſt aperture, and the ſhorteſt eye glaſs, without colouring or dimneſs, is the beſt glaſs.

If ſeveral teleſcopes of the ſame length be compared together, theſe are the beſt, with which you can read the ſame print, at the greateſt diſtance. And this may be a rule for thoſe that buy teleſcopes, by which they may know how to chuſe the beſt.

There are four principal things in a teleſcope to be conſidered. 1. Magnifying power. 2. Diſtinctneſs. 3. Brightneſs. 4. The viſible angle, or linear view it takes in. And all theſe have

Fig. been determined in the foregoing Problems, for refracting teleſcopes.

PROB. XIII.

To make a ſolar teleſcope; or to fit up a teleſcope, for obſerving eclipſes, ſpots of the ſun, &c.

114. This is no more than a common refracting teleſcope with two glaſſes, where the eye glaſs may be either convex or concave. Thro' this teleſcope the ſun's image is projected on a plane ſurface, or on the oppoſite wall of a dark room, and magnified. If a teleſcope be uſed for this, as it is fitted up for looking at diſtant objects, the ſun's image will be projected at an infinite diſtance, becauſe the rays emerge parallel out of the eye glaſs. Therefore to have his image at any given diſtance, the glaſſes muſt he placed after this manner.

Let ABCD be a tube in which is the object glaſs G, and its focus H. ECDF another tube ſliding within the former, in which is the eye glaſs L, whoſe focus is I. Let K be the place where the ſun's image is to fall, upon the plane MN. Now whether the glaſs L be convex or concave, it muſt be placed further from G, than in the common teleſcope, by the diſtance HI. And H, K muſt be the conjugate foci of the glaſs L, which is done by making HI, HL, HK in continual proportion. Or if L be a concave, *h* the focus of G, *i* the focus of L, then *hi*, *h*L, *h*K muſt be in continual proportion. Then if the teleſcope AD be put thro' a hole in the window ſhut of a dark room, and directed to the ſun, you'll have his image at K, upon the plane MN. Or if the plane MN be fixt, then ſlide the tube CF, back and forward, within the tube FA, till you find the image diſtinct at K, upon that plane.

For if H be the focus of the lens AB, the ſun's image will be at H inverted. And as H, K, are conjugate foci; the image of K or the ſecond image of the ſun will be at K erect. But in the concave lens CD, the firſt image would be at *h*, and the ſecond at K inverted. Fig. 114.

Cor. 1. *To find the diameter of the image* MN; *it will be as* HL : LK : : *as the diameter of the image at* H : *to* MN *the diameter of the image at* K.

For the ſun and the image at H, ſubtend equal angles at the center of the lens G (by Prop. XXXI. of B. III.); and likewiſe the image at H and the image at K, ſubtend equal angles at the lens L; whence the foregoing proportion.

Cor. 2. *The larger the image* MN *is made, the fainter it will be.*

For if it be as big as the object glaſs G, it will be as bright as if the ſun ſhined upon it. And therefore if it be bigger, its brightneſs will be reciprocally as the area.

Cor. 3. *After this manner may be viewed the ſun's face, his ſpots, his eclipſes, the tranſits of the planets, clouds; likewiſe the moon's face, &c. And this teleſcope is uſed by aſtronomers for this purpoſe.*

PROB. XIV.

To fit up a teleſcope for taking ſmall angles, by placing croſs hairs in the focus of the eye glaſs. 115.

Take a braſs plate AB, in which make a round hole, whoſe diameter is ſomething leſs than that of the eye glaſs; this being placed in the focus P of the eye glaſs (fig. 108, 110.), ſerves for a ſtop to cut off the ſuperfluous rays; and in this the hairs are to be fixt, or rather ſeveral pieces of ſmall ſilver wire. In this plate is fixt the ſmall wire *pq*, going

Fig. 115. thro' the center C of the circular hole; and perpendicular to it is fixt the hair *rs*, near the ſide of the hole. There is a ſlider BDE, in which is fixt the hair or wire *tv*, parallel to *rs*, ſo that when the ſlider is thruſt up cloſe to the ſide EB, the hair *tv* may coincide with *rs*; and when it is drawn out to the extent, the hair may be near the other ſide of the hole, at EB. Then the ſpace BF is divided into 10 or 100 parts, which is the utmoſt extent between the hairs *rs* and *tv*. Then the plate ADE muſt be fixt exactly in the plane of the image made at P (fig. 108, 110,) which may eaſily be done after a few trials; for when it is right, the hairs will appear very diſtinctly thro' the eye glaſs, then they muſt be covered with a thin lantern horn. When the place of the plate is had, one may find the focus of the object glaſs by meaſuring from the hairs to the glaſs: and this is the exacteſt way. The plate muſt alſo be placed ſo, that the center C of the hole may be in the axis of the teleſcope. Then we muſt find what arch is repreſented by BF or *no*, the extreme diſtance of the hairs *rs* and *tv*, when the ſlider is drawn out to its uttermoſt.

To find the arch or angle repreſented by no.

Here the croſs hairs muſt be illuminated by a candle, made to ſhine on them. Direct the teleſcope to ſome ſtar, and turn it round, till you find the ſtar moves along the hair *pq*; then the teleſcope is in a right poſition; therefore keeping the ſame ſide uppermoſt; draw out the ſlider to the end of the diviſions, and direct the teleſcope ſo to the ſtar, that it may fall upon the interſection of the croſs hairs at *n*. Then the teleſcope being fixt there, obſerve by a clock or a watch, how long the ſtar is moving from *n* to *o*. Then ſince one degree paſſes thro' any point in four minutes time, or fifteen minutes of the equinoctial in one minute of time or ſixty ſeconds. Whence one ſecond of time correſponds to fifteen ſeconds of meaſure. Therefore dividing the number of ſeconds obſerved by the clock by four, gives the minutes of a degree anſwering to *no*. But if the ſtar is out of the equinoctial, this number muſt be leſſened in the ratio of radius to the coſine of the ſtar's declination. Fig. 115.

Otherwiſe.

Set up two marks againſt a wall at a good diſtance from you, ſo that looking thro' the teleſcope, the two hairs *rs*, *tv*, ſet to their utmoſt extent, may juſt cover the two marks. If they do not, ſhift the teleſcope further back or nearer hand; or elſe ſhift the marks, till they do. Then meaſuring the diſtance of the teleſcope, as likewiſe the diſtance of the two marks from each other, you will find by plane trigonometry the angle included by the viſual rays; and that is the angle repreſented by *no*.

To take an angle with the teleſcope.

Direct the teleſcope, ſo that the fixt hair *rs* may juſt touch one ſide or term of the angle to be meaſured; and draw out the ſlider EF, till the moveable hair *tv* touches the other ſide; then mark the diviſion of the ſcale at B; and knowing before what arch *no* repreſents: find by the rule of three what angle the number of the ſcale at B amounts to.

If there be no moveable ſlider, then the two hairs *rs*, *tv*, muſt be both fixt. But then in taking an angle, when one ſide is ſet to *rs*, the other ſide will interſect *no* in ſome point, whoſe diſtance from *n*, in reſpect of *no*, can only be taken by gueſs, as there will be no ſcale to meaſure it by.

But the moſt exact way of all for taking ſmall angles, is by the *micrometer*, which is an inſtrument, Fig. 115. ment, with the croſs hairs like theſe deſcribed; and beſides there is a machine made of clock-work, that by turning the handle, moves the hair *tv* always parallel to itſelf; and by one or two indexes, ſhews the diſtance of the hairs very exactly.

There is another way of fixing the hairs, by placing them diagonally, making half a right angle with one another. And theſe are of uſe for finding the difference of right aſcenſion, or of declination of any two ſtars, which the teleſcope can take in.

Theſe croſs hairs, and micrometers, are only applied to refracting teleſcopes, but have not yet been uſed with reflecting ones, ſince they do not magnify the angle ſo much.

Cor. 1. *Hence the diameters of the planets, may be truly meaſured; and all the particulars of an eclipſe determined; or any ſmall angles accurately taken; and applied to other aſtronomical uſes.*

Cor. 2. *The line* no *ſubtends an angle at the eye, which is greater than the angle it repreſents; in proportion to the magnifying power of the teleſcope.*

For the line *no* being placed with the image, they are both magnified alike.

Cor. 3. *Hence,* no *repreſents the angle which it ſubtends at the center of the object glaſs.*

For it is magnified in the ratio of the focal diſtance of the object glaſs to the focal diſtance of the eye glaſs; that is, as the angle it ſubtends at the eye glaſs, to that it ſubtends at the object glaſs.

Fig. 115.

PROB. XV.

To manage teleſcopes without tubes.

To do this, a long pole muſt be ſet upright in the earth, in a wide place, where there is nothing to obſtruct the ſight. In this pole is made a grove, in which a piece of wood is to be fitted to ſlide freely up and down, this is to be drawn up by a rope going over a pulley at the top, when it is uſed. The object glaſs is put into a ſhort tube AB, and this tube is fixt to a piece of machinery, made to move about two axes CD, lying acroſs one another, and this machinery is fixed to the ſliding piece of wood. On the under ſide of the tube a ſmall rod AF is fixt parallel to it, reaching ſix or eight inches beyond the tube; to the end of this a ſtring G is tyed, reaching to the ground. Then any perſon on the ground pulling at the ſtring, can draw the tube into a poſition parallel to the ſtring, by means of the croſs joints; ſo that the tube will point directly to the perſon that draws the ſtring.

The eye glaſs is alſo put in a ſhort tube H, on the under ſide of which is alſo fixt a ſmall rod, at the far end of which I, the low end of the ſtring is to be fixt; and at the nearer end a handle K is fixt to take hold of.

When an obſervation is to be taken, the ſliding piece of wood, with the object glaſs, is to be drawn up the pole to a proper height, and there fixt. Then drawing the ſtring IG, which is fixt to the tail of the object tube, to ſet it ſtreight on you; tie the low end of it to the end of the ſmall rod at the eye tube, ſo as the length of it between the glaſſes may be of a due length for diſtinct viſion, which in the aſtronomical teleſcope, is equal to the ſum of the focal diſtances. Then the ſtring

Fig. 115. drawing the two tubes, ſets them in a right line, and if they alſo be in a right line with the ſtar you want to obſerve, then applying your eye to the tube, you will ſee the ſtar thro' the glaſſes. But if they are not in a line, you muſt alter your poſition, till they are ſo.

It is neceſſary that the obſerver have a reſt to lean on, or to lay the eye tube upon, like a chair back. For without it, the tube cannot be held ſteady. And the obſervations muſt be made in calm fair weather, for the wind will ſhake the tubes, and prevent any thing from being ſeen diſtinctly. And beſides in windy weather, the air is always foul, which makes it unfit to make any obſervations in. And the ſame thing often happens in calm weather, when the air is full of groſs aqueous vapours.

And after this manner perſpectives may be uſed without cloſe tubes; which may be done by holding the eye tube, applied to one end of a long ſtick, with one hand; and the object tube in the other hand, applied to the other end of the ſtick; and ſliding it back and forward on the ſtick to a proper diſtance; which is ſoon found by a few trials. Obſerving to keep the tubes ſtreight on one another, and parallel to the ſtick. After this manner one may uſe the glaſſes of a perſpective as long as ones arm, or longer, by tying the object tube to the ſtick, at a proper diſtance from the nearer end of it; and after this manner, the goodneſs of glaſſes may be tried, without the trouble of putting them into long tubes.

And the ſame way one may make uſe of a long moveable pole, inſtead of one fixt in the ground; which may be uſed for glaſſes not exceeding ſix or eight yards focal diſtance, by tying the eye tube to one end, and the object tube towards the other, at a proper diſtance. And it may be raiſed upon a wall, or the ſide of a houſe, by the help of a hay fork, or ſome ſuch thing. Fig. 115.

Sometimes the ſight is diſturbed, by the ſide light coming into the eye tube, which a cloſe tube would have kept out. To prevent which, two ways may be uſed; either to lengthen the tube of the eye glaſs; or to put an umbrella or border round the tube with the object glaſs; and then to take ſight truly at the object, you muſt raiſe your eye above the eye tube, as high as the breadth of the umbrella.

To enlighten the object tube, to take ſight; a lantern may be uſed, tied to the end of a long pole, and ſo raiſed up on high.

PROB. XVI.

To make a reflecting teleſcope.

A reflecting or catadioptric teleſcope may be made ſeveral ways. I ſhall firſt deſcribe the *Gregorian* one, becauſe they are moſt in uſe.

The Gregorian Teleſcope.

ABCD is a tube open at the end AB, FG a concave reflector, having a hole in the middle, and whoſe focus is *n*. *arb* a ſmall concave ſpeculum, whoſe focus is *t*. This is ſo placed, that the focus *n* of the great reflector, and the place of the image *o*, may be conjugate foci, or that *tn*, *tr*, *to* may be continual proportionals. HI is a ſhort braſs tube, ſcrewing into the end of the large one. In this tube is the convex lens E, ſo placed, that EO may be its focal diſtance, and EI the ſame; and at the end I is a ſmall hole for the rays to come thro' to the eye. But I or the place of the eye is found more exactly, by making *ro*, *r*E, *r*I continual proportionals. 117.

Fig. 117. Now the open end AB being ſet directly towards the object, the rays coming from all the points of the object, are parellel; and being reflected by the concave ſpeculum FG, will form the image of that object at *n*, which will be inverted (by Prop. XVIII. B. II.) And for the ſame reaſon, rays flowing from *n*, and reflected by the ſmall concave *ab*, will make the ſecond image at *o*, which is inverted in reſpect of the firſt image, and erect in regard to the object. Laſtly, the rays flowing from the image at *o*, which is the focus of the glaſs E, and being refracted by it, will emerge parallel to the eye; and therefore the object will appear erect thro' the teleſcope. The path of an oblique ray coming from the upper part of the object, is KF*xbsw*I.

To ſuit all ſorts of eyes, there is a ſmall rod ML, with a ſcrew at the end M; this ſcrew goes thro' the arm which is fixt to the ſmall reflector *ab*, ſo that turning the end L it brings it nearer, or ſets it further, from the other reflector FG. It has a ſhoulder N to keep it ſteady.

The two ſpeculums FG, *ab*, are made of a mixture of copper and tin, but any white hard metal will do that will take a good poliſh. But no mixture has been found out yet that will poliſh truly without pores, ſcars, and ſcratches, and dark ſpots in them. And the ſpeculum *ab* muſt be a little bigger than the hole in the middle of the ſpeculum FG.

118. Inſtead of a ſingle eye glaſs, they are now generally made with two; which increaſes the viſible area a little, and prevents the object being coloured at the edges of the aperture. In order to this, the conjugate focus *o*, muſt be taken in the leſſer tube HI, a good diſtance beyond the hole; and then place a lens P near the hole, to make the rays converging to *o*, to converge ſooner, as to *q*;

and there to form the image. And then place another lens (a meniſcus) at E, ſo that E*q* and E*v* may equal the focal diſtance of it. Then the image which before was at *o*, by the refraction of the lens P, will now be at *q*. Therefore rays flowing from the image at *q*, and refracted by the lens E, will emerge parallel to the eye at *v*. But there is one inconvenience attending two eye glaſſes, that the object will be fainter by ſo many reflexions from their ſurfaces. Fig. 118.

An eye ſtop is to be placed at the laſt image *q*, to circumſcribe the image, and to cut off the ſuperfluous rays, which is a thin baſs plate, with a hole in it, leſs than the reflector *ab*. Alſo a very ſmall hole muſt be made at *v*, to let the rays paſs thro' to the eye, which muſt be no bigger than the breadth of a pencil of rays at *v*.

Since *ro*, *r*E, *r*I are continual proportionals, 117. I will be the focus of rays flowing from *r*; whence the image of the reflector *ab* will be painted diſtinct at I; and therefore a hole is to made in the plate at I, juſt the bigneſs of that image, for the rays to paſs thro' to the eye. If that hole be any bigger than the picture, the light coming by the ſide of the reflector *ab*, will go thro' the hole, and make the picture confuſed. And if the hole be leſs than the picture, rays from the extremities of *ab* will not paſs thro', and ſo the whole cannot be repreſented. Therefore to find exactly the diameter of the eye hole; it will be (by Prop. XVI. B. III), as *r*E : *ab* : : EI : breadth of the image at I, or the diameter of the hole required.

If you hold a thin paper at I, inſtead of the plate, the image of *ab* will be painted on it, and will be viſible thro' the paper, being a black ſpot very diſtinct, ſurrounded with light; and thus one may find the bigneſs of the hole experimentally. And the like for two eye glaſſes.

Fig. 117. To ſee objects that are near hand diſtinctly; or for long-ſighted people, to ſee far diſtant objects; the little concave *ab*, muſt be moved a little from the large one; but a little nearer for ſhort-ſighted people. And all this is done by turning the end L of the rod LM. For if an object comes nearer, its image at *n* will come nearer *t*; and as *tn* grows leſs, *to* will grow greater; and *o* will come nearer the lens E. Therefore to reduce it back, the glaſs *ab* is to be ſet further off. But for ſhort-ſighted people, the focus *o* muſt be ſet nearer E, to make the rays diverge more; that is, *to* muſt be longer, and conſequently *tn* ſhorter, or *ab* brought nearer FG. And the contrary for long-ſighted people. Therefore for far diſtant objects, and ſhort-ſighted people, ſcrew right about. And unſcrew for near objects, and for old men.

The properties of this teleſcope.

1. It ſhews the objects erect; but not ſo bright as in a refracting teleſcope; becauſe metal reflects leſs light than glaſs tranſmits. For by obſervation, it appears that all reflecting metals loſe more than a third part of the rays.

2. The viſible area of an object is as the breadth of the eye glaſs. For (fig. 117.) if the image at *o*, and the eye glaſs E be increaſed, the image at *n* will alſo be increaſed, becauſe the angles of incidence and reflexion at *ab* are equal; and conſequently the viſible part of the object is increaſed.

3. The brightneſs of an object is greater when the aperture is greater. For then more light comes to the eye.

118. 4. The brightneſs of the image is leſs towards the extreme parts; becauſe the ſhadow of the ſmall ſpeculum falls on the outſide; but on the middle it only covers the hole. And this diminution is greater in long teleſcopes.

5. It is found by experience, that a concave reflector, whoſe focal diſtance is 9.6 inches, will bear an aperture of 2.3 inches; and requires a ſmall ſpeculum, whoſe focal diſtance is 1.5 inches, and breadth 0.6 inches; and two eye glaſſes, whoſe focal diſtances are 3.8 and 1.1 inches. Whence the other requiſites are, the breadth of the hole = 0.5 inches; diſtance of the little ſpeculum and the greater eye glaſs = 14.2 inches; diſtance of the eye glaſſes 2.4 inches. The magnifying power of this is 60. Fig. 118.

6. A teleſcope of this ſort is far ſhorter than a dioptric teleſcope that has the ſame magnifying power. For one of theſe reflectors 16 inches long, is found to magnify as much as a dioptric one of 15 or 16 feet long.

Mr. Caſſegrain's *teleſcope.*

ABCD is a tube open at the end AB; FG a concave ſpeculum whoſe focus is *n*, and in the middle of it, is a round hole for the reflected rays to paſs thro'; *arb* a ſmall convex ſpeculum whoſe focus is *t*; this is placed between *n* and FG, ſo that the focus *n* of the great concave, and the place of the image *o*, may be conjugate foci, or that *tn*, *tr*, *to*, may be continual proportionals. HI is a ſhort braſs tube, ſcrewing into the end of the larger, having a convex lens E in it, ſo placed, that E*o*, EI may be each equal to its focal diſtance. At I is a ſmall hole for the rays to come thro' to the eye, which is ſomething leſs than in Gregory's. 119.

Now when the open end is ſet towards the object, parallel rays coming from the object are reflected by the great concave FG, to the focus *n*; and there would make the image of it inverted (by Prop. XVIII. B. II.); but before they come at *n*, they are reflected by the little convex *ab*, Fig. 119. to the conjugate focus *o*, which being on the other ſide of the reflector, will (by the ſame Prop.) make there the ſecond image, erect in reſpect of that at *x*, but inverted in reſpect of the object. Then *o* being the focus of the lens E, the rays flowing from *o*, and being refracted at E, will emerge parallel to the eye at I; where the object will appear inverted; but may be made erect, by cauſing the focus *o* to fall without the tube HI, and applying a concave lens, as in Galileo's teleſcope.

To move the ſmall convex back and forward to ſuit all eyes and objects, there is the ſame contrivance as in the Gregorian teleſcope, by the rod LK. And inſtead of one eye glaſs two may be uſed, which muſt be placed as in the Gregorian. At *o* an eye ſtop is to be placed as uſual, to keep off ſtraggling rays.

The tract of a ray RG coming from the upper part of the object, will be RG*asw*I; where the part G*a* tends to *x*.

The properties of this teleſcope.

1. It ſhews the object inverted; and therefore is more proper for the night, where the inverſion of the objects is not regarded.

2. It is ſhorter than Gregory's teleſcope, by twice the focal diſtance of the leſſer ſpeculum; and has the ſame magnifying power, or rather more, and ſhews the object as bright.

3. The viſible area is as the breadth of the eye glaſs. For increaſing the eye glaſs, more rays will come from the outſides of the object.

4. The greater the aperture is, the brighter the object will appear. For then more light is brought to the ſame parts of the retina. This will alſo appear by ſtraitening the aperture, by putting a paper before it, with a hole in it.

5. The brightneſs of the object is ſomething leſs towards the extreme parts, occaſioned by the ſhadow of the ſmall ſpeculum.

Fig.

The Newtonian teleſcope.

ABCD is a tube open at the end AB; FG a concave reflector, whoſe focus is *n*. *arb* a plain metal reflector, ſet to an angle of 45 degrees with the axis *rz* of the concave. Or inſtead of metal, you may have a glaſs priſm *abm*, the ſide *bm* being parallel, and *am* perpendicular to the axis *rz*, and *am* equal to *bm*, and the ſide *ab* muſt not be quickſilvered over; for it will reflect all the light without it. And whether this reflector be metal or glaſs, it muſt be an oval, whoſe length *ab* muſt be to its breadth, as $\surd 2$ to 1; becauſe it ſtands obliquely. This reflector is cemented to an arm L, which is fixed in the ſide of the tube, ſo that the axis of the ſpeculum may paſs thro' the middle of *ab*. Let *ro* be made equal to *rn*. And let *o* be the focus of the glaſs lens *e*, placed in the ſide of the tube; and a thin plate laid over the glaſs, with a ſmall hole in it; or rather a ſmall tube going from thence, of the length of *eo*, or rather more, as EI; at the end of which I is a ſmall hole for the rays to paſs thro' to the eye. The reflector *ab* muſt not be made too great, and the reflector FG may either be made of metal, or of glaſs, according to Prob. V. 120.

The tube AD being turned with its open end AB towards the object; parallel rays coming from all the points of it, will be reflected from the concave FG to the focus *n*, where would be made the image of it inverted. But by the interpoſition of the ſmall reflector *ab*, they are hindered from coming to *n*, and are reflected thereby to the focus *o*, where they form the ſecond image equal to that at *n* Laſtly, rays proceeding from this image, which is alſo in the focus of the lens *e*, will be refracted by the lens, and emerge parallel to the eye placed at I. Therefore if the face be ſet towards the

Fig. 120. the open end AB, and the eye applied at I, the object will appear inverted. Or if you ſet your face towards the other end CD, it will appear erect to the eye at I; but it is troubleſome to ſtoop down, except the teleſcope be nearly in a horizontal poſition.

The image may be made erect, by making the ſides *um* and *bm* of the glaſs priſm not plain but convex, that the rays may croſs, as well before they come at it, as afterwards between it and the eye glaſs. Or it may be erected, by projecting the focus *o*, without the tube, and applying a concave at *e*, of a proper focus.

The tract of an oblique ray RZ coming from the upper part of the object, is RZ*aswI*, where the part Z*a* tends to *x*, but is reflected in direction *as* by the plane *ab*.

The inſide of this tube, and indeed of all teleſcopes, muſt be made very black, to abſorb the rays of light that fall upon them.

All theſe ſorts of reflecting teleſcopes are ſet upon a ſtand with three feet; and are moveable in it every way, ſo that they may be directed to any object; which done, they are fixt there with a ſcrew, which keeps them ſteady in that poſition.

In ſuch of theſe teleſcopes as magnify much, there is a ſmall teleſcope fixt to the tube, by the help of which, any object may be found out, by directing it to the ſame object.

The properties of this teleſcope.

1. It will bear a greater aperture and eye glaſs than either of the others; and conſequently will magnify more by about a fifth part. It has likewiſe other advantages, for leſs light will be loſt by reflexion from the oval plane, than from a ſpherical reflector. And will alſo reflect the light more truly. In the other teleſcopes, the diſtance of the eye

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eye glaſs and little reflector is greater, which aug- Fig.
ments the errors of refraction. The errors of the 120.
object metal will be more augmented by reflexion from the ſmall convex, which therefore requires a more exact figure; and hence it will not bear ſo large an aperture; alſo being convex it magnifies too much; and therefore will repreſent the objects obſcure, dark, and confuſed.

2. The object in this teleſcope appears inverted, and therefore is fitter for the night: beſides, it appears in a quite different place, which is not ſo natural, as when it is ſeen in its true place, by looking towards it thro' the teleſcope. And this makes it hard to find.

3. The viſible area is greater, the broader the eye glaſs is. And the object is brighter, the larger the aperture is.

4. This and other ſorts of reflecting teleſcopes, are ſhorter than dioptric ones of the ſame magnifying power. Becauſe the images made by refraction, are confuſed and imperfect, by reaſon of the different refrangibility of the rays, which has no effect in reflections. And conſequently theſe images will not bear to be ſo much magnified by refractors.

5. There is more care required in giving a true figure to a reflecting ſurface than to a refracting one. For an error in the figure of a reflector will produce almoſt ſix times as great an irregularity in the image, as it will do by the refractor.

PROB. XVII.

To find the magnifying power of a reflecting teleſcope; alſo the indiſtinctneſs and brightneſs, and viſual angle.

Let rV be the axis of the teleſcope, and let an 121.
object appear to the naked eye under the angle

Fig. 121. RVr or RVn, n being the focus of FG; and let nx be the firſt image made by reflexion from FG; and os the ſecond image made by reflexion from ab. Then drawing sE to the center of the eye glaſs, the angle sEo (by Prop. XVII. B. III.) is the angle which the object appears under thro' the teleſcope; and therefore it is magnified in the ratio of Rvr to oEs. But the angles of incidence and reflection being equal, we have $RVr = nVx$, and $nrx = ors$.

Now the ratio of RVr or nVx to oEs, is compounded of (RVr or) nVx to nrx, and of (nrx or) ors to oEs; that is, of rn to nV, and oE to or; or of $rn \times oE$ to $nV \times or$. And therefore the magnifying power is $\frac{nV \times or}{rn \times oE}$. But ſince t is the focus of ab, and (by conſtruction of the teleſcope) $tn : tr :: tr : to$, therefore $nr : tr :: or : to$, and $\frac{or}{rn} = \frac{to}{tr}$, and therefore the magnifying power is $\frac{nV \times to}{oE \times tr}$. This is in the Gregorian and Caſſegrain's teleſcopes. But in the Newtonian one, where the images nx and os are equal, tr and to have nothing to do; and its magnifying power is $\frac{nV}{oE}$ or (fig. 120) $\frac{nZ}{oE}$.

For the indiſtinctneſs.

The confuſion or indiſtinctneſs of the image in the focus of the reflector, is as the circle of aberration; and (by Cor. 1. Prop. XXIV. B. II.) the diameter of the circle of aberration is as $\frac{B^3}{4RR}$, that is, as the cube of the aperture directly, and the ſquare of the focal diſtance inverſly. And the apparent

Fig. 121. apparent magnitude of that diameter ſeen thro' the eye glaſs, is as the diameter itſelf directly, and the focal diſtance of the eye glaſs inverſly. And the indiſtinctneſs in the eye is as the apparent area of that circle, that is as the 6th power of the aperture directly, and the 4th power of the focal diſtance of the great reflector inverſly, and alſo as the ſquare of the focal diſtance of the eye glaſs inverſly. Therefore if A be the aperture, and F the focal diſtance of the large reflector, g the focal diſtance of the eye glaſs; then the indiſtinctneſs is as $\frac{A^6}{F^4gg}$.

For the brightneſs.

If the area of the image in the eye be given, the brightneſs will be as the quantity of light, that is as the ſquare of the aperture. And if the light, or the aperture be given, the brightneſs will be as the area inverſly. Therefore in general the brightneſs is as the ſquare of the aperture directly, and the area inverſly, but the area is as the ſquare of the magnifying power; therefore the brightneſs is as the ſquare of the aperture directly, and the ſquare of the magnifying power reciprocally. Therefore if M be the magnifying power, the brightneſs is as $\frac{A^2}{M^2}$.

Cor. 1. *If* F, f, *be the focal diſtances of the great and ſmall reflector*; g = *focal diſtance of the eye glaſs, to* = d. *Then in the Gregorian and Caſſagrain's teleſcope, the magnifying power is* $\frac{Fd}{fg}$. *And in the Newtonian, it is* $\frac{F}{g}$.

Cor. 2. *An object appears equally diſtinct in reflecting teleſcopes of the ſame ſort; when the focal diſtances*

of

Fig. 121. *of the eye-glaſſes, are as the cubes of the apertures of the great reflectors, divided by the ſquares of their focal diſtances.*

For if $\frac{A^6}{F^4g^2}$ be given; A^6 is as F^4g^2, and A^3 as F^2g, whence g is as $\frac{A^3}{F^2}$.

Cor. 3. *In reflecting teleſcopes of the ſame ſort; an object appears equally bright, when the aperture is as the magnifying power.*

For if $\frac{A^2}{M^2}$ be a given quantity, then A^2 is as M^2, and A is as M.

Cor. 4. *In ſeveral reflecting teleſcopes of the ſame ſort, an object will appear equally bright and diſtinct; when their apertures and magnifying powers are each as the fourth roots of the cubes of the focal diſtances of the great concaves; and the focal diſtances of the eye-glaſſes as the fourth roots of theſe focal diſtances.*

For in the Newtonian teleſcope M is as $\frac{F}{g}$; and ſince A is as M or $\frac{F}{g}$, therefore Ag is as F, and g as $\frac{F}{A}$, but g is alſo as $\frac{A^3}{F^2}$; therefore $\frac{F}{A}$ is as $\frac{A^3}{F^2}$, and F^3 as A^4, and A as $\sqrt[4]{F^3}$; and therefore M is as $\sqrt[4]{F^3}$. Whence g is as $\frac{F}{\sqrt[4]{F^3}}$ or as $\sqrt[4]{F}$.

And in the Gregorian and Caſſegrain's teleſcopes, we have M as $\frac{Fd}{fg}$, wherein if we ſuppoſe $\frac{d}{f}$ or (which is the ſame thing) $\frac{tr}{tn}$ a given ratio, which ſuits them very well; then M will be as $\frac{F}{g}$, and all things will follow as before in the Newtonian telescope. Fig. 121.

Cor. 5. *In reflecting teleſcopes the viſual angle is as the breadth of the eye glaſs, divided by its diſtance from the ſmall reflector.*

For the angle of reflection at r being equal to the angle of incidence, the ray rs will miſs the lens E, if it is not broad enough. Therefore diminiſhing the lens, diminiſhes the angle srV and Vrx; and alſo rVx, and rVR the viſual angle; and $< sxo$ is as $\frac{so}{or}$ = breadth of the eye glaſs, divided by Er.

Cor. 6. *If* N *be the angle, which the eye glaſs ſubtends at the little reflector. Then* $\frac{Nf}{F}$ *is the angle the teleſcope takes in, nearly; ſuppoſing the hole as big as the eye glaſs. But in Newton's, the angle taken in, is that which the eye glaſs (placed in the focus) ſubtends at the great concave.*

For ſuppoſing the ray rs to touch the edge of the eye glaſs, then the angle N = ors or xrn, is to be diminiſhed in the ratio of rn to nV to get the angle nVx or RVn, that is nearly in the ratio of rt to nV, or f to F. And in Newton's, nx ſubtends the angle nZx = RZn (fig. 120.) The angle N is the linear view; and that diminiſhed in the ratio of f to F nearly, is the real angle, the part ſeen of the object, appears under, to the naked eye.

Cor. 7. *In ſimilar reflecting teleſcopes, the magnifying power, diſtinctneſs and viſual angle, are the ſame. And the brightneſs as the ſquare of the length of the teleſcope.*

For F, f, g, are in given ratios to one another; and the magnifying power is as $\frac{Fd}{fg}$ or $\frac{F}{g}$, which

Fig. 121. is a given ratio. And the diſtinctneſs (or its ſquare root) is as $\frac{A^3}{F^2 g}$, and this is a given ratio. Alſo the viſual angle is as $\frac{b}{F+f}$ or as $\frac{b}{F}$ (putting b = breadth of the eye glaſs), which is alſo a given ratio. And laſtly, the brightneſt is as $\frac{A^2}{M^2}$ or as A^2 (becauſe M is given) that is, as F^2.

Cor. 8. *From what has been laid down concerning the nature and conſtruction of theſe teleſcopes, it will not be difficult to conſtruct one of any length. I ſhall here inſert ſome tables conſtructed by the preceding rules.*

The Gregorian Teleſcope.

F		f	A	a	g	M
f.	in.	inches	inches	inches	inches	
0	6	1.14	1.61	0.32	1.25	25
0	9	1.46	2.23	0.38	1.53	36
1	0	1.76	2.70	0.42	1.73	47
1	6	2.32	3.62	0.51	2.04	68
2	0	2.67	4.52	0.56	2.31	93
3	0	3.43	6.26	0.65	2.56	148

Mr. Caſſegrain's.

F		f	A	a	g	M
f.	in.	inches	inches	inches	inches	
1	6	2.17	3.89	0.42	1.67	90
3	0	3.57	6.57	0.59	2.34	155
5	0	5.17	9.61	0.76	3.03	230

Fig. 121.

Sir J. Newton's.

F	A	*g*	M
feet	inches	inches	
½	0.86	0.16	36
1	1.44	0.20	60
2	2.45	0.23	102
3	3.31	0.26	138
4	4.10	0.28	17
5	4.85	0.29	202
6	5.57	0.31	232
8	6.89	0.33	287
10	8.16	0.35	340
12	9.36	0.36	390

SCHOLIUM I.

The breadth of the ſpeculum *ab* is not determined by the foregoing calculations, and therefore is left to the judgment of the artiſt. But it muſt be conſidered, that if it is too broad, it will intercept too many of the beſt rays, from coming to the large concave FG; and if it be two ſmall, many of the rays reflected from FG, will paſs by it. Therefore to avoid both inconveniences, it will be proper to make its breadth to that of FG, as their focal diſtances, or rather as *rn* to V*n*. For then it will juſt reflect the whole pencil of direct rays that falls upon FG, (ſee fig. 118.)

Likewiſe $\frac{d}{f}$ was aſſumed a given ratio; but as it is unlimited, it may be taken otherwiſe; and then this will cauſe a variation in all the other quantities, and their proportions to one another. And of this experience will be the beſt guide.

In the tables before laid down, it is ſuppoſed that the focus *o* falls at or about V, and on that

Fig. 121. ſuppoſition the magnifying powers are calculated; which would be greater if *o* falls beyond it.

Inſtead of a ſmall concave or convex reflector *ab*, a plane reflector may be uſed. But then to have the focus *o* to fall at V, it muſt be placed in the middle of *n*V; and then its breadth muſt be half of FG, or elſe it cannot reflect all the direct rays falling on FG, and reflected by it. Such a plane reflector would ſtop a quarter of the direct pencil of rays, and the beſt of them too. And beſides, the image at *o* would be no bigger than that at *n*; and therefore this conſtruction will not anſwer the end.

Very long teleſcopes ſhould not be uſed to magnify terreſtrial objects. For they equally magnify the particles of the atmoſphere, and all groſs vapours ariſing in it. And thro' ſuch teleſcopes the air appears in a perpetual tremor; and the more ſo, the more the teleſcope magnifies. And this trembling motion of the atmoſphere makes every object appear confuſed and indiſtinct. The only remedy is a ſerene and quiet air.

If it was not for the different refrangibility of the rays, the errors in refracting teleſcopes, would only be ſuch as are cauſed by the ſphericalneſs of the figure of the glaſſes; and then the dimenſions of the ſeveral parts would be proportioned by the ſame rules, as in reflecting teleſcopes. But as experience ſhews the contrary, it is plain, that the aberration ariſing from the ſpherical figure, is extremely ſmall, in compariſon of that proceeding from the different refrangibility.

I ſhall here add the algebraic expreſſions for the ſeveral parts of Gregory's teleſcope, ſuch as have been found to anſwer experience.

Let focal diſtance of the great concave — = F.
its breadth — — — = A.
magnifying power — — = M.

$\left.\begin{array}{l}\text{tan. angle the hole in the concave}\\ \text{ſubtends at the eye glaſs}\end{array}\right\} = n.$ Fig. 121.

$$\frac{nF}{M} = c, \quad \sqrt{Ac + cc} = R.$$

then, c = breadth of the image at n, of ſo much of the object as the eye takes in; ſuppoſing the eye glaſs as big as the hole.

$\left.\begin{array}{l}\text{Breadth of the hole in the concave,}\\ \text{or of the ſmall ſpeculum } (q)\end{array}\right\} = c + R.$

focal diſtance of the ſmall ſpeculum (f) $= \dfrac{F \times \overline{c+R}}{A+c+2R}.$

diſtance of the ſpeculums (Vr) $= F + \dfrac{F\sqrt{c}}{\sqrt{A+c}} = l.$

focal diſtance of the eye glaſs ($g = Eo$) $= \dfrac{c+R}{n}.$

diſtance of the eye (EI) $= \dfrac{l+g}{l}g = b.$

breadth of the hole in the plate at the eye $= \dfrac{b \times \overline{c+R}}{l+g}.$

Note, day teleſcopes require an eye glaſs of twice the focus of night teleſcopes; and magnifies but half as much.

SCHOLIUM II.

Beſides the teleſcopes deſcribed before; there is alſo one called a *binocular or double teleſcope*. This is no more than two equal teleſcopes ſet in a frame, parallel to one another; and theſe may be ſet at a proper diſtance from one another, by the help of ſcrews; and that diſtance is to be the ſame, as the diſtance of the two pupils of the eyes. When that is adjuſted, a perſon is to look thro' them both at once; thro' one with each eye, to any object; which will then be ſeen by both eyes, and appear far brighter than thro' a ſingle teleſcope.

All teleſcopes in general repreſent all terreſtrial objects to be nearer hand, but not larger; and this nearneſs, vicinity or ſeeming approach of the object, is as the magnifying power of the teleſcope. Thus looking at a man 100 yards off, with a teleſcope that magnifies 100 times; the man will appear no bigger, but will ſeem only to be a yard off; and the like of other objects ſituated on the earth.

PROB. XVIII.

To find the magnifying power of a teleſcope, by experiment.

Make two equal circles of paper of an inch diameter or more, and fix one of them upon a wall 100 or 200 yards diſtant; and the other at a ſmall diſtance, in a line with the firſt. Then look at the further circle thro' the teleſcope with one eye, and at the near circle with the other eye naked. Move the nearer circle (or elſe the teleſcope) back and forward, till the two circles appear equal, or coincide. Then meaſure the two diſtances, from the eye glaſs of the teleſcope to the two cirles; then divide the greater diſtance by the leſſer, and you have the magnifying power of the teleſcope.

Or thus,

If you be in a room where there is a caſement or ſaſh window; cauſe a ſtaff, or ſuch a thing of two or three feet long, to be ſet upright at a good diſtance from the houſe. Open the window, and place yourſelf at a convenient diſtance therefrom, ſo that you may be in a line with the ſtaff, and the ſide or frame of the window. Then looking thro' the teleſcope with one eye, to the ſtaff, mark the extremities of the ſtaff where they appear on the window frame. Alſo with the other eye naked, (or with the ſame eye afterwards, placed where the eye glaſs of the teleſcope was) look at the ſtaff, and mark the extremities of it alſo on the window frame. Meaſure how oft this laſt diſtance is contained in the firſt, and that is the magnifying power. Fig.

Inſtead of a ſtaff, an accidental object may preſent itſelf, which will do as well; as a chimney top, a window or door of a houſe, a tree, &c. By trying thus ſeveral times to find if your marks be rightly ſet, you may do the thing pretty exactly.

In a refracting teleſcope, when you have a large object to look at, you may do the ſame thing, by looking the contrary way thro' the teleſcope to diminiſh it; and making marks as before; and likewiſe looking with the naked eye, and making marks likewiſe; for theſe will give the ſame reſult. Inſtead of marks, you may ſtick pins into the window frame.

PROB. XIX.

To make a ſingle microſcope.

A *microſcope* is an optical inſtrument, made of one or more glaſſes, or ſometimes with a reflecting metal; to view very ſmall objects. 122.

To make a ſingle one, take a very ſmall lens L, fix it in a frame to take hold on with the fingers; and place any ſmall object PR in the focus of it; and applying the eye cloſe to it on the other ſide, at Q, and looking thro' the lens; and it will view the object diſtinctly and magnified. Or if the focus is not known, move the object to and fro', till it appears moſt diſtinctly; and then it is in the focus; and therefore the rays will be refracted parallel to the eye.

In this microſcope, put D = the leaſt diſtance, that the naked eye can ſee diſtinctly; then the mag-

Fig. 122. nifying power of the microſcope is $\frac{D}{QL}$. And it is ſeen direct.

For the rays from P and R enter the eye under the angle PLR, but to the naked eye the object appears under the angle which PR ſubtends at the diſtance D. And theſe angles are reciprocally as the diſtances D and QL. And therefore the apparent magnitudes are in that ratio.

If the lens L was taken away, and a plate with a ſmall hole in it put in its place; the object PR will appear under the ſame angle PLR as before.

Otherwiſe thus.

123. Since lenſes cannot be formed ſmall enough to view very minute objects; there is a neceſſity of making uſe of very ſmall ſpheres for that purpoſe. Let ABD be a ſmall ſphere, C its center, PQR an object, placed in the focus Q, where $AQ = \frac{1}{2}AC$. Then applying the eye cloſe behind the ſphere, it will ſee the object magnified. For the rays flowing from the focus Q, and falling on the ſphere, will emerge parallel to the eye, and the object will appear diſtinct, and erect.

If D be the leaſt diſtance that the naked eye can ſee diſtinctly; then the magnifying power will be $\frac{D}{CQ}$.

For drawing PCG, RCF thro' the center C, the rays from P and R come to the eye under the angle FCG or PCR. And that angle is to the angle which the object appears under at the diſtance D, as that diſtance D to CQ; and therefore that is the ratio of the magnifying power.

Whence this ſphere has the ſame magnifying power as a lens whoſe focal diſtance is CQ. But theſe ſpheres are inferior to lenſes; becauſe in the lens the object is three times as far diſtant from it, Fig. as it is from the ſphere; for which reaſon the 123. ſphere hides almoſt all the light from the object.

Theſe ſpheres are fixt between two braſs plates, or rather one plate doubled in the middle; and having a ſmall hole drilled through it, to take up about thirty degrees of the ſphere; and in this hole the ſphere is lodged.

Otherwiſe thus.

AB is a tube, having a ſmall lens or ſphere fixt 124. in the end A, and the end B muſt be covered with a very thin paper. P is the object, which muſt be placed a little further off than the focus; ſo that the other focus (conjugate to P) may fall juſt on the paper at B; which is eaſily known by moving P back and forward, till the image at B be diſtinct, where it will appear magnified, in the ratio of the length of the tube to the focal diſtance of the glaſs ſphere or lens, and inverted. The object P ought to be illuminated.

A microſcope may be made, by placing two plano convex glaſſes in a frame, to touch at their convex ſurfaces. This will ſhew the objects flat and not crooked.

Cor. 1. *In ſingle microſcopes, the object will be equally diſtinct, when the apertures are as the focal diſtances.*

For then they are ſimilar parts of the whole ſpheres, and conſequently there is the ſame ſeparation of the rays in both, upon which the indiſtinctneſs depends.

Cor. 2. *The magnifying power of a ſingle lens is reciprocally as the focal diſtance; or it is* $\frac{D}{QL}$.

For D the diſtance to ſee diſtinctly, is given.

Fig. 122. Cor. 3. *An object would appear as great to the naked eye as thro' a ſingle microſcope, if it was placed at the focal diſtance from the eye, and the eye could ſee diſtinctly at that diſtance.*

For then D = QL, and the magnifying power $\frac{D}{QL}$ is 1; and it would appear under the ſame angle to the naked eye as thro' the microſcope.

Cor. 4. *The brightneſs of an object ſeen through a ſingle microſcope is as the ſquare of the aperture, divided by the ſquare of the focal diſtance.*

For the focal diſtance and aperture being ſuppoſed in a given ratio, the angle which the glaſs ſubtends at the focus is given, and conſequently the quantity of light contained in that angle (or in a cone deſcribed by its revolution) will be given. But when a given quantity of light enters the eye, the brightneſs of the image will be reciprocally as its bigneſs, that is reciprocally as the ſquare of the magnifying power, or reciprocally as the ſquare of the focal diſtance. Now increaſe the aperture, and the light will be increaſed in the ratio of the ſquare of the diameter of the aperture.

Cor. 5. *The more convex the lens is, the greater is the image; but then it is alſo the more obſcure. And therefore we cannot magnify as far as we pleaſe, unleſs more light be borrowed to illuminate the object; and that would fail at laſt.*

For the power of the microſcope would be limited by the breadth of the pencils, that enter the pupil, and theſe are equal to the breadth of the aperture; and if that be leſs than $\frac{1}{5}$ or $\frac{1}{6}$ of a line; the edges of the object will begin to appear indiſtinct. And to preſerve the ſame diſtinctneſs, the aperture cannot be increaſed, but muſt be in a given ratio to the focal diſtance,

Fig.

SCHOLIUM.

There is a way to make water microſcopes, 124. which is thus; take a thin braſs plate, and dril a hole in it, about the 30th part of an inch. Then take a little water upon the point of a needle or pin, and fill the hole with it, and this water will form a double convex lens or ſphere. Then a ſmall object being placed upon a thin glaſs, fixed in the other end of the braſs turned up, and drawn into the focus by a ſcrew. Then the object will appear magnify'd thro' the drop of water, more or leſs according to the ſmallneſs of the hole. But this ſphere of water will exhale, and vaniſh away in a little time.

If an object be illuminated by ſome homogeneal colour in a dark room, the microſcope will bear a deeper charge and a larger aperture, with diſtinctneſs.

To make the little glaſs ſpheres for microſcopes. Take ſome ſmall pieces of broken glaſs, and hold them in the blue flame of a candle, till they be red hot, and run into globes, or rather in a lamp made with rectified ſpirit of wine, with a wick made of ſilver wire laid cloſe together. Any ſmall piece of glaſs taken upon the end of a ſilver needle wetted with ſpittle, and this held in the flame till it is melted, and becomes round; then it muſt be immediately taken out and cooled. Or take a clear piece of glaſs and heat it in the lamp, and draw it out into a long thread; then holding the end of the thread in the flame, till it run into a round globule hanging at the end of the thread; it muſt then be taken out. Theſe will magnify vaſtly if they are as big as a grain of muſtard ſeed, or of a ſmall pin head. The chief uſe of theſe globules, is to look at tranſparent particles of mat-

Fig. ter; and the leaſt globules have the greateſt power of magnifying.

Drops of liquors which are to be viewed, muſt be ſpread upon a thin glaſs, contrived to ſlide every way, that all the parts of it may be brought ſucceſſively to the focus.

By help of theſe inſtruments one may ſee the circulation of the blood in animals; the animalcula that ſwim in drops of water impregnated with ginger, pepper, and ſuch like things; alſo in vinegar. Their forms are various, and their motion odd and ſurpriſing; theſe in vinegar are like eels, and ſwim like them; and generate their own kind. It is likely theſe animals are drawn out of the air by the ſmell of the infuſion in which they are found. For none appears in a veſſel that is cloſe covered. Milk appears to conſiſt of tranſparent globules, ſwimming in a tranſparent liquor, but of a different degree of refraction. The animalcula that ſwim in water are of a globular form, but tranſparent; they have two motions, one progreſſive, and the other a rotary one; they are exceedingly ſmall, 30,000 of them being but equal to a grain of ſand. In rivers after rain there are plenty. Rain water has many, and ſnow water more. The dew and the air is full of them.

PROB. XX.

To make a compound Microſcope with two or more convex lenſes.

125. Let the microſcope be made with the two lenſes A and B; A the object glaſs, which muſt be a very ſmall one, and very convex; F its focus. B the eye glaſs, being larger than the other. PR is the object, which muſt be placed a little beyond the focus F, of the glaſs A. Make QF, QA,

Q*q* continual proportionals; and make *q* the focus of the glaſs B. Alſo make A*q* AB, AO continual proportionals, and O is the place of the eye, where looking thro' the glaſs B, it will ſee the object at *q* inverted. Theſe glaſſes are incloſed in a tube, which may be brought nearer, or ſet further from the object PR, as there is occaſion, by ſliding within another tube. Generally there are ſeveral ſizes of the lens A, fixt in little cells or buttons, which ſcrew upon the end of the tube, as one has occaſion to uſe a greater or leſſer magnifier. The objects are laid on thin pieces of glaſs or talk, to which they cleave, and theſe are fixed in ſliders, to be uſed occaſionally. It is beſt to ſet the tube upright, and look down perpendicularly upon the object, thro' a ſmall aperture made at O. There is a concave or convex glaſs in moſt of theſe machines, to throw light upon the object PR. Fig. 125.

Since QF, QA, Q*q*, are in continued proportion, Q and *q*, are conjugate foci; (by Prop. XXI. B. III.) and *pr* is the image of PR. And ſince *q* is the focus of B; the rays flowing from *pr*, and refracted by B, will emerge parallel to the eye at B; where it will ſee the image *pr* diſtinct, and in a contrary poſition to PR. Likewiſe ſince A*q*, AB, AO are continual proportionals, A and O are conjugate foci; and (by Prob. III.) therefore O is the place where all the pencils meet, and conſequently is the place of the eye, to ſee the moſt of the object.

If D = the leaſt diſtance the naked eye can ſee diſtinctly, then the magnifying power is $\frac{D \times qA}{qB \times QA}$.

For in the ſingle microſcope B (Prob. XIX.), the image *pr* is magnified in the ratio of D to *q*B. And by the lens A, the object PR is magnified (to *pr*), in the ratio of A*q* to AQ, (by Prop.

XXXII.

Fig. 125. XXXII. B. III.) ; therefore the whole magnifying power is $\frac{D\times Aq}{qB\times AQ}$. Alſo ſee the Corollaries of Prob. II.

To ſuit the microſcope to ſhort-ſighted people, the glaſſes A and B muſt be placed a little nearer together, that the rays of each pencil may come a little diverging to the eye.

The lens B muſt be greater than A, otherwiſe many of the rays paſſing from the object through A, would miſs B ; therefore it muſt be broad to receive them ; and the axes of all the pencils will meet at O. This conſtruction is nearly the ſame as in the aſtronomical teleſcope ; only the object glaſs A is here leſs than the eye glaſs, which muſt be greater in the teleſcope.

In many of theſe ſorts of microſcopes, there are two eye glaſſes ; and the middle one is the largeſt ; and is placed before the image *pr*, between *q* and A. By this means the microſcope receives more light. And theſe two eye glaſſes bring the rays ſooner to a focus, and gives a larger view of the object.

Cor. 1. *The magnifying power of a double microſcope* AB, *is* $\frac{D\times qA}{qB\times QA}$. *And is ſomething leſſer in a treble microſcope: which is ſhorter, where* D *is the diſtance to which the eye can ſee diſtinctly.*

Cor. 2. *The apparent brightneſs of an object* PR, *is as* $\frac{A^2\times qB^2}{qA^2}$; *putting* A *for the aperture of the object glaſs* A.

For the light falling upon A in the plane PAR, is as the angle it ſubtends at Q, that is, as $\frac{A}{AQ}$; and that cone of light is as $\frac{AA}{AQ^2}$; and that

light is tranſmitted to the image *pr*, and ſo to the Fig.
bottom of the eye; and its brightneſs there will be 125.
as the denſity, or reciprocally as the area of the image; that is, reciprocally as the magnifying power. Therefore the brightneſs is as the ſquare of $\frac{A}{AQ} \times \frac{Bq \times AQ}{D \times Aq}$; that is, becauſe D is given, as the ſquare of $\frac{A \times Bq}{Aq}$.

Cor. 3. *The viſual angle is as the breadth of the eye glaſs.*

For if the eye glaſs be contracted, ſome of the extreme rays will miſs it, and ſo the view is contracted.

Cor. 4. *It is found by experience, that a double microſcope performs well when the aperture of* A *is* $\frac{1}{10}$ *of an inch*, AF $= \frac{7}{10}$. Aq $= 7$, *and* Bq $= 2$. *And the magnifying power* $= 36$.

SCHOLIUM.

Altho' the aberration occaſioned by the figure of the glaſs is of no moment in teleſcopes, when compared with the aberration ariſing from the different refrangibility of the rays; yet in ſuch ſmall lenſes as are uſed for microſcopes, where the apertures are large, in reſpect to the focal diſtance; the aberrations by the figure become ſenſible, and often cauſe the image to be indiſtinct. For example, let there be a ſingle microſcope, whoſe lens is $\frac{1}{10}$ of an inch in breadth, and its focal diſtance near $\frac{4}{10}$. Then (by Cor 6. Prop. XLI. B. III.) the aberration by the figure, will be to the aberration by the colours; as 232 to 256, almoſt equal. So that in ſmall lenſes, the indiſtinctneſs is cauſed by both ſorts of aberration, and therefore cannot well be brought under any rule.

Fig. 125. A ſmall aſtronomical teleſcope may be turned into a microſcope, by drawing the eye glaſs ſomething further from the object glaſs; and looking thro' it the contrary way, at the object placed near the end of it.

PROB. XXI.

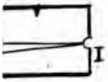
To make a reflecting microſcope.

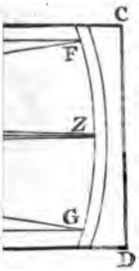
126. Let FG be a reflecting metalline concave, whoſe focus is P, BC a refracting lens for the eye glaſs, placed upon the ſame axis. O an object, placed near the focus O. Take PO, PV, PA continually proportional, and place the lens BC ſo, that A may be its focus. Then take VA, VE, VI, continually proportional, and I is the place of the eye, which thro' the eye glaſs BC, will ſee the object at A diſtinct, inverted and magnified.

For O and A being conjugate foci, the object at O will be repreſented at A inverted. And A being the focus of BC, the rays flowing from A will emerge from BC parallel to the eye at I, and cauſe diſtinct viſion.

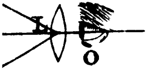
The eye glaſs E is incloſed in a tube KL, which ſlides within another KH, which is cloſe as far as DH, but open in the ſpace DFGH, to let the light in upon the object O. A ſtop muſt be ſet at A to cut off the ſtraggling rays. To illuminate the object, it will be proper to throw the light upon it, with a concave reflector.

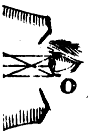
Cor. 1. *If* D *is the leaſt diſtance that the eye can ſee diſtinctly (as eight inches); then the magnifying power is* $\frac{VA \times D}{VO \times AE}$.





122.





Pl. XI. *pa.222.*

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for the object is magnified at A, in the ratio $\frac{1}{5}$ (by Prop. XVI. B. II.) and this image at A again magnified by the lens BC, in the ratio $\frac{D}{AE}$ by Cor. 2. Prob. XIX.) Fig. 126.

Cor. 2. *The apparent brightneſs in ſeveral microſcopes, is as* $\frac{AA \times AE^2}{VA^2}$. *Where* A *is the aperture of* FG.

For the light falling on FG, is as the ſection of the cone FOG, or as $\frac{AA}{VO^2}$; and that light carried to the image at A, and thence to the eye, has the brightneſs diminiſhed in the ratio of the area of the image in the eye, or as the magnifying power. Therefore the brightneſs is as the ſquare of $\frac{A}{VO} \times \frac{VO \times AE}{VA \times D}$ or of $\frac{A \times AE}{VA \times D}$ or of $\frac{A \times AE}{VA}$.

Cor. 3. *It is found by experience, that a microſcope anſwers very well, that has the following dimenſions*; FG = 1 *inch*, VP = $\frac{7}{3}$, AE = $\frac{9}{5}$. PO = $\frac{4}{14}$ PV.

SCHOLIUM.

A reflecting microſcope may alſo be made after the manner of the Gregorian teleſcope, with two concave ſpeculums, the object being placed without the tube at a ſmall diſtance from the end of it. Or it may be made after the form of Mr. Caſſagrain's teleſcope, with a concave and a ſmall convex reflector; and the object to be placed between the principal focus and center of the large concave, and a little without the ſmall reflector; and then each of the reflectors muſt have a hole made thro' it, to let the rays paſs from the object.

PROB.

Fig.

PROB. XXII.

To make a ſolar microſcope.

The uſe of this inſtrument is to magnify tranſparent objects which are very ſmall; and to expoſe them to view by the help of the ſun's light thrown upon them, and caſting their image on the oppoſite wall of the room.

127. The conſtruction is this; CD is a microſcope-glaſs, F its focus, its focal diſtance GF, which may be $\frac{1}{4}$ of an inch or $\frac{1}{2}$ an inch. The glaſs CD is fixt in the tube EK, which ſlides within another tube HI about two inches diameter, and eight or ten inches long. O is a very ſmall object, placed on a ſlip of Muſcovy glaſs or talk, in a ſlider, which goes thro' the tube HI, where a ſlit is made on each ſide to receive it. By ſliding the tube EK containing the microſcope glaſs CD, within the other tube HI back and forward, the glaſs is ſet to any diſtance from the object O, which is to be placed ſomething beyond the focus F. The place of the object O, is adjuſted thus; let L be the oppoſite wall of the room; make OF, OG, OL continual proportionals; and O, L will be conjugate foci, by Prop. XXI B. III.

To illuminate the object, a lens AB is fixt in the end of the tube HI, to collect the ſun beams, and throw them upon the object at O, which muſt be placed between the glaſs AB and its focus; for if the object was in the focus of CD it would be burnt or diſperſed by two much heat.

This inſtrument is only uſed in a dark room, to which purpoſe the end of the tube HI goes thro' a ball and ſocket, which fits into a hole in the window ſhut MN; then the tube being directed towards the ſun; the lens AB will caſt the ſun's rays upon the object at O, by which it is ſtrongly illuminated. Then rays flowing from it as a luminous body, will be refracted by the lens CD, and converge to L, where it will paint the image of that body. For O and L being conjugate foci, the image will appear diſtinct at L. And therefore the tube EK muſt be ſlid in or out, till the image at L appears diſtinct. Fig. 127.

There is a contrivance of placing a looking-glaſs without the hole of the window ſhut, which may be ſet at any angle. And by turning ſome ſcrews or pins, it is ſet to ſuch an angle as to reflect the ſun's rays horizontally thro' the tube; ſo that it need not be elevated to the ſun.

By the help of this inſtrument may be viewed the animalcula in pepper water, in vinegar, or any other liquor; and likewiſe any other ſmall animals that are tranſparent.

It is by the means of the ſun's light that the picture at L becomes viſible; for ſeeing it is very much magnified; the brightneſs would decreaſe in proportion to its magnitude, and it could not be ſeen at all with common light.

Any object O is magnified in proportion of GL to GO, or the diſtance of the image from the lens CD, to the diſtance of the object from it. And therefore the ſmaller the lens CD is, or the leſſer FO is, the more the object will be magnified.

PROB. XXIII.

To make a magic Lanthorn.

ABPQR is a tin lanthorn a foot diameter; or it may be a ſquare box of wood; HI a round tube four inches diameter, and eight inches long, fixt in the ſide of it. CK another tube four inches long, ſliding within the firſt, and having a 128.

Fig. convex lens D of three inches focus, fixed in it.
128. O an object whoſe image is to be projected on the oppoſite wall of the room at L. F is the focus of D, and O is to be placed beyond F, ſo that OF, OD, OL, be continual proportionals; and then L will be the place of the image. This inſtrument is only uſed in a dark room; and to illuminate the object, a candle or lamp is ſet at E, and a concave reflector GN placed on the oppoſite ſide; ſo that its vertex may be in the ſame line with E, O, F, D; and that E and O may be conjugate foci, to the reflector.

The object at O is generally ſome ludicrous or frightful figure, on purpoſe to divert the ſpectators, it is painted upon a piece of plain thin glaſs, in dilute and tranſparent colours; for if they were opake they could not be repreſented at the focus L. They are beſt done with oil of ſpike, mixt with ſeveral ſorts of colours; they penetrate the glaſs and dry preſently. The pieces of glaſs the figures are painted on, are put into ſliders, three of them in one ſlider; which are to be put in at a ſlit at O in the tube HI, when uſed, and in an inverted poſition; they muſt be placed in a line between E and D. Small living animals may alſo be uſed; and ſome of them make a moſt terrible appearance.

This machine carried into a dark room, and ſet on one ſide, and the candle or lamp E being lighted, and the box made cloſe, the candle partly by its direct light, but more by that reflected from the ſpeculum GN, will ſtrongly illuminate the object at O, which being tranſparent, the light will paſs thro' it, and being refracted by the lens CD to the oppoſite wall, will give there the image erect, becauſe the object was inverted. If the image at L is not diſtinct; then to adjuſt it, draw the tube CD a little in or out, till the image appears diſtinct,

tinct, for it may be projected to any diſtance. As Fig. there is a deal of ſmoak produced within the lan- 128. thorn, there is a hole R at the top, to convey it away.

Inſtead of the concave reflector GN, one may place a deep convex lens AB, in the end of the tube HI, which will throw the light of the candle upon the object at O, and illuminate it. Likewiſe one may place two lenſes in the tube CK, which will make the rays converge ſooner; and then there muſt be placed a ſtop between them with a hole to let the light thro', and cut off the ſuperfluous rays. The whole length of the machine may be about a foot and a half long; and the focus of the glaſs CD, ſix or eight inches. This inſtrument and the ſolar microſcope are both upon the ſame theory.

PROB. XXIV.

To make a Camera Obſcura for taking the draught of a country, a town, or any building, &c.

Let ABCD be a wooden box, about a foot and 129. a half ſquare, open only on the ſide AD; and *efgh* a ſmaller box about ſix inches ſquare, and open only on the under ſide *gh*. This leſſer box is let into a hole in the top of the great box AB, and made cloſe and fixt there; *eopi* is a plain reflector or looking-glaſs, fixt diagonally in the ſmall box, with the reflecting ſide downwards, making an angle of 45 degrees with the perpendicular ſide, or face, *eh*. There is a tube K going thro' the ſide *eh* of the ſmall box, which ſlides in and out; in this tube is contained a convex lens about three inches diameter, and focal diſtance about two feet; this tube ſtands even with the middle of the reflector *ep*. The focal diſtance of the lens ought to be equal to the height of the middle of the reflector

Fig. 129. above the bottom CD of the great box, added to the diſtance of the ſaid middle point from the lens. On the top of the ſide AD, along AL there hangs a curtain; by the help of this, the box may be made entirely dark within.

When this machine is to be uſed, it is carried into the fields, to the top of a hill, or ſome high piece of ground, where the place, whoſe draught is to be taken, may be ſeen to the beſt advantage. Then placing it upon a table, or upon a frame made on purpoſe; ſet it level upon its baſe, and let it exactly face the objects that are to be drawn. Then the draught's man is to put his head and his hands into the open ſide AD, and draw the curtain cloſe about him, that no light get in; then laying a ſheet of paper upon the bottom CD, and faſtening it that it cannot move; ſee if the objects appear diſtinct upon the paper; if they do not, move the tube a little inward or outward, till the objects do appear diſtinctly; and then it is time to draw the pictures of the objects within. Therefore having your head and hand in the box, and the curtain cloſe drawn, run over all the lines of the ſeveral objects, as they appear on the paper, with a black lead pencil, or one of chark; till you have gone thro' the whole. Then you may either keep the draught as it is, or run the lines over again with ink, and then it will ſtand in no danger of being rubbed out. Theſe inſtruments generally take in about 45 degrees.

This machine is on the ſame principle as that of darkening a room, and letting the light in thro' a lens in the window ſhut. For here the rays flowing from any object, are refracted by the lens in the tube K, and would paint the image of that object, in its focus; but being reflected by the ſpeculum *ep*, directly down to the baſe DC, they will there paint the image thereof, at the ſame diſtance from the reflector, as it would have been beyond it, in their true ſituation, and proper colours. Here the lens muſt not be too broad, for then the images will not be diſtinct; nor ſhould it be too narrow, for then they will appear dark. And of all things a clear ſunſhiny day muſt be choſen for this work, ſo as to ſhine on that ſide of the objects that appears to your view. Fig. 129.

Inſtead of having a tube with the lens in the ſide of the box; the tube may be fixt perpendicular under the reflector, and then the rays will be firſt reflected, and then refracted. This requires a greater height of the box; but all the effects will be the ſame.

Alſo inſtead of having the ſide AD open, one may have it cloſe, and near the bottom make a hole to put the hand thro' with a pencil; and near the top make another hole to look thro; and there the ſide of the box ſhould be ſlanted off. The part where the hand goes thro', ſhould ſlide back and forward, that the hand may move to any part of the paper.

PROB. XXV.

To make a portable Camera Obſcura, to view diſtant objects.

130. ABCD is a ſquare box eight inches long, open at top and on the ſide AD. ALMD a ſquare hollow fruſtum of a pyramid five inches long, fixed cloſe to the ſide AD of the box. In the end of this pyramid LM, there ſlides a tube KI, in which is fixt a broad convex lens, whoſe focus is near the end BC. FG is a plain reflector or looking-glaſs going diagonally thro' the box, and is ſet to an angle of 45 degrees. EB is a glaſs plane, having the upper ſide unpoliſhed; AH is a lid that opens up when the machine is uſed; this lid has two wings,

Fig. 130. on each ſide, hanging perpendicularly, and they reſt upon the ſides of the box when the lid is up; and ſerve to ſhade the image, made in the rough glaſs EB, from the light.

When this inſtrument is uſed, the lid is opened, and the ſide AD ſet towards the object. Then the rays coming from thence, are refracted by the lens in the tube KI, and would form the image upon the ſide BC; but being reflected upwards by the looking-glaſs FG, they form the image upon the glaſs plane EB, with the right end uppermoſt, when one faces the objects; but the left ſide is to the right, and the right to the left; where it may be ſeen diſtinctly with the eyes. If the image does not appear diſtinct, draw the tube KI out or in, gradually, till it does appear diſtinct. Then the image may be traced out on the glaſs with a black lead pencil. This operation is beſt performed, when there is a ſtrong ſunſhine upon the objects.

Or thus,

131. ABCED is a cheſt in form of a long parallelopipided, nine inches broad, and two feet long; in the end AC is a tube, with two convex glaſſes F, G; this tube may be drawn out or put in, at pleaſure. H is the focus of the lens F, and FH its focal diſtance; alſo HG is ſomething leſs than twice the focal diſtance of the lens G. The lenſes F, G ought to be pretty broad, the diameter of F about $\frac{6}{10}$; of G, $\frac{4}{10}$ of a foot. IKLM is an oiled paper, or a plane glaſs, parallel to the end DE, and whoſe diſtance from the lens G is ſomething more than twice its focal diſtance, and from the end DE, ſix or eight inches. N is a ſlit big enough to look thro' with both eyes. The whole cheſt is cloſe and dark within; no light entering, but what comes thro' the tube. To make the rays converge ſooner, three lenſes may be uſed.

Then the tube being directed to any objects, the rays of light proceeding from them, will be refracted by the lens F to its focus H, where the images will be made, and from thence paſſing thro' the lens G, will be refracted again to their focus upon the paper IKLM, being the conjugate focus to H; and therefore a perſon looking in at the ſlit N, will perceive them painted upon the paper in their true and natural poſition. If the images appear not diſtinct, draw the tube FG, in or out, till they are ſo. Fig. 131.

SCHOLIUM.

The pictures of external objects may be made in a dark room, by holding a concave ſpeculum directly before a ſmall hole in the window-ſhut, at a diſtance equal to the focal diſtance. For the ſpeculum will paint the images upon the window-ſhut, or a paper held there, but inverted.

Or the pictures of objects may be made in a dark room with a convex lens placed in the window ſhut, or in a ſky-optic ball which goes into a hole in the window ſhut; but then the images will be inverted. But receiving them upon a white apron, and looking down at them, they appear erect.

But to have them direct, hold a plain reflector, ſlanting from the lower ſide of the hole, upwards, to reflect the light upwards; and ſtanding with your back to the window, you'll ſee the images upon the cieling, or on a paper held at a proper diſtance above the reflector, in their proper ſituation. This is like fig. 130, in this Problem.

Or thus,

Place the reflector in a contrary poſition; that is, cloſe to the lens on the upper ſide, and ſlanting downwards; and it will project the images upon a horizontal paper near the floor, in their proper ſituation; which may be ſeen, by ſtanding ig. 31 with the back towards the window, for this has the ſame effect, as fig. 129, in Prob. 24.

Theſe two laſt methods are preferable in this, that they repreſent objects in their natural ſituation. But they have this diſadvantage, that they want light. Therefore one muſt have very good glaſſes, a dark room, and a ſtrong ſunſhine on the objects.

PROB. XXVI.

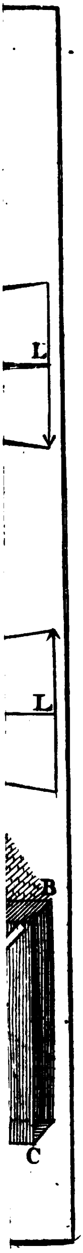
To make a machine to magnify a perſpective draught.

32. Here ABCDE is a wooden box eight inches ſquare, open only at the bottom DI, L is a convex lens three or four inches diameter, and about two feet focal diſtance; this is placed in the foreſide of the box CD; alſo CIEF is a plane reflector, ſet at an angle of 45 degrees, and with the reflecting ſide downwards; G, G, G, G, are the legs, the inſtrument ſtands on, whoſe height muſt be ſuch, that the diſtance from the lens to the middle of the reflector, and thence to the baſe, muſt be equal to the focal diſtance of the lens; P is the draught to be viewed, laid on the baſe.

When you would view any draught or piece of perſpective, lay it flat on the baſe at P, with the upper end towards you, and looking into the box, thro' the lens L, you will ſee it diſtinctly, and magnified.

For the rays iſſuing from the draught at P, being placed in the focus, and reflected to the lens, and then emerging parallel to the eyes, will repreſent the objects vaſtly diſtant, and conſequently as much magnified; having the very ſame appearance as the natural objects themſelves would have. Only the right hand is to the left, and the left to the right.

For



a232.

For as the draught is turned upſide down, it Fig. will appear contrary in the machine; that is, it 132. will appear the right way up. If it was laid the right way up, the right ſide would appear to the right; but as it is wrong way up, the right ſide appears to the left. And therefore one cannot read any writing this way, unleſs one can read it backward.

This machine is the ſame as that in fig. 129, taking away the box ABD. For as the external objects give the picture thereof on this baſe CD; ſo on the contrary the picture placed there, would ſend the rays back again to the ſeveral objects; but the eye takes them before they come there, and they paint their images upon the retina; but upſide down, becauſe the face is turned the contrary way.

But inſtead of legs, one may have it all cloſe 133. below, the better to conceal it; as BH, which is a hollow box, cloſe below, but above having the ſame apparatus as the laſt; at the bottom a dozen views are laid flat upon one another at P; which are all ſhewn ſucceſſively, and then drawn back, by the rulers O, into the part GK behind. EG a thin cloth to let the light thro'; which is ſhut by the ſlider BE, when the inſtrument is not uſed. There may be two or three lenſes L, on the ſide CH, for different people to look thro' at once; which are ſhut up by ſliders when not uſed.

Otherwiſe thus.

ABE is a wooden box open on the top, the ſide 134. AC a foot long, or more, and almoſt as broad; D a concave reflector, 9, 10 or 12 inches broad, fixt perp. with its back leaning a little againſt the ſide BCK; its focal diſtance 12 inches, equal to the length AC. HG a ſlit in the end AE to look thro'. EH, FG two ſtops or bearers, faſtened to the oppoſite

Fig. 134. poſite ſides of the box, againſt theſe the picture or draught is to reſt.

When a piece of perſpective is to be viewed, you muſt ſlide it down upon the pieces EH, FG, as low as the ſlit, and with the right end upwards; and facing the reflector; and then looking in at the ſlit HG, it will appear magnified and erect, but the right hand will be to the left, and the left to the right.

For ſince the print leans with the upper end towards the reflector, and the reflector leans a little back, the rays iſſuing from the picture will not be reflected back upon the picture, but will paſs below, thro' the ſlit HG, to the eye; and the picture being in the focus of the reflector, the rays are reflected parallel upon the eye, and therefore it will ſee the picture diſtinctly, and very much magnified.

Or thus.

But the beſt way of all is, to place the view upon a table, to ſtand perpendicular againſt a wall, or hang it on a pin; and then to place a lens at its focal diſtance from it; then looking thro' the lens, you will ſee the proſpect in its natural poſition every way; and magnified ſo as to appear like the natural place itſelf; eſpecially if the view be painted in its proper colours, as it ought to be.

To have the proſpect to the beſt advantage, the lens ſhould not be leſs than four inches diameter; and two feet, or two feet and two or three inches focal diſtance. And in order to confine the view to your proſpect, the lens ſhould be ſet upright in the middle of a little box or tube ſix or eight inches long; at the end of which, a ſlit muſt be made an inch wide, to look thro' with both eyes, towards the object; and therefore this box muſt be placed horizontally to face it. The picture ſhould be ſo large, as to ſubtend an angle of 25 or 30 degrees, at the focal diſtance of the glaſs. But when your Fig. picture is given, you muſt ſuit it with a glaſs, ſo that the focal diſtance of it, may be about twice the breadth of the picture. A glaſs ſhould never be of ſo ſmall a focal diſtance, that the ſcratches and deformities of the picture may appear. For at too ſhort a diſtance the glaſs will magnify theſe errors and make them viſible, and ſpoil the proſpect. The way to judge of a proper glaſs for any picture, is this; obſerve at what diſtance the picture makes the beſt and fineſt appearance to the naked eye; and that ought to be the focal diſtance of the glaſs. So that altho' a picture placed in the focus of any lens, will be magnified and ſeem at a vaſt diſtance; yet to prevent any blemiſhes or irregularities in the drawing to appear, the lens ought to be choſen by the former rule. A ſhort glaſs will not take in all the view; and a long glaſs makes the picture have but a ſmall appearance; and the eye cannot diſtinguiſh the minute parts.

I ſhall further obſerve, that to have any perſpective view appear exactly ſimilar to the place it was drawn for; the lens ought to be placed in the point of view in the air, of that picture or perſpective draught; and conſequently the focal diſtance of the lens muſt be equal to the principal ray (or line of diſtance); and when this is ſo, if it be viewed thro' that glaſs, it will give the true appearance of that place in all reſpects, in regard to the magnitude, and ſituation of the ſeveral parts. And therefore all theſe perſpective draughts ought to be ſo drawn, that the point of view be at the due diſtance, that is, equal to the focal diſtance of the glaſs proper to be uſed. The longer the focal diſtance of the glaſs is, the longer and more extended, the place appears to be, whoſe perſpective you view. And the ſhorter the glaſs, the ſhorter and more contracted is the apparent view. Both

theſe

Fig. theſe diſtort the view, the longer glaſs by making it too long; and the ſhorter by making it too ſhort, but they do not affect the breadth. And nothing but a glaſs of a juſt length will make it appear in its juſt proportion. If a proſpective draught be well taken, and then viewed according to theſe rules, you'll have the exact appearance of the place, juſt the ſame as if you was looking at it with your naked eyes.

It may be thought, that inſtead of a lens, two ſmall glaſſes of the ſame focal diſtance, for the two eyes, may do as well. But theſe will not anſwer the ſame end, as one large glaſs; becauſe the axes of the two pencils of rays, that fall on theſe two glaſſes, interſect at the picture, and no further off, and there make an angle; and ſo enter the eye; whereas they ought to be parallel. And this makes the picture not appear far enough off, and quite ſpoils the view.

Many other optical inſtruments might be inſerted, but theſe that have been here explained, are the principal; and from theſe, all others of a like nature are eaſily underſtood, and their effects demonſtrated by a like way of reaſoning.

Likewiſe by the known properties of light, the phænomena of all ſorts of meteors may be explained, and the reaſon of their colours ſhewn; ſuch as the Rainbow, Corona's, Halo's, Parhelia's, Paraſelene's, &c. But I ſhall here only explain the colours and appearances of the rainbow, becauſe it is a phænomenon ſo common, that it comes under every body's obſervation.

PROB.

PROB. XXVI.

To explain the magnitude, poſition, and colours of the Rainbow.

The rainbow never appears but when it rains in the ſun-ſhine; and is cauſed by the refraction and reflection of the ſun's light paſſing thro' the drops of falling rain. And it may be made artificially, by ſpouting up water, and cauſing it to fall down in drops, in the ſun's rays; in theſe drops the colours will appear to a ſpectator ſtanding in a due poſition. And the like may be ſhewn in globes of glaſs filled with water, and placed in the ſun in a proper poſition.

To underſtand how the bow is made; let BNFG be a drop of rain of a ſpherical figure, C its center, and let AN be one of the ſun's rays incident upon it at N, and then refracted to F, where let it either go out of the drop by refraction to V, or be reflected to G; and at G let it either go out by refraction to R, or be reflected to H; and at H let it either go out by refraction to S, or be reflected to I, &c. On the incident ray AX, and the refracted one NF, let fall the perpendiculars CD, CE. Draw the diameter BQ parallel to the incident ray AN. And let *m*, *n* be the ſines of incidence and refraction out of air into water. Draw NC, CF. 135.

Now in the progreſs of the ray AN, thro' the points N, F, G, H, &c. it is continually changeing its direction, the ſame way about the circle, till at laſt it emerges out of the drop, making the angle of refraction, equal to that at its incidence; for all the lines NF, FG, GH &c. are equal. Then to find the direction in which it at laſt emerges, produce NF to W, FG to T, GH to Y, Fig. 135. &c; and then we muſt find the ſum of the angles XNW, WFT, TGY &c. and YHS, which makes the whole change of direction, or the revolution of the ray in its progreſs. Draw CZ perp. to FG. Then in the quadrilateral CEFZ, the angles at E and Z are right, therefore the angle $ECZ + EFZ$, $=$ two right angles $= EFZ + ZFW$, therefore $ECZ = ZFW$, or $ZFW = NCF = 2NCE$. But $t =$ number of reflections, F, G, H, &c. Then ſince all the external angles at F, G, H are equal, the ſum of them all is $= t \times ZFW = 2t \times NCE$, to which add $XNF + YHS$ or $2XNF$, and the ſum is $2t \times NCE + 2XNF$, for the whole change of direction of the ray. But $XNF = XNC - ENC$, and $NCE = R - CNE$ (putting $R =$ a right angle), therefore the whole change of direction is $= 2tR - \overline{2t + 2} \times CNE + 2CNX$, or twice the quantity $tR - \overline{t + 1} \times CNE + CNX$. And this being known, it is eaſy to find the angle the emerging ray makes with the incident ray AN or BQ. This is true of any ray whatſoever, but to our purpoſe there is but one poſition of the incident ray AN, which will bring the colours, and this is next to be found.

Let AN be an incident ray of any colour, and let it emerge at H, deſcribing the tract ANFGHS. Now that the rays of any colour emerging at H, may affect the ſenſe ſtrongly, they ought to emerge parallel, for if they emerge and fall upon the eye, in different directions, they mix with other rays of a different colour, and cannot appear ſenſible. And there will be ſo few rays of that colour in any one direction, that they cannot excite any ſenſation. And ſince to excite a ſtrong ſenſation, the rays of any one colour ought to emerge parallel, and as they are alſo parallel at their incidence; therefore we muſt find what poſition, or what angle of incidence, the ray AN muſt have, that all the rays of that colour, emerging at a ſmall diſtance from H, may be all parallel to HS. Now this can only be when the ſum of all the angles $2tR - \overline{2t+2} \times CNE + 2CNX$ is a maximum or minimum. For by the nature of a maximum or minimum, ſuch a quantity remains the ſame for a ſmall variation of the arch BN. Now we may obſerve, that when the arch BN is very ſmall, the arch FQ is very ſmall; and as BN grows greater, QF grows greater, and BF leſſer; and when BN approaches a quadrant, QF grows leſs again, and BF bigger. So that BF firſt decreaſes to a certain limit, and then increaſes again, and at that limit it is at its minimum. But the quantity $2tR - \overline{2t+2} \times CNE + 2CNX$ depends much upon the ſituation of F, and therefore will be at its minimum when the rays at H emerge parallel. Now ſince $2tR$ is given, $2tR - \overline{2t+2} \times CNE + 2CNX$, will be at is minimum when $\overline{2t+2} \times CNE - 2CNX$ is a maximum, or when $\overline{t+1} \times CNE - CNX$ is a maximum. But (by Prob. 83. Sect. VII. Algebra) $\overline{t+1} \times CNE - CNX$ is a maximum, when $\overline{t+1} \times$ increment of $CNE =$ increment of CNX. Fig. 135.

Let $y = CD$ or NM the ſine of incidence; $CN = 1$; m, n the ſines of incidence and refraction, and take Nn a very ſmall increment or decrement of BN, and drawing no parallel to AN, No will be the increment or decrement of MN; then $\frac{n}{m}y = CE$ the ſine of refraction, and its increment $= \frac{n}{m} \times No$. And by the nature of the circle, coſine $(\sqrt{1-yy})$: radius $(1) :: No : Nn = \frac{No}{\sqrt{1-yy}}$, the increment of BN or $\angle CND$. Alſo coſine $\left(\sqrt{1-\frac{nnyy}{mm}}\right)$: radius

Fig. 135. dius (1) : : increment of the ſine of refraction $\left(\frac{n}{m} \times No\right) : \frac{n \times No}{m\sqrt{1-\frac{nnyy}{mm}}}$, the increment of the arch or angle of refraction CNE. Therefore $\overline{t+1} \times \frac{n \times No}{m\sqrt{1-\frac{nnyy}{mm}}} = \frac{No}{\sqrt{1-yy}}$. Put $p = t + 1$, then $\frac{pn}{\sqrt{mm-nnyy}} = \frac{1}{\sqrt{1-yy}}$, and $pn \times \sqrt{1-yy} = \sqrt{mm-nnyy}$; and ſquaring, $ppnn - ppnnyy = mm - nnyy$; and reducing, $ppnnyy - nnyy = ppnn - mm$; and $y = \sqrt{\frac{ppnn-mm}{pp-1.nn}}$. And y being known, the angles CND and CNE are known, and conſequently the quantity of the angle of deviation of the ray, $2tR - p \times CNE + 2CNX$. And the angle in which HS cuts the incident ray AN or BQ, will be known; for if the deviation be 180 degrees, the incident and emergent rays will coincide, and the emergent ray will return back to the ſun.

Since y depends upon the quantities of m and n, it is evident that the rays of different colours, having a different degree of refrangibility, will have different limits of BN, and will emerge parallel in different points H, and will cut BQ at different angles. And theſe ſeveral ſorts of rays being ſeparated from one another, will appear each in their proper colours.

For the firſt rainbow; if light paſſes out of air into water, m and n (by Schol. Prop. II. B. III.) will be 108 and 81 for the leaſt refrangible rays, or the red; and 109 and 81 for the moſt refrangible rays, or the violet. And therefore in this bow, where $m = 108$, $p = 2$, y comes out $= 86036 =$ S. 59 22 = CNX, and CNE = 40 11. And 2R — 4 CNE + 2CNX = 137 58, and this or its ſupplement 42 2, is the angle the emergent ray makes with the incident ray for the red. And in like manner when $m = 109$, the angle which the emergent ray makes with the incident ray, will be found 40 17, for the violet. Fig. 155.

Again in the ſecond bow where $p = 3$, $m = 108$, we ſhall find $y = 95015 = \text{S. } 71\ 50 = \text{CNX}$, and $\text{CNE} = 45\ 27$; and $4\text{R} - 6\text{CNE} + 2\text{CNX} = 230\ 57$ which exceeds 180 by 50 57, the angle made with the incident ray, for the red. And when $m = 109$, the angle the violet emergent ray makes with the incident ray will be 54 7.

Now ſuppoſe O to be the eye of the ſpectator, 136.
and OP a line drawn thro' the ſun and the eye. And let the angles $\text{POE} = 40\ 17$, $\text{POF} = 42\ 2$, $\text{POG} = 50\ 57$, and $\text{POH} = 54\ 7$. And let theſe angles be turned about their common ſide OP; then ſhall the other ſides OE, OF, OG, OH, deſcribe the extreme arches of two rainbows AFBE, and CHDG. For if E, F, G, H be drops of water placed any where in the conical ſurfaces deſcribed by OE, OF, OG, OH, and be illuminated by the ſun's rays SE, SF, SG, SH; the angle SEO or POE of 40 17 will be the greateſt angle in which the violet rays will after one reflexion be refracted to the eye; and therefore all the drops in the line OE, ſhall ſend the moſt refrangible rays moſt copiouſly to the eye, and thereby ſtrike the ſenſes with the deepeſt violet colour. And in like manner the angle SFO or POF of 42 2, will be the greateſt angle under which the red rays can emerge out of the drops, and therefore will come moſt copiouſly to the eye and ſtrike the ſenſes with that colour. And the rays that have intermediate degrees of refrangibility, ſhall come moſt copiouſly from drops between E and F, and ſtrike the ſenſes with the intermediate colours; that is, from the inſide to the outſide of the bow, the colours will

Fig. 135. appear in the order violet, indigo, blue, green, yellow, orange, red.

Again in the ſecond bow, the angle SGO or POG of 50 57 ſhall be the leaſt angle under which the red rays, after two reflexions can emerge out of the drops, and come moſt copiouſly in the line OG, and ſtrike the ſenſe with that colour. And the angle SHO or POH of 54 7 ſhall be the leaſt angle in which the violet rays after two reflexions, can emerge out of the drops, and therefore they will come moſt copiouſly to the eye; and ſtrike the ſenſe with that colour. And by the ſame argument the drops between G and H, ſhall ſtrike the ſenſe with the intermediate colours; therefore from the inſide to the outſide of the bow, the colours will appear in this order, red, orange, yellow, green, blue, indigo, violet. And ſince theſe four lines OE, OF, OG, OH, may be ſituated any where in the conical ſuperficies, what is ſaid of the drops and colours in theſe lines, is to be underſtood of the drops and colours every where in thoſe ſurfaces. But the ſecond bow, by reaſon of the light loſt by two reflexions does not always appear, and is always fainter than the firſt.

Theſe would be the angles if the ſun was but a point, but from his apparent diameter, the breadth of the bows will be increaſed half a degree, whence the breadth of the inner bow will be 2 15, and of the outer one 3 40; and their diſtance 8 25; the radii of the inner bow 40 2 and 42 17, of the outer one 50 42 and 54 22; and this appears to be ſo by obſervation, when their colours are ſtrong and lively.

136. It may be obſerved, that all the rays but the violet in the line SE, will emerge from E in a greater angle than SEO made by the violet, and will therefore all paſs below the eye: and all the rays but the red, in the line SF, will emerge from F in a leſſer angle than SFO made by the red, and will therefore

paſs above the eye; by which means, only red will appear in the line OF, and only violet in the line OE. Fig. 136.

They that have a mind to meaſure the bow, may obſerve that its radius is equal to the height of the bow and the height of the ſun together; and therefore is eaſily meaſured with a quadrant. And if the ſun's altitude be greater than 42 17, the inner bow cannot be ſeen; nor the exterior one, if his altitude exceed 54 22.

In the third and fourth rainbows y is .97373 and .98366 reſpectively, and CNX $=$ 76 50 and 79 38. 135. Whence $2tR + 2CNX - 2p \times CNE =$ 318 20, and 400 31 reſpectively, putting m and n as 4 to 3 for the red rays. And therefore in the 3d rainbow, the emerging ray cuts the incident ray at an angle of 41 40; and in the 4th, at an angle of 40 31, both moving from the ſun. And therefore looking towards the ſun, the red rays in theſe rainbows would appear at the diſtance from him of 41 40 and 40 31 reſpectively, and nearly coinciding. And in the 3d the violet would be towards the ſun; but in the fourth the violet is from the ſun. But their light is weakened ſo by reflexions, that they never appear.

Cor. 1. *The angle of revolution* $2tR + 2CNX - 2p \times CNE$ *will come to its limit when* ND *is to* NC :: *as* $\sqrt{mm-nn}$ *to* $\sqrt{\overline{pp-1} \times nn}$. *And* ND *is to* NE :: *as* m *to* pn.

$$\text{For } ND^2 = 1 - yy = 1 - \frac{ppnn-mm}{\overline{pp-1}.nn} = \frac{mm-nn}{\overline{pp-1}.nn}.$$

$$\text{And } NE^2 = 1 - \frac{nn}{mm}yy = 1 - \frac{ppnn-mm}{\overline{pp-1}.mm} = \frac{ppmm-ppnn}{\overline{pp-1}.mm}.$$

Therefore ND : NC :: $\sqrt{\frac{mm-nn}{\overline{pp-1}.nn}} : 1 :: \sqrt{mm-nn} : \sqrt{\overline{pp-1}.nn}$. Alſo ND : NE :: $\sqrt{\frac{mm-nn}{\overline{pp-1}.nn}} : \sqrt{\frac{ppmm-ppnn}{\overline{pp-1}.mm}} :: \sqrt{\frac{1}{nn}} : \sqrt{\frac{pp}{mm}} :: \frac{1}{n} : \frac{p}{m} :: m : pn$.

Cor.

Fig. 136. Cor. 2. *When the angle of revolution (or change of direction) of the emerging ray differs not much from a ſemicircle or any odd number of ſemicircles; one muſt look for the bow towards the ſun's oppoſition. But when it differs not much from a circle or any number of circles, one muſt look towards the ſun for the bow. Thus the firſt and ſecond bows are ſeen oppoſite to the ſun, and the third and fourth towards it.*

For if the revolution be a ſemicircle, or 3, 5, 7, &c. ſemicircles, the direction of the ray is towards the ſun; and therefore, that it may enter the eye, one muſt look the contrary way. But if the revolution be 1, 2, 3 &c. whole circles, the ray moves directly from the ſun, and therefore to receive it into the eye, one muſt look towards the ſun; for the face muſt be ſet againſt the motion of the ray.

Cor. 3. *If the angle of revolution falls ſhort of a ſemicircle or any number of ſemicircles, the violet colour will be on the inſide the bow. But if it exceed them, the violet will be on the outſide. Thus in the firſt and third bows, the violet is within, and in the ſecond and fourth, without.*

For the violet rays by a greater refraction, make a greater revolution than the red; and therefore when the revolution angle falls ſhort, the violet ray will come nearer a ſemicircle, and make a leſs angle with the incident ray; and therefore will be neareſt the center of the bow. But the contrary will happen when the revolution is greater than a ſemicircle, for then the violet recedes farther off, and makes a greater angle with the incident ray; and ſo it will be furtheſt from the center of the bow. Thus in the firſt bow the violet is within, in the ſecond without, in the third within, and in the fourth without. But after 3 or 4 reflexions the light is ſo weak as not to affect the ſenſes; and ſo the third and fourth bows, and thoſe that follow, never appear; and therefore I ſhall ſay no more about them.

FINIS.

nd.

UNIV OF

Emerson, William

PERSPECTIVE:

OR, THE

ART of drawing the Repreſentations of all OBJECTS upon a Plane.

In TWO SECTIONS.

SECT. I. Demonſtrates the Principles whereon this Art is founded.

SECT. II. Gives the practical RULES for Operation; with great Variety of Examples.

Decipimur ſpecie recti: ——— HOR.

THE

PREFACE.

2-27-37 ACM.

THE *Art of* Perſpective *is properly a branch of Optics; for it depends on this optic principle, that all the rays of light flowing from the ſeveral parts of an object, move in right lines to the eye. And therefore the images of theſe parts muſt be ſomewhere in theſe viſual rays; and conſequently muſt be where theſe lines interſect the picture. Perſpective is a kind of projection, for here the picture being placed at a convenient diſtance between the eye and the object, (or it may as eaſily be placed beyond the object;) the eye projects the ſeveral points, and parts, of the body, upon that picture, by lines drawn from the eye to the parts of the body; and that by certain rules, which are to be explained in this book.*

Perſpective is an art abſolutely neceſſary to be underſtood by all thoſe that practice drawing. From this it derives all its beauty and juſtneſs. It is alſo of great uſe in Architecture, Fortification, Engraving, Carving, and all the mechanical arts; and particularly in Painting. It is of admirable uſe for deſcribing the figures of ſolids, or of any buildings, and of all ſorts of machines. An artiſt cannot put his deſign into execution without this; for all neat and curious drawing ſignifies nothing without it, if the deſign is not executed according to the rules of this art. By this, the figures and ſhapes of all natural objects may be imitated to the life, which no other art can equal. By *this are contrived thoſe deceptions in viſion, which ſo agreeably delight and pleaſe the eye.*

I took notice in the Optics *how delightful a ſtudy that is; but the greateſt ſhare of that pleaſure belongs to Perſpective. For what greater pleaſure can one enjoy by the eye, than to ſee a grand building, a large country, or great city, ſo truly drawn and finely coloured, which being placed at a due diſtance, ſhall appear to the eye in its true proportions, and in all its pomp and beauty, even to vie with Nature itſelf; and even to out-do it in ſome caſes. For if it be drawn in ſummer, with all the beauty of flowers, and verdure of leaves; ſuch a draught look'd at in winter will appear gay; whilſt the place itſelf, now deſtitute of all theſe beauties and natural colourings, gives but a diſagreeable proſpect? Can it fail to pleaſe and charm the beholder to ſee a whole country have the very ſame appearance, and the ſame beauty, as the place it was drawn for, and even of the ſame magnitude when ſeen thro' a lens at a due diſtance. And if it be a place one knows, how agreeably one is ſurprized to find himſelf in it. Who would not be ſtruck at ſo agreeable a deluſion?*

Hence it has been objected by ſome, that this art does not ſhew us truth, but deceives us with falſe appearances; alledging, that nothing appears to us as it really is, but in a quite different form. It is true, if we regard the picture itſelf as an object, which is a piece of paper, being nothing but a plane ſurface perpendicular to the horizon, with ſeveral lines and colourings laid on it; then it gives us a falſe appearance of itſelf. But nobody that draws a picture, ever intends that it ſhould repreſent itſelf, and therefore this objection will be nothing to the purpoſe. For the picture is always deſigned to repreſent a thing that is out of the picture; and therefore if the picture truly repreſents that thing, then it ſhews us truth. And if it does not in all reſpects, repreſent it truly; then ſo far as that fails, it does not repreſent the thing intended, but ſomething elſe; and then indeed it ſhews us falſhood. So that if the picture is truly drawn, it ſhews truth; but it never ſhews falſhood, but when it is falſely drawn.

As the art of painting has always been in great eſteem, therefore we need not wonder that men of excellent talents, have taken great pains in perfecting that art. But without the knowledge of Perſpective, that art will always be deficient and imperfect; and therefore every painter that is ignorant of Perſpective will unavoidably run into errors. Perſpective then is the compleating of the art of painting, without which no painting can be perfect. And therefore no perſon ſhould engage in the art of painting, except he reſolve to make himſelf acquainted with the rules of Perſpective.

The inventive part of painting, is nothing but the original deſign, contrived in the imagination of the artiſt; but the executive part is wholly tied to the rules of art, which muſt not be tranſgreſſed upon any account. Therefore he that deſigns to be a painter, muſt firſt of all learn Perſpective; and when he has done this, and informed his judgment what alterations happen to figures, when drawn upon a plane; he ought then to be put to drawing by ſight, and exerciſe himſelf in this along with Perſpective, till he becomes ſufficiently perfect in both. Nothing ought to be more familiar to a painter than Perſpective; for it is the only thing that can make the judgment correct; and will help the fancy to invent with ten times more eaſe, than it could do without it.

Yet it is not neceſſary or even poſſible, that the ſtrict rules of Perſpective ſhould be applied to the drawing every minute part of a body. For there would be ſuch a confuſion of lines, and their interſections would be ſo near together, that no exactneſs could be expected from thence; ſuch parts as theſe ought to be drawn by the *eye. Nevertheleſs, the rules of Perſpective muſt direct ones judgment, and be a guide to his eye, how they are to be drawn.*

In the firſt ſection of this book, I have laid down the fundamental propoſitions on which the practical part of Perſpective is founded; and given their demonſtrations, that their truth and certainty may appear. And in the ſecond ſection, I have given all the rules of practice, for performing the ſeveral operations, in drawing any points, lines or ſolids in Perſpective. And have given the reaſons and demonſtrations of theſe rules, exceeding ſhort; being eaſily deduced from the propoſitions in the firſt ſection. And have alſo given great plenty of examples to illuſtrate the rules, and to ſatisfy ſuch as have a great deſire for obtaining the arts of drawing and painting. And by frequent uſe and practice ſeveral compendious methods of working will occur of themſelves to every practitioner. All this is principally for upright pictures, the only ſort that are uſeful. For as to inclined pictures, they require more lines and more labour; and no object drawn in them ever appears ſo well as in an upright picture; ſo that they are nothing but a piece of curioſity. Yet leaſt it ſhould happen that any object requires to be drawn upon an oblique plane, which cannot be put into any other poſition; I have alſo ſhewn the properties of theſe pictures, and the methods of drawing upon them, in a general way; with ſeveral examples thereto, which will make the method very plain and clear, to any body, that is deſirous to know how to draw upon this kind of pictures.

W. Emerſon.

ERRATA in the OPTICS.

page	line	read
85	5	$\dfrac{Cfxx}{\dfrac{rr}{2Cf} - x}$.
169	14	$\overline{\dfrac{q-s}{q+s}}\Big|^{2} t$.
173	15	thro' the lens B,
220	25	microſcope, which is ſhorter. Where
224	29	focus of AB.

ERRATA in the PERSPECTIVE.

In the Plates.

Fig.	
19	For *t* put *l*.
57	The pricked lines 41, 1S, and ſome others wanting.

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PERSPECTIVE.

DEFINITIONS.

DEF. I.

PERSPECTIVE is the art of repreſenting any object upon a plane; juſt as it appears to the eye, in any given ſituation. When the plane is perpendicular to the horizon, it is called *common Perſpective.* When the eye is infinitely diſtant from that plane, it is called *military Perſpective.*

DEF. II.

1. The *Picture* is that plane upon which the object is repreſented, as ABCD; which is perpendicular to the horizon; and is ſuppoſed as tranſparent as glaſs. This is alſo called the *Table*, or *perſpective Plane.*

DEF. III.

The *geometrical Plane* is a plane parallel to the horizon, in which the viſible objects are ſituated as FGHI. And the *ſituation* of any point of an object, is where a perpendicular from that point, cuts this plane.

DEF. IV.

The *horizontal Plane* is a plane paſſing thro' the eye, parallel to the horizon, as KLMN; or rather in the horizon. Therefore this is parallel to the geometrical plane, and perpendicular to the picture.

DEF. V.

The *vertical Plane* is a plane paſſing thro' the eye, perpendicular to the picture, and to the horizon, as OPQR.

Fig. 1.

DEF. VI.

The *directing Plane* is a plane paſſing thro' the eye, parallel to the picture, as KNHI: And its interſection IH with the geometrical plane, is the *Line of Extremes.*

DEF. VII.

The *ground Line*, is the ſection of the picture, and the geometrical plane, as DC. It is alſo called the *Baſe Line*, or *fundamental Line.*

DEF. VIII.

The *horizontal Line* is the interſection of the picture, and the horizontal plane, as TV. Whence this is parallel to the ground line. It is alſo called the *vaniſhing Line.* But in a general ſenſe, the *vaniſhing Line* of any plane, is the ſection of its parallel plane drawn thro' the eye, with the picture.

DEF. IX.

The *principal Ray*, is a line drawn from the eye, perpendicular to the picture, as ES. Therefore this lies in the horizontal plane.

DEF. X.

The *Line of Station*, is the ſection of the vertical plane with the geometrical plane, as RQ.

DEF. XI.

The *Line of Diſtance* is that part of the line of ſtation, contained between the directing plane, and the picture; as RX. Hence the line of diſtance is parallel and equal to the principal ray.

DEF. XII.

The *vertical Line* is the line in which the vertical plane cuts the picture; as SX.

DEF. XIII.

The *Hight of the Eye* is a perpendicular paſſing from the eye to the geometrical plane, as ER. This is equal and parallel to the vertical line.

DEF.

DEF. XIV.

The *Point of View* is the point where the eye is fixed, as E. 1.

DEF. XV.

The *Point of Sight* is the point where the principal ray cuts the picture, as S. Therefore this is in the horizontal and vertical lines. It is alſo called the *principal Point*, or *Center of the Picture*.

DEF. XVI.

The *Point of Diſtance* is a point in the horizontal line, whoſe diſtance from the point of ſight is equal to the principal ray, as V or T; ſuppoſing VS or TS, equal to ES.

DEF. XVII.

The *accidental Point* of a right line is the point, where a line, drawn thro' the eye, parallel to the line propoſed, cuts the picture. This is alſo called the *vaniſhing Point*. Hence all lines parallel to one another, have the ſame accidental point. And all lines perp. to the picture, have the point of ſight for their accidental point. And all lines parallel to the picture have no accidental point.

DEF. XVIII.

The *Plan* is the projection of an object upon the geometrical plane, by lines drawn from all points of the object, perpendicular to that plane. This is alſo called the *Ichnography* of the object, or the *Baſe* of it.

DEF. XIX.

The *Front* is the fore view of an object, or the ſection of it parallel to the picture; or it is the projection of it upon a plane parallel to the picture.

DEF. XX.

The *Profil* is the ſide view, or ſection of an object, parallel to the vertical plane. It is taken for the appearance of any ſide.

Fig.

DEF. XXI.

1. *Schenography* is the repreſenting ſolids, or ſuch objects as are elevated in the air.

DEF. XXII.

The *optic Cone* is the cone of rays proceeding from the eye to all the points of an object, and cutting the picture.

SCHOLIUM.

In general, the things to be drawn, are called *Objects* or *Originals*; and their pictures when drawn, are called *Repreſentations*, *Appearances*, *Figures*, *Images*, or *Projections*.

AXIOMS.

1.

The appearance or repreſentation of any point of an object, is where the right line, drawn from the eye to that point of the object, cuts the picture.

2.

If any part of an object touches the picture; its appearance will be in that part of the picture which it touches.

3.

All parts of objects lower than the eye, or below the horizontal plane, will be repreſented below the horizontal line, in the picture: and thoſe that are higher, will be repreſented above it.

4.

All objects on the right hand of the vertical plane, will be repreſented on the right hand of the vertical line; and thoſe on the left hand, on the left ſide of it.

5.

If a right line in the object paſſes thro' the eye, it will be repreſented in the picture by a ſingle point.

6.

Any plane (in the object) paſſing thro' the eye, will appear only a right line in the picture.

SECT.

Fig.

SECT. I.

The fundamental Principles of PERSPECTIVE.

PROP. I.

IN military perſpective, lines that are parallel in the object, are repreſented by parallel lines in the picture. And they are ſhortened in the picture, in the ratio of the radius to the ſine of inclination of the viſual rays.

For let there be two parallel lines in any object; and ſuppoſe two planes to paſs thro' theſe two lines and the eye. Now ſince the eye is ſuppoſed infinitely diſtant, in this ſort of perſpective; therefore theſe planes, which only interſect at an infinite diſtance, may be looked upon as parallel. But (Geom. V. 11.) if two parallel planes be cut by a third; their common ſections are parallel. And therefore the two planes cut the picture in two parallel lines; which repreſent the two parallel lines in the object. And it is equally the ſame if there is never ſo many of theſe parallel lines. And it is evident that their lengths are diminiſhed in the ratio of radius to the ſine of obliquity.

Cor. 1. *If the eye be at a great diſtance; lines that are parallel in the object, are repreſented by lines nearly parallel in the picture.*

Cor. 2. *Viſual rays from the eye, cut all parallels, and all parts of them, at the ſame angle.*

This follows from the paralleliſm, both of the rays and the lines.

Cor. 3. *Equal parallel lines have their images equal.*

PROP. II.

2. *In common perſpective, a right line in the object, which paſſes not thro' the eye, is repreſented by a right line in the picture.*

Let GH be the right line, E the eye; draw EG, EH, cutting the picture in *g* and *h*; then *g* and *h*, repreſent the points G and H (by Ax. 1.). And thus all the points between G and H are repreſented by right lines drawn from thence to E; and all theſe lines are in the plane EGH. And ſince that plane interſects the plane AC in the right line *gh* (Geom. B. V. Prop. 3.) therefore all the points in GH are repreſented by all the points in *gh*; or the right line GH by the right line *gh*.

Cor. *Hence the lines* GH *and gh, are in one and the ſame plane.*

PROP. III.

If a right line be parallel to the picture, its repreſentation in the picture, will be parallel to that right line.

2. For let GH be a line parallel to the picture, and ſuppoſe a plane to paſs thro' it parallel to the picture, then the plane EGH being cut by the two parallel planes GH, and AC; the ſections GH, *gh* will be parallel (by Prop. XI. B. V. Geometry.)

Cor. 1. *If a right line be parallel to the ground line; its appearance in the picture will be parallel to the ground line.*

Cor. 2. *If a right line be parallel to the vertical line; its appearance in the picture will be perpendicular to the ground line.*

Cor. 3. *If a line parallel to the picture interſect the geometrical plane, and make any angle with it; its figure or appearance in the picture will make the ſame angle with the ground line.* Fig. 2.

Cor. 4. *If two lines parallel to the picture, interſect; their repreſentations in the picture, will make the ſame angle, as the original lines.*

For each of them makes the ſame angle as its original; and therefore the differences of each two are equal; that is, the angles which the originals and their repreſentations make, are equal.

Cor. 5. *All theſe things hold good, when the picture is inclined to the horizon.*

PROP. IV.

The figure or appearance of any right line in the picture, tends to the accidental point of that line.

Let GH be the original line, *gh* its figure or appearance in the picture, P the accidental point of it, E the eye. Then (Def. 17.) ſince EP and HG are parallel; EP and HG are in the ſame plane, which plane paſſes thro' E and P. Likewiſe (Cor. Prop. II.) *gh* is in the plane EGH; therefore all the points P, *g*, *h* are in one plane, which plane interſects the picture in the line P*gh*; therefore *gh* produced, paſſes thro' P. 3.

Cor. 1. *The repreſentations of all lines parallel to* GH, *do all meet in the accidental point* P.

Cor. 2. *The appearances of all lines perpendicular to the picture, tend to the point of ſight.*

Cor. 3. *The appearances of all lines, in or parallel to, the geometrical plane, making an angle of 45 degrees with the ground line or picture; tend to the point of diſtance on that ſide.*

Fig. 3. Cor. 4. *The appearance of a right line, paſſes both thro' the accidental point, and thro' the point where the original cuts the picture.*

PROP. V.

If two equal right lines be parallel to, and equidiſtant from, the picture; their repreſentations will be equal.

4. Let the lines GH, KI, be equal, and both parallel to the picture AC, and at equal diſtances from it; and gh, ki their repreſentations. Draw HI and hi E is the eye. Then by reaſon of the parallel planes GHIK and AC, the triangles EGH, egh; alſo EHI, ehi; and EIK, Eik, are ſimilar each to each. Therefore, GH : gh :: EH : Eh :: HI : hi :: EI : Ei :: IK : ik. But GH = IK, therefore $gh = ik$.

Cor. 1. *If ſeveral lines are parallel to, and equidiſtant from, the picture; their repreſentations will be to one another, in the ſame proportion as the original lines.*

For we have GH : gh :: IK : ik; or GH : IK :: gh : ik.

Cor. 2. *If a right line parallel to the picture, be divided into equal parts; the images or repreſentations of theſe ſeveral parts, will be equal.*

Cor. 3. *Two equal right lines proceeding from one point, and both parallel to the picture; will have their appearances equal in the picture. And the angle they make in the picture, will be equal to the angle made by the originals.*

For the repreſentations are parallel to the originals, by Prop. III.

Cor. 4. *All theſe things hold equally for an inclined picture.*

PROP.

PROP. VI.

If a circle be parallel to the picture, its figure in the picture will be a circle, in any ſort of projection.

Let E be the eye, BD the circle. Then ſince 5.
the optic cone EBD, is cut by the plane of the picture AC, parallel to the baſe of the cone, therefore (by Cor. Prop. LXXXIX. B. I. Conic Sections) the ſection *bd* will be a circle.

Cor. 1. *If a circle be ſo placed, that the optic cone, be cut by the picture, ſo that the angles on oppoſite ſides may be equal, which are made by the picture and by the baſe; then alſo the appearance in the picture will be a circle.*

For then it is cut in ſub-contrary poſition; and therefore (by the ſame Prop.) the ſection is a circle.

Cor. 2. *In other caſes when a circle is not parallel to the picture; its appearance is an ellipſis; ſuppoſing the optic cone to be cut on both ſides by the picture.*

PROP. VII.

The appearance of any plane figure parallel to the 5.
picture, will be ſimilar to the original.

Let GHIK be the plain figure whoſe repreſentation is *ghik*. Then ſince the optic pyramid EGHIK is cut by the plane of the picture AC, parallel to the baſe GHIK; therefore the ſection *ghik* will be ſimilar to GHIK (by Cor. 1. Prop. XVII. B. VI. Geometry.)

PROP. VIII.

The vaniſhing or accidental points, of all lines drawn 6.
in any plane, are in the vaniſhing line of that plane.

Let FG be any line in the plane CFG, and EL drawn from the eye at E, parallel to FG, cutting the

Fig. 6. the picture in L. Then ſince all lines as FG are in the plane CFG, all the parallels EL, are in the parallel plane AEL; and therefore all the points L, are in the interſection AL; that is, in the vaniſhing line. And it is the ſame thing whether the plane AC be right or oblique to the plane CG, ſince the planes CFG and EAL are parallel in all caſes.

PROP. IX.

7. *All lines perpendicular to the picture, will be projected into ſome parts or other of the ſame lines in the picture; in whatever point of the principal ray, the eye is placed.*

Let FG be a line perpendicular to the picture, E the eye, ES the principal ray; produce GF to cut the picture in N, and draw SN. Then ſince ES, NG are parallel, they and the line SN are in one plane. Therefore FG appears ſomewhere in the line SN. Then let the eye remove to K, in the ſame line ES, and K is ſtill in the ſame plane ESNG; therefore FG appears ſtill in the line SN, which has not changed its ſituation. But then the repreſentation of FG does not fall in the ſame part of the line SN, when the eye is at E, as when it is at K; for the farther the point K recedes back, the leſs angle the line FG appears under; and the further the repreſentation of it is from S.

Cor. 1. *The further the eye recedes along the principal ray, the leſs any line appears, which is perpendicular to the picture.*

Cor. 2. *Wherever the eye is taken in the principal ray, the repreſentations of all lines, perpendicular to the picture, will converge to the point of ſight. And conſequently the picture will appear a regular piece of perſpective, to an eye placed any where in that line.*

Cor. 3. *Likewiſe by the ſame reaſoning, the repreſentations of all parallel oblique lines will tend to their accidental point, if the eye recedes in the line drawn thro' that accidental point.* Fig. 7.

Cor. 4. *Theſe things hold true in an inclined picture, ſubſtituting lines parallel to the principal ray, inſtead of lines perp. to the picture.*

PROP. X.

If a light be put in the place of the eye, and the ſhadows of any objects be projected on a plane parallel to the picture, placed behind the objects; theſe ſhadows will form a piece of perſpective ſimilar to that made on the picture, when the eye is placed in the point of view; and will have the ſame appearance.

For the image made by the ſhadows, is formed by rays drawn from the light thro' all the points of the objects, till they cut the parallel plane. And the image on the picture, is formed by the very ſame lines cutting the picture. Therefore theſe two images, upon theſe parallel planes, muſt needs be ſimilar; and will appear equal to an eye in the point of view.

Cor. 1. *The images of objects, formed by their ſhadows, on a plane; will make a regular piece of perſpective.*

Cor. 2. *The images formed, by the ſhadows of the ſun's light, make a piece of military perſpective.*
For the ſun's light may be ſuppoſed to come from an infinite diſtance.

Fig.

PROP. XI.

8. *If the back ſide of the picture* AC *be a plain reflector; and if the eye be placed at* O *in the principal ray* ES, *produced as far beyond the picture. Then the eye looking towards the reflector, will ſee all objects in the ſame places of the reflector; as an eye at* E *will ſee them in the picture.*

Let F be any point of an object, and draw FIG perpendicular to the picture cutting it in I, and make GI = FI; and draw SI from the point of ſight S. Then ſince both EO and FG are perpendicular to the picture, the point F ſeen by reflection from O, or directly from E will appear ſomewhere in the plane FEO, and therefore ſomewhere in the line SI, the ſection of it with the picture. But (Optics, B. II. Prop. II.) F is ſeen by reflection in the line GO, and F is ſeen directly in the line EF. But both theſe lines interſect at V in the line SI. For the lines GI = FI, and ES = OS, and SI perpendicular to them all. Therefore their interſection muſt be in the line SI. But in looking towards the reflector, with the back towards the objects; the right ſide will appear on the left; and the left on the right; being contrary to what they are when the face is turned the contrary way. Now what is proved of the point F, holds of all points of all objects whatever.

Cor. *Tho' all objects and all points of them appear in the ſame places by reflection as when ſeen directly, yet the face being turned the contrary way, what is on the right will appear on the left, and what is on the left will appear on the right hand.*

PROP.

Fig.

PROP. XII.

In viewing any piece of perſpective of a country, city, &c. the further the eye is placed from it, in the principal ray; the longer the draught appears to be, and that in proportion to the diſtance of the eye.

I call that the length which is taken along the 9. geometrical plane, as it runs directly from the picture.

Let AC be the picture, EA the principal ray, CV the horizontal plane. Let E be the point of view, for which the proſpect was drawn. Then 'tis plain CB repreſents the length CF, to the eye at E. Let the eye be removed to D, in the principal ray AED; then the ſame CB repreſents the length CV. But by ſimilar triangles, CF : AE : : (CB : AB : :) CV : AD; and therefore the apparent length of the draught is greater at D than at E, in proportion of AD to AE.

Cor. 1. *Hence in any given piece of perſpective, the further the eye is placed from the picture; the larger and more extended it ſeems to be; and the nearer, the leſs it appears.*

Cor. 2. *No perſpective view will appear exactly like the original, but when it is ſeen from the true point of view for which it was made.*

For when it is ſeen from a point nearer than the true point, it gives too ſhort a view; and if from a point further off, the view is too long. And in no poſition of the eye but one, can all lines come from the objects to the eye, to cut the picture in the ſame points. And therefore the ſituation of all objects will not be the ſame, when viewed from different points. Whence no view can appear ſo natural, as when ſeen from its own point of view. And

Fig. 9. And then all rays coming to the eye will come in their true directions, and make an appearance exactly ſimilar to the original.

Cor. 3. *Tho' the objects ſeem greater when the eye is further off, yet they really appear under leſs angles.*

For in this caſe the reaſon of their appearing greater, is not becauſe the ſeveral parts of the view appear under greater angles, for they really appear under leſſer; but becauſe the view appearing longer, we judge the diſtance to be greater, and therefore the objects ſeem greater, and the view more extenſive. But this apparent greatneſs ariſing merely from judgment, it may be different in different people.

PROP. XIII.

If any print or perſpective view, be looked at thro' a lens, whoſe focal diſtance is equal to the principal ray, and the print placed in its focus; it will be ſo magnified, as to have the very ſame appearance, as the place it was drawn for.

10. Let L be the lens, AC the picture placed in its focus, whoſe focal diſtance is LS = the principal ray. Then (Cor. 10. Prop. XX. B. III. Optics), all the rays iſſuing from the picture AC, in its focus, and refracted by the lens L, will emerge parallel to the eye; therefore they come to the eye as they do from the objects at a great diſtance; therefore the view is magnified. Alſo (by Prop. XXXVII. ib.) the apparent magnitude of the view, or any part of it (that is the angle it appears under), ſeen thro' the lens; is the ſame as that ſeen by the naked eye from L, that is from the point of view of the print. But the apparent magnitude of the view or any part of it ſeen from L, is the ſame as that of the country itſelf (or other object) ſeen

from L, ſince the ſame lines go to all the correſpondent points. Therefore the angles, the whole and every part of the print is ſeen under, thro' the lens, are the ſame as thoſe of the object or objects ſeen with the naked eye from L. And therefore both muſt neceſſarily have the ſame appearance. Fig. 10.

Cor. *Glaſſes which are not of a due focal length, will not give the exact appearance of a place. Shorter glaſſes make the diſtances leſs, and ſo contract the view. And longer glaſſes make the diſtances greater, and extend the view.*

For by Prop. XII. the further the eye is off, the larger and more extended the draught appears. And when 'tis ſeen thro' a lens of any focal diſtance, it appears equally large, as ſeen at that diſtance with the naked eye. Therefore it follows that a print ſeen thro' a lens of a long focus, appears larger than when ſeen thro' one of a leſſer focus.

SCHOLIUM.

Since glaſſes of a long focal diſtance, give a large and extenſive proſpect of a country, therefore they are better than ſhorter glaſſes. And when the proſpects are well drawn, and properly coloured; it is very delightful to view them thro' a good glaſs, as they ſo nearly imitate nature. And tho' there is but one focal length that will give a true appearance, yet the draught will always appear a regular piece of perſpective, tho' it may not exactly repreſent any place in the world, ſuppoſing the eye placed ſomewhere in the principal ray. And the draught will ſeem longer in proportion to the focal diſtance of the glaſs made uſe of; or in proportion to the apparent diſtance of the neareſt part of the picture.

As there is nothing more pleaſant than viewing the draughts of countries, towns, cities, magnificent

Fig. 10. cent buildings, and other grand objects, when well drawn: to ſee them to the beſt advantage, the focal diſtance of the glaſs ſhould be juſt ſo long, as not to ſhew the ſcratches, and coarſeneſs of the engraving; or not much longer; for then the view will be narrow, and the parts too ſmall to be ſeen ſo far off. And if it be far off, it will hide the beauties as well as deformities. And to get a proper glaſs, obſerve at what diſtance the ſcratches diſappear to the naked eye, and that is the focal length of the glaſs. Perſpective views ſhould be drawn, ſo that the point of view be further off, than is generally practiſed, if you would have the piece to be a true copy of nature. The principal ray ſhould not be leſs than two foot, and then the draught being looked at thro' a lens of that focal diſtance, it will appear in perfection, and give a true repreſentation of the place it was drawn for. The view ſhould be ſo large as to ſubtend an angle at the lens of about 30 degrees. And it is proper to put the lens in a ſhort ſquare tube; which will confine the ſight, and direct it to the perſpective draught; which, to compleat its beauty, ought to be coloured with the ſame colours as the natural objects appear in. For which purpoſe water colours need only be uſed, as being ſufficient for that purpoſe.

PROP. XIV.

11. *If* G *be any point in the geometrical plane,* GF *perpendicular to the ground line* DC, S *the point of ſight, and* SF, EG *drawn. The repreſentation of* G, *ſhall be at* P *in the line* SF; *and ſo placed, that* ES : SP : : GF : FP.

For ſince GF and ES are parallel, they are both in one plane, which plane cuts the picture in the line SF. And all lines as EG, drawn from E to any point of the perpendicular GF, will interſect the picture in the line SF; thus EG interſects it in P, and therefore P is the picture of G. Now the triangles ESP, GFP are ſimilar, for by the parallels, angle SEP = FGP, and the vertical angles at P are equal; therefore GF : FP : : ES : SP. Fig. 11.

Cor. 1. *If G be any point in the geometrical plane,* GF *perp. to the ground line, and* SF *drawn, and* EG *to interſect* SF *in* P. *Then* P *the image of* G *will be ſo placed, that the ſum of the principal ray and perpendicular* ES + FG : *to the line* SF : : *as the principal ray* ES : *to* SP *the diſtance of the image* P *from* S: *when the point* G *is beyond the picture. But when* G *is on this ſide the picture, you muſt take* ES — FG.

For ſince ES : SP : : GF : FP, therefore ES + GF : SP + PF (SF) : : ES : SP. And ES — *g*F : S*p* — *p*F (SF) : : ES : S*p*.

Cor. 2. *If the picture* AC *is oblique to the geometrical plane, the ſame proportions will hold good.*

This is evident, becauſe ES and FG will ſtill be parallel, and the triangles ESP, and GFP ſimilar.

Cor. 3. *Hence, if the given point be beyond the picture, its image will be above the ground line; if on the ſame ſide as the eye, it will appear below the ground line.*

PROP. XV.

The length of a line GF *ſtanding upright upon the geometrical plane, is to the length of its image* gf *in the picture; as the hight of the eye* ER, *to the diſtance of the image of its foot from the horizontal line,* gn. 12.

Let E be the eye, RCB the geometrical plane, ESTNV the horizontal plane. Produce GF and

Fig. its repreſentation *gf*, to cut the horizontal plane 12. at N and *n*, and draw NT parallel to VA. Then ſince GFN, and *gfn* are parallel, the triangles EGF, E*gf*, as alſo EGN, E*gn* are ſimilar, whence FG : *fg* : : EG : E*g* : : GN : *gn* : : ER : *gn*.

Cor. 1. *As the diſtance of the line from the eye, to the diſtance of its image from the eye; ſo the length of the line, to the length of its image.*

For by ſimilar triangles, FG : *fg* : : EG : E*g* : : DN : E*n*.

Cor. 2. *As the diſtance of the eye from the plane* TNG *(in which the line is), to the diſtance of the eye from the picture* ES, *ſo the length of the line* FG, *to the length of its image fg.*

For FG : *fg* : : EN : E*n* : : ET : ES.

Cor. 3. *The ſame is true on an inclined picture, provided the line* FG *be parallel to the vertical line* SX.

For the triangles EGF, E*gf* will be ſtill ſimilar; as alſo EGN, and E*gn*.

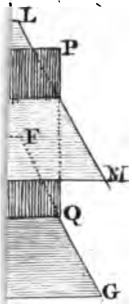
SCHOLIUM.

This propoſition is the foundation of all ſchenography. By its help all elevations are performed, the poſitions and hights of all ſolids truly laid down. And the ſchenographic repreſentations of all bodies exactly delineated.

PROP. XVI.

If a point be elevated above the geometrical plane, it is the ſame thing as if it was in a new geometrical plane raiſed ſo much higher, as is the hight of that point. The point of diſtance remaining the ſame.

13. Let E be the eye, S the point of ſight, AC the picture, IHQ the geometrical plane, and GF a line



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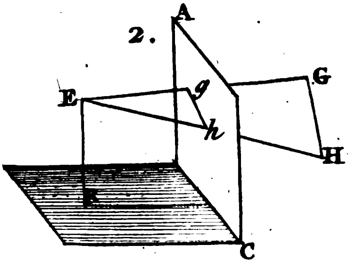

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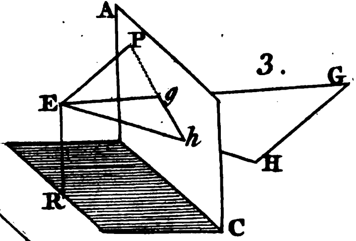

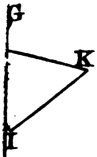

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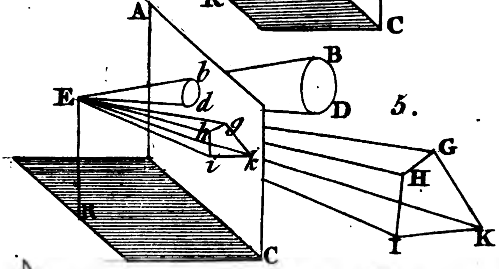

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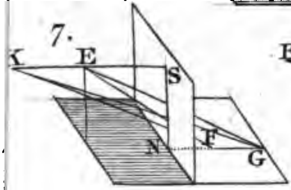

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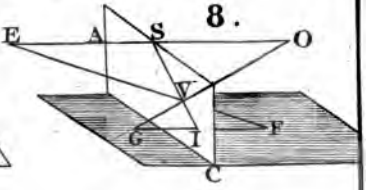

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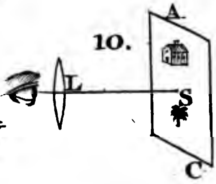
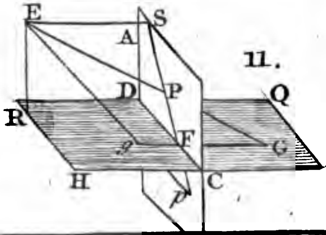


11.



Pl. I. pa. 18.

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Fig. 13. line ſtanding perpendicularly upon it. Thro' the top F draw the plane KLMN, which will cut the picture perpendicularly in the line *dfc*, and the hight of the eye in *r*. Therefore if we make this new plane the geometrical plane, the point F will be a point in it. And the hight of the eye will be E*r*, and ground line *dc*. And the hight of the eye, and the picture will be ſhortened by the hight R*r* or FG or *fg*; while the point of diſtance, and principal ray ES remains the ſame.

Cor. *And thus every point, elevated above, or depreſſed below, a given geometrical plane; may be aſſumed in ſome other geometrical plane, drawn thro' that point, parallel to the firſt.*

PROP. XVII.

14. *If* E *be the eye,* AC *the horizontal plane,* ER *the geometrical plane. And if any objects (ſituated in the plane* GR *perpendicular to the horizon) are to be drawn upon the plane* AC, *for the picture. They will appear in the ſame places, as when the hight of the eye, and the diſtance, are changed for one another.*

For let G be a point to be repreſented in the plane SC. Draw the line EG to cut SC in *g*; and *g* will be its place in that plane; where ES is the hight of the eye, and SC its diſtance. But this is the very ſame thing as if ER was the hight of the eye, and ES its diſtance, AC the picture, and RG the geometrical plane, in which the point G is ſituated. This will be evident, by turning the whole figure about, till RG becomes the baſe; for then E being the eye, ER is its hight, ES its diſtance, &c.

Cor. *Hence the ſame rules that ſerve for drawing upon a vertical plane, will alſo ſerve for drawing upon a horizontal one.*

Fig.

PROP. XVIII.

The ſhadow of a right line, upon a plane cauſed by any light, is alſo a right line.

For if a plane be drawn thro' the light, and the given right line, and extended to the plane, it will cut that plane in a right line (Geom. V. 3.); and it is evident this line of ſection is the ſhadow of the given line.

Cor. 1. *If the right line given be parallel to the plane, its ſhadow will be parallel to that line.*

For the line being parallel to the plane will be parallel to the ſection, (Geom. V. 11. Cor.).

Cor. 2. *If the right line go thro' the light, its ſhadow is only a point.*

Cor. 3. *The image of the ſhadow (of a right line upon a plane) will alſo be a right line.*

This is evident by Prop. II. conſidering the ſhadow as an object.

Cor. 4. *If ſeveral parallel lines be parallel to a plane, all their ſhadows will be parallel to one another and to theſe lines.*

This is plain by Geom. V. 8. and Cor. 1. of this.

Cor. 5. *And if the light be at a great diſtance; the ſhadows of any parallel lines, will be parallel to one another.*

For then the planes paſſing thro' theſe lines, and the light may be eſteemed parallel. And therefore (Geom. V. 11.) theſe ſhadows, or their ſections with any other plane, will be parallel.

PROP.

PROP. XIX.

The images of all rays, proceeding either from the ſun or a ſmall light, tend to the image of that light in the picture.

For the image of the light is the image of one end of any ray proceeding from that light; and the ray being a right line, its image will be a right line, proceeding thro' the image of the light, or tending to it.

Cor. 1. *The image of the ſhadow of any point, is in the right line drawn thro' the image of that point, and the image of the light.*

Cor. 2. *The images of all bodies are terminated, by lines drawn thro' the image of the light, and thro' the images of the extremes of theſe bodies.*

PROP. XX.

If a right line FG *be parallel to any plane* PQ; *the image ik of its ſhadow* IK *upon that plane, (by any ſort of light), ſhall tend to the accidental point* O, *of that right line.* 15.

For (by Cor. 1. Prop. XVIII.) the line FG and its ſhadow IK will be parallel; and (Cor. 1. Prop. IV.) the images of theſe parallel lines FG, IK, tend to the accidental point of FG; that is to O.

Cor. 1. *If a line be perpendicular to the picture, the image of its ſhadow, upon the geometrical or vertical planes, or their parallels, tends to the point of ſight.*

For then the line is parallel to both the geometrical and vertical planes.

Fig. 15. Cor. 2. *If a line be perpendicular to the vertical plane; the image of its ſhadow upon the geometrical plane, or the picture, or their parallels, is parallel to the ground line.*

For then it is parallel both to the geometrical plane, and the picture.

Cor. 3. *If the line be perpendicular to the geometrical plane; the image of its ſhadow upon the picture and vertical plane, and their parallels, is parallel to the vertical line.*

For then the line is parallel to the vertical plane and the picture.

PROP. XXI.

16. *If* L *be the image of the ſun in the picture, which is below the horizontal line* AS, *when the ſun is before the picture; or above it, when beyond. And if* LI, LN, *be drawn parallel to* AS, SX; *cutting the vertical line* SX *in* I, *and the horizontal line* AS *in* N, *and* SL *be drawn. Then if a right line be exposed to the ſun's light; the image of the ſhadow, which ſuch a line, perpendicular to the picture, or vertical plane, or geometrical plane, or their parallels, caſts upon the ſaid plane, is parallel to* SL, *tends to* I, *or tends to* N, *reſpectively.*

1. The ſhadow of the principal ray ES will be LS. For L is the image of the ſun in the picture, and a line drawn thro' L and the eye at E, paſſes thro' the ſun; and therefore L is the ſhadow of E. But planes drawn from the ſun, thro' all the parallels to ES, will cut the picture AC in parallel lines; becauſe the ſun is at an infinite diſtance, and therefore theſe planes are parallel, and their ſections with the picture, parallel (Geom. V. 11); that is, the ſhadows of the lines perpendicu-

lar to AC will be parallel to one another on the Fig. 16.
picture, and to SL.

2. If ſeveral lines be perpendicular to the vertical plane EPQR; their ſhadows upon the plane will be parallel to one another, as appears from what is demonſtrated juſt now. But theſe ſhadows being taken for objects, their appearances in the picture (by Prop. IV. Cor. 1.) will tend to their accidental point, which is I. For the ſhadow of a line paſſing thro' E, parallel to AS, will fall upon LI (Cor. 1. Prop. XVIII.), and that ſhadow falling on the vertical plane, will be parallel to the other ſhadows on that plane, and alſo in the line EI. Therefore EI being parallel to all the ſhadows, I will be their accidental point.

3. Since L is the ſhadow of E, the ſhadow of ER perpendicular to the horizontal plane, will fall in the line GLN. And (Cor. 5. Prop. XVIII.) the ſhadows of all lines perp. to the geometrical plane will be parallel to RG, which is the ſhadow of ER. And ſince EN is parallel to RG, N will be the accidental point of RG, and of all the ſhadows, upon the geometrical plane. And therefore their repreſentations in the picture tend to N.

And the ſame is true of the images of the ſhadows upon any parallel planes; ſince theſe ſhadows are parallel to the former.

Cor. *If the ſun is in the plane of the picture, the appearance of the ſhadow of any line perp. to it, is parallel to a ray of the ſun falling on it. If in the plane of the vertical plane, they tend to* I. *If in the horizontal plane, they tend to* N. *And the ſame for their parallels.*

Fig.

PROP. XXII.

The image of the ſhadow, which a line perpendicular to any plane caſts upon that plane, tends to the image of the point of ſituation of the light upon that plane; that is, to the image of the point where a perpendicular from the light falls upon that plane.

17. Let D be any light, BF a line perpendicular to the plane KH, draw DI perp. to the ſame plane; then I is the point of ſituation of the light upon the plane KH. Then ſince BF, and DI are parallel, they will be in one plane, DBFI; and the ſhadow of BF will be in the ſame plane. But this plane cuts the plane KH in the line IFG. But the image of that line in the picture (by Prop. II.) is alſo a right line *ifg*. Therefore *fg*, which is the image of the ſhadow FG, tends to *i*, which is the image of I, the ſituation of the light.

Cor. 1. *If the ſhadow of a line perp. to a plane, fall on that plane and others parallel to it; the images of the ſeveral parts tend to the accidental point of any part.*

For theſe parts being parallel, their repreſentations (by Cor. 1. Prop. IV.) tend to the accidental point.

Cor. 2. *If many parallel lines ſtand at any inclination upon the plane* KH; *and if* DI *be drawn from the light* D *parallel to theſe lines, cutting the plane in* I. *Then the images of all the ſhadows will tend to the image of* I *in the picture.*

For the plain paſſing thro' DBF will ſtill cut the plane KH in the right line GFI.

PROP.

PROP. XXIII.

The properties of an inclined picture ABC *are the ſame as thoſe of an upright one, in regard to the places, poſitions, and magnitudes of any points, lines or figures; taking* ER *for the hight of the eye, parallel to the vertical line* SX. *And inſtead of lines perpendicular to the picture or to the geometrical plane, taking the ſame lines parallel to* ES *or* SX. 18.

Let NG be a line in the geometrical plane parallel to the ground line XC; *ng* the image of it in the picture. Then the plane ENG, drawn thro' the eye at E, and the line NG which is parallel to the picture, will cut the picture in the line *ng* parallel to NG. And ſo would it do in the upright table, by Prop. III.

Let GF be a line ſtanding upon the geometrical plane at G, parallel to SX. Then *fg* its image will alſo be parallel to SX, or perpendicular to CX. And ſo it is in the upright picture, by Cor. 2. Prop. III.

Draw RG which will meet *fg* at I in the ground line CX. Then by ſimilar triangles, GF : *gf* : : GE : *g*E : : RG : RI : : RE : I*g*; which are the ſame proportions we have in Prop. XV. for the upright picture. Likewiſe NG : *ng* : : EG : E*g* : : ER : *g*I; which is the ſame porportion, as in the upright picture.

So that whether the picture and the lines ER, FG, ſtand ſloping, or upright upon the line CX, at the points R and G; the repreſentations of the lines NG, GF, will fall in the ſame places of the picture, and be of the ſame magnitudes in it.

Cor. *Hence if a perpendicular ſolid, as a priſm or cylinder, &c. as* DM, *is to be drawn; before this can be*

Fig. 18. *be done, by the rules for upright pictures, lines muſt be drawn from the top* D *to the geometrical plane, parallel to* SX, *as* D*m*. *Then by having the length of* *m*D, *the image of* D *may be found; and then the image of* M, *and the ſolid compleated.*

SCHOLIUM.

Hitherto I have been laying down ſuch principles as are the foundation of the whole practice of perſpective; which being well underſtood, the reaſon of the ſeveral operations in the practice will eaſily be ſhewn and made intelligible.

And tho' objects may be drawn in perſpective various ways, either upon a plane perpendicular to the horizon, or parallel to it, or inclined in any given angle; yet ſince almoſt all objects ſtand perpendicular to the horizon, and all views and perſpective draughts, are hung againſt a wall, which is alſo perpendicular to the horizon. And ſince the rules for drawing are the moſt ſimple and eaſy, when the picture is in that poſition. Therefore I have given all the principles in full for that ſort of projection. And in order to extend the rules to other kinds, I have given ſome propoſitions, ſhewing how the other ſorts may be reduced to this.

Alſo as it is moſt natural to look at a draught of any thing directly, rather than obliquely. And all towns, cities, caſtles, towers, churches, edifices of all ſorts, ſtand in a perpendicular poſition, this ſort muſt be eſteemed the moſt natural of any, as well as the moſt ready, familiar, and eaſy.

And tho' the objects I have been deſcribing, are ſuch as fall below the horizontal line, and beyond the picture. Yet all the propoſitions are equally true, tho' the objects are never ſo high; or when they are between the eye and the picture. But to have drawn them ſo, would have required larger figures and more room.

And ſince ſhadows ſerve to diſtinguiſh the different ſides of a body, and to ſhew us what ſides are towards the light, and what from it, and thereby enable us to judge of the poſitions of all the parts of it; and upon this account they give life to a piece of perſpective, and are the chief ornament and beauty thereof. And as this light may either come from the ſun, whoſe rays are to be eſteemed parallel to one another, as coming from an infinite diſtance. Or they may proceed from a ſmall light as a lamp or a candle; and then they diverge from a point near hand. And ſince a perſpective draught is imperfect without ſhadows; therefore I have laid down a few propoſitions, ſhewing the direction and poſitions of the ſhadows of bodies upon all ſorts of planes, by any ſort of light; from whence the method of laying them down in perſpective is eaſily deduced. Fig.

SECT.

SECT. II.

The Practical Rules for drawing in PERSPECTIVE.

HAVING in the firſt ſection treated of the theory or fundamental principles of perſpective; I come now to lay down the rules by which all the operations, in the practical part, are performed. All which are eaſily deduced and demonſtrated from the foregoing theory.

The nature and foundation of drawing in perſpective is eaſily underſtood, by looking thro' a glaſs window, and viewing the ſeveral objects without, and whilſt you keep your eye fixed ſteady in one place, imagine theſe objects painted on the glaſs in the ſame places they appear in; that is, in the places where the rays coming from the ſeveral objects to the eye, meet the glaſs. For if ſuch pictures of the ſeveral objects were actually deſcribed upon the window, it would be really a perſpective draught. But then it would only be drawn mechanically, without any rules of art. Whereas, by the practical rules, which will be here laid down, the very ſame draught may be made, without looking at the objects; and that from certain data, ſuch as the diſtance, poſition, altitude, &c. of the ſeveral objects to be deſcribed.

The practical part of perſpective is either direct or inverſe; the *direct* method is that by which the appearance of any given object is repreſented in the plane of the picture. And the *inverſe* method is that which finds out the object, by having its image given in the picture. In what follows we ſhall ſuppoſe the picture directly oppoſed to the eye, and by the help of ſuch lines and points, as we have given, we ſhall find the appearance of any object upon that plane. In what went before concerning the demonſtrations, we were obliged to view it ſideways; that the eye might ſee all the interſections of the ſeveral lines and planes concerned therein; many of which, in any other poſture, would be hid from the eye; ſo that each of theſe figures is really itſelf a piece of perſpective. But theſe principles being eſtabliſhed, we ſhall have no further occaſion for it in that poſition; but ſuppoſe it placed directly before the eye, as it is to be ſeen from the point of view.

In the direct method, we find firſt how to place the image of a point in the geometrical plane, in the picture. Then the image of a line in the geometrical plane is found, by finding the images of the extreme points. Then the image of any plain figure is found, by finding the repreſentations of all the ſides. Thoſe of curve lines are found, by finding a good number of points, and drawing a curve regularly thro' them. The image of a line perpendicular to the geometrical plane, is eaſily found by having the point or baſe given. And then the image of a ſloaping line. The image of a ſolid is had by finding the image of its baſe, and then erecting perpendiculars at the angles, of a proper hight; and joining the tops of them by right lines.

In general, the images of all lines in the geometrical plane, which are perp. to the ground line, converge to the point of ſight. And thoſe that cut it obliquely tend to their accidental point. And the images of all lines parallel to one another, except ſuch as are parallel to the picture, converge to ſome point or other.

All lines parallel to the ground line, have their repreſentations on the table parallel thereto. And the images of all lines perpendicular to the geometrical plane, are perpendicular to the ground line. All theſe things will be exemplified particularly in what follows.

PROB. I.

To repreſent an object by military perſpective.

Military or orthographic perſpective is the drawing lines thro' all the points of the object perpendicular to the plane of the picture, placed in a proper poſition; and then connecting all the points by right lines as they are in the object. Here all lines that are not directly expoſed to the eye, muſt be made leſs than they are, ſo much as the obliquity is greater. Angles may appear either bigger or leſſer than the truth, according to the poſition of the plane of their ſides. All lines that are parallel in the object, muſt be parallel in the picture. And thoſe perpendicular to the horizon, muſt be perp. to the ground line.

The practice is eaſy. Draw any line for the ground line, and the object being placed, or ſuppoſed to be placed, in a convenient ſituation; let fall perpendiculars from every point or angle of it upon the plane of the picture; drawn in obſcure lines; all theſe will be perpendicular to the ground line. Then lines muſt be drawn from one point to another as they are in the object.

Or thus, Draw the plan or baſe of the object, and draw from all the angles, the ſeveral altitudes of the parts of the object in theſe places, parallel to one another; and perpendicular to the ground line, all in obſcure lines. Then join the tops of theſe lines, by drawing other lines, as they are in the object, and all lines belonging thereto that are viſible; rubbing out thoſe that are hid from the eye. But ſtrictly ſuch lines as are not directly oppoſed to the eye muſt be diminiſhed, according to the ſine of the angle of incidence, from the eye. And the angles will be increaſed or decreaſed, in conſequence of that obliquity. Laſtly, the ſeveral parts or planes compoſing the ſurface of the body, that lie from the eye muſt be ſhaded; the more, the more oblique they are. Fig.

Sometimes you muſt draw the front or profile as well as the plan, to ſhew the ſeveral altitudes.

Examp. 1.

Let OPQRS be a piece of fortification ſeen in front. ABEF is the plan, where all the lines BA, CD, &c. perpendicular to the ground line BE, are reduced in length in the proportion of *a*B to AB, by reaſon of the obliquity of the rays, upon the lines AB, CD, in the baſe. GHIKLMN is the front, where all the lines H*b*, I*i*, &c. are ſhortened in proportion of the radius to the ſine of inclination of the rays upon the lines H*b*, I*i*, &c. in front, or plane GLMN. Then the figure OV*l*RS, being made equal to GHLMN, and the lines OP, V*v*, &c. and RQ, be made equal to BA, CD, &c. and EF; and the lines joining on the ſeveral planes be drawn, as P*v*, OV, *l*R, &c. the ſolid will be formed; which being ſhaded, gives the true appearance thereof OPQRS, according to this method of drawing. 19.

Ex. 2.

ABCD is a table, ſeen both in front and profile. Let EC be the ground line, EF a line parallel to the front. MG the length of the table, MF the length contracted by the obliquity of the lines in the front, parallel to the ground line; and MK perp. to MF, is the breadth; and MH the contracted 20.

Fig. 20. tracted breadth, by the obliquity of the lines perp. to the line EF. Make HI = HK, and draw IN parallel to MF, and make ML = MH, and draw FN parallel to ML; then LNFM is the plan. Make the figure OCDP equal and ſimilar to LMFN. And on the points O, C, D, P, raiſe the perpendiculars OQ, CR, DS, PT, equal to the contracted hights of the table by the obliquities of the lines OQ, CR, &c. Draw a plane ABV, thro' the points Q, R, S, T, whoſe ſides ſhall be parallel to QR, RS, ST, TQ; and finiſh your table as in the figure.

SCHOLIUM.

Drawing by this ſort of perſpective, is no more than drawing the ſhadows of all the lines of an object, which the ſun, (or a light at an infinite diſtance), caſts upon a plane; and then ſhadowing the parts.

The point of ſight in this kind of perſpective is any where. For the eye is ſuppoſed to project any parts of the object, by lines perpendicular to the picture; which comes to the ſame thing as viewing them from an infinite diſtance.

This ſort of perſpective is the beſt for drawing ſmall bodies, as things within a houſe; and particularly all ſorts of engines; for ſtanding at a diſtance, the ſides that are parallel in the object, ſcarce appear otherwiſe than parallel. And upon account of its eaſineſs, it is an excellent method to learn to draw by, and very proper to be practiſed by young ſtudents that are learning to draw; for by this they acquire a habit of judging of the poſitions of the ſeveral parts of an object, and by that means readily learn to draw by hand. The principal difficulty is to know how much any line, (or any other parallel to it) is contracted in the draught, and to do this, either the angle which the viſual ray makes with that line muſt be given; or it muſt be found

by

pa 32.

by experiment; and theſe being had, the thing will be eaſy. But it muſt be obſerved, that the method here uſed of laying down the plan, is not accurate, but only an approximation; and is uſed only to avoid a more tedious and troubleſome conſtruction. And therefore it gives not the angles exactly true. But for ſuch as would be exact in finding the true repreſentation of an angle, I ſhall lay down the following problem. Fig. 20.

PROB. II.

To find the apparent magnitude of an angle in military perſpective.

21. Let ABCD be the plane, BAD the given angle in that plane. At the point A erect the perpendicular AP, to that plane; thro' AP and the eye (at an infinite diſtance) draw the plane PAFG, which will be perp. to the given plane AC. Let the angle AGF be the elevation of the eye above that plane, and let AF be perp. to FG. Then thro' AF and FG draw the two planes AFE, and FGBE, both perp. to AFG; then ſince the three planes AFE, AGB, FGB, are all perp. to the plane AFG, their interſections will be all perp. to AFG, and conſequently parallel to one another; and therefore FE parallel to GB. Therefore if a ray paſſing thro' F fall upon G, another ray parallel to it falling on B will paſs thro' E making FE = GB. Therefore it is evident, the angle BAG in the plane ABCD, is projected into the angle EAF in the perpendicular plane EAF. And the angle BAG is the declination of the plane AFG (or of the eye) from the ſide BA. Therefore in the triangle AFG, rad : $s.AGF :: AG : AF = \frac{AG}{rad} \times S.AGF$, and in the

Fig. 21. triangle AGB, rad : tan. GAB : : AG : GB = $\frac{AG}{rad} \times t$.GAB; therefore in the triangle AFE, rad : tan. FAE : : AF : FE or GB : : $\frac{AG}{rad} \times$ S.AGF : $\frac{AG}{rad} \times t$.GAB : : S.AGF : tan. GAB.

Cor. 1. *Hence, as ſine of elevation of the eye* AGF *above the plane* ABCD : *radius* : : *tan. declination* BAG *of the eye or vertical plane from the ſide* BA : *tan. of the apparent angle* FAE, *from the ſame vertical plane* AFG.

Cor. 2. *After the ſame manner that the angle* FAE *is found for the repreſentation of the angle* GAB *on one ſide of the vertical plane* AFG; *the repreſentation of the angle* GAD *on the other ſide, is found; and conſequently the whole angle* BAD.

PROB. III.

Any figure or figures being given in the geometrical plane, to lay them down in a proper ſituation for drawing them in perſpective.

22. Let AB be the ground line, CDEF the given figure. From every point of the figure, C, D, E, F, draw perpendiculars to the ground line CG, DH, F*f*, EK, and continue them till G*c* = GC, H*d* = HD, I*f* = IF, and K*e* = KE. Then draw *cd*, *de*, *ef* and *fc*. And the figure *cdef* is in a proper ſituation for drawing.

For if the figure CDEF was to remain in its proper place, the projection of it in the picture would interfere with the figure itſelf, and one of them would confound another. Therefore the original is transferred to the other ſide of the ground Fig. line in an inverted poſition, as if the whole figure 22. was turned upſide down over the line AB into the poſition *cdef*. And this is juſt as it would appear by reflexion in a looking-glaſs; and therefore the inverted figure, will by reflexion ſhow the true figure. And as the diſtances of the ſeveral points *c*, *d*, *e*, *f* are the ſame as of the points C, D, E, F, and in the ſame perpendiculars to the ground line; therefore they anſwer exactly the ſame ends, whether the true or inverted figure is made uſe of.

SCHOLIUM.

Hence in what follows, all figures placed in the geometrical plane, are ſuppoſed to be tranſpoſed to the other ſide of the ground line, at the ſame diſtance, and there drawn in an inverted poſition. But it is not always neceſſary that the figures be actually drawn; it is ſufficient in many caſes, to have only the diſtances of their ſeveral points from the ground line, and the points where the perpendiculars cut it. For by having theſe things, all the operations may be performed as well as if the figures were drawn.

PROB. IV.

To find the vaniſhing line of any plane, whoſe interſection and inclination to the picture are given.

Let S be the point of ſight, PQ the interſection 23. of the plane with the picture. Draw RSP perpendicular to PQ, and SE parallel to it, and equal to the principal ray. Make the angle SER equal to the compliment of the angle, which the given plane makes with the picture, interſecting RP in R; draw RV parallel to PQ, for the vaniſhing line of the plane.

The

Fig. 23. The vaniſhing line is the interſection of the picture, and a plane paſſing thro' the eye parallel to the given plane. Juſt as the interſection of the picture and the horizontal plane, is the vaniſhing line of the geometrical plane.

For ſuppoſe the plane RES to be raiſed up perpendicular to the plane of the picture. Then as E is the eye, and RV parallel to PQ; a plane paſſing thro' ERV, will then be parallel to the plane paſſing thro' PQ; and inclined to the picture in the angle ERS. Therefore RV is the vaniſhing line, ſought.

PROB. V.

24. *To find the accidental point of a right line; having given its elevation on the geometrical plane, and the angle its baſe makes with the ground line.*

I call that its baſe, which is a line drawn from the point where it interſects the geometrical plane, to the point where a perpendicular interſects the geometrical plane, which is drawn from the top of it.

Let S be the point of ſight, SP the horizontal line, XQ the ground line. Then as it is all one where the line ſtands, if it be but in a parallel poſition; therefore ſuppoſe it cuts the geometrical plane at X. Make the angle BXC the declination of the baſe, or the angle it makes with the ground line, and DXF its elevation above the plane; which here is towards the eye. Let SE, equal to the principal ray, be perp. to SP, and make the angle SEP = BXC the declination; thro' P, draw PQ parallel to the vertical line SX. Then make PG = PE, and the angle PGR = DXF the depreſſion, here below the geometrical plane; and R is the accidental point.

For imagine the plane ESP to be raiſed up till it Fig. be perpendicular to the plane of the picture SXQP, 24. then the angle SEP being equal to BXC, the plane paſſing thro' EPQ will be parallel to the plane in which the given line is, whoſe baſe is XC; therefore PQ is the vaniſhing line of that plane. Then ſince PG or PE is the diſtance of the line PQ from E; and the angle PGR being equal to DXF, therefore ER muſt be parallel to DX (ſuppoſing ER to be right over S, as was ſaid before); and therefore R is the accidental point of DX, or of the given line, which is parallel to it, by Prop. VIII.

It is eaſy to know, from the poſition of the given line, whether the angle SEP is to be taken to the right or to the left; and whether the angle PGR is to be taken upwards or downwards.

Cor. 1. *If the given line have no declination, its accidental point is in the vertical line* SX.

Cor. 2. *If the given line be parallel to the geometrical plane; its accidental point is in the horizontal line* SP.

Cor. 3. *Two lines proceeding from one point, make the ſame angle, as two lines drawn, from the eye to their accidental points do.*

PROB. VI.

A point in the geometrical plane being given; to find its appearance in the picture.

1. Let AB be the ground line, DD the horizontal line, S the point of ſight; D, D, the points 25. of diſtance. And let C be the point given, tranſpoſed to the other ſide of the ground line. Draw CF perpendicular to AB; make FG = FC, draw FS, then draw GD to the oppoſite point of

Fig. 25. diſtance D, if the original point is beyond the picture, to cut SF in *c*, then *c* is the image of C in the picture.

But if the original point be on this ſide the picture, you muſt draw a line from G to the point of ſight on the ſame ſide; and then the interſection with SF, gives the image of C below the ground line.

For the triangles D*c*S and F*c*G are ſimilar, whence DS : S*c* : : FG : F*c*. But (by Prop. XIV.) the image of C is in the right line SF, and ſo placed, that the principal ray : S*c* : : diſtance of the point : F*c*. But DS is equal to the principal ray (by Def. XVI), and FG is equal to FC the diſtance of the point. Therefore *c* is the image of C.

2. *Otherways thus.*

Set off the diſtance FC both ways from F to G and A, and draw from A and G, two lines to the oppoſite points of diſtance D, D; and their interſection *c* is the place of the image, if C is beyond the picture. Otherwiſe draw lines to the points of diſtance on the ſame ſide.

For ſince AF = FG, and DS = SD, therefore DG and DA interſect SF in the ſame point *c*, and conſequently they interſect one another in the very ſame point.

3. *Or thus.*

When the diſtance of the eye, or the diſtance of the point, is very great, or when the point D or the point G, falls without the picture. Take any aliquot part of SD from S, and the ſame part of FC or FG from F; and from theſe points draw a line to interſect SF, which interſection will ſtill fall upon the ſame point *c*, becauſe ſimilar parts are proportional to the wholes.

4. *Other-*

Fig.

4. *Otherwiſe.*

Let SD be the horizontal line, AB the ground line, S the point of ſight, C the given point tranſpoſed. Draw SE perpendicular to SD, and equal to the principal ray; and draw FC perp. to AB. Draw SF, and EC to interſect it in *c*, the image ſought. If the given point be on this ſide the picture, FC muſt be taken upwards. 26.

For the triangle ES*c*, and CF*c* are ſimilar, whence SE : S*c* : : CF : F*c*. Therefore (Prop. XIV.) *c*, in the line SF is the image of C. Hence alſo, SE : E*c* : : CF : C*c*.

5. *Otherwiſe thus.*

Let SD be the horizontal line, S the point of ſight, AB the ground line, C the given point tranſpoſed. Draw SE perpendicular to SD, and equal to the principal ray. Draw the line EC, then draw any line CB and EL parallel to it, draw LB to interſect EC in *c*, the image of C. 27.

If the given point be on this ſide the picture, draw FC upwards.

For draw CF perpendicular to AB. Then by the ſimilar triangles ESL, CFB, and E*c*L, CB*c*; we have, ES : CF : : EL : CB : : E*c* : C*c* : which comes to the ſame thing as the 4th method.

6. *Or thus.*

From the point C tranſpoſed, draw two lines CB, CG, to the ground line AB; and two lines EL, ED parallel to them, from the eye at E, to the horizontal line LD. Draw LB, DG to interſect in *c*, which will be the image of C.

For by the ſimilar triangles, the lines LB, EC interſect one another in the ſame point, in which DG, EC interſect. And therefore LB, DG interſect in the ſame point.

Fig.

7. *Otherwiſe.*

28. Let SD be the horizontal line, S the point of ſight, AB the ground line, C the given point tranſpoſed, CF its perpendicular. Draw from any point D, two lines DB, DG, making any angle; make DG = principal ray, and EG = CF; draw GB, and EK parallel to it, and KC parallel to AB, cutting SF in *c*, the repreſentation of C.

For by the parallels, DE : EG : : DK : KB : : S*c* : *c*F, according to Prop. XIV. therefore *c* is the image of C.

Cor. 1. *From hence the accidental point of any right line may be found as follows.*

29. Let DE be the horizontal line, S the point of ſight, AB the ground line; then ſince the poſition of the line is given, its interſection with the geometrical plane, and alſo with the picture, is given. Let P be the point in the geometrical plane tranſpoſed, and I that in the picture. Find the appearance of P in the picture as at *p*. And thro' I, *p*, draw EI*p*F. Make *as* the diſtance of the foot of the line from the picture, PC : to the principal ray DS : : So is *p*I : to *p*K; and K is the accidental point of the given line.

30. For let PI be the given line, and let *p* be the appearance of P, in the picture; thro' *p* and I, and the eye at D, draw the plane DIP*p*, in this plane draw DK parallel to PI, then it is evident (Def. XVII.) K is the accidental point of PI. But the triangles P*p*I and D*p*K are ſimilar, whence *p*K : *p*I : : D*p* : *p*P. Let the plane DIPK revolve about the line DP, till it coincide with DS, then the plane DSCP is perp. to the picture, and PC perp. to the ground line; and the triangles D*p*S and P*p*C are ſimilar, whence D*p* : *p*P : : DS : PC. Therefore *p*K : *p*I : : DS : PC.

Cor. 2. *Let* Ip *interſect the horizontal line in* E, *and the ground line in* F. *Then if* pF : pE : : pI : pK, K *will be the accidental point of* PI. Fig. 30.

For the triangles S*p*E, C*p*F are ſimilar, therefore DS : PC : : S*p* : *p*C : : E*p* : *p*F, and (Cor. 1.) DS : PC : : *p*K : *p*I ; whence *p*E : *p*F : : *p*K : *p*I.

SCHOLIUM.

To avoid confuſion with many lines, if two threads be fixt at the points of ſight and diſtance, and be extended to the diviſions of the ground line, their interſection will give the points in the picture. It may be ſufficient only to apply the threads.

PROB. VII.

Any right line in the geometrical plane being given; to find its appearance in the picture.

Find the image of one end of the given line (by the laſt Prob.) and in like manner find the image of the other end of it. Then a right line drawn from one of theſe images or points to the other, will be the image of the given line.

Ex. 1.

Let FG be the given line tranſpoſed; find the image of F at *f*, and the image of G at *g*; then draw *fg*, which is the repreſentation of FG. 31.

Ex. 2.

Let the given line FG be parallel to the ground line PQ, draw FP, GQ perpendicular to the ground line. Then find the image of F at *f*, and of G at *g*. And draw the line *fg* for the image of FG. 32.

Or ſhorter thus, draw SP and SQ, and having found the point *f* as before, draw *fg* parallel to PQ,

to

Fig. 32. to cut SQ in *g*. Then draw *fg* for the image of FG. For (by Prop. III.) the image *fg* will be parallel to FG or to PQ. And (by Prop. XIV.) it will be terminated in the line SQ.

PROB. VIII.

A right lined figure in the geometrical plane being given; to find its repreſentation in the picture.

Find the appearances of all the angular points thereof in the picture (by Prob. VI.) and draw lines from one to another, as in the original.

Or find the appearances of all the lines that compoſe it, one after another (by Prob. VII.) and that will give the image of the whole figure.

And here all lines, perpendicular to the ground line, muſt have their images drawn to the point of ſight; and all lines parallel to the picture, will have their repreſentations parallel. By this means ſeveral ſhort methods of working, eſpecially in irregular figures, will appear of themſelves.

Ex. 1.

33. Let NOPR be a ſquare, whoſe ſide NR is parallel to the ground line AB. DE the horizontal line, S the point of ſight, Make CF = CN, and CA = CO. OC and PI being perp. to AB. From S draw SC, SI, and FE to interſect it in *n* and *p*; draw *nr*, *op* parallel to AB, and *nopr* is the image or repreſentation of the ſquare NOPR.

For ſince CN = CF, *n* is the image of N; and CA being = CO = IP = IF, therefore *p* is the image of P, and therefore *nopr* is the image of NOPR.

Ex. 2.

34. Let LMNOP be a regular pentagon, DE the horizontal line, S the point of ſight, AB the ground line.

12.

ne. From the ſeveral angles L, P, O, N, M, Fig.
et fall perpendiculars upon the ground line, L*f*, 34.
g, M*h*, O*i*, and N*k*. To the point of ſight S draw the lines *f*S, *g*S, *h*S, *i*S, *k*S. Make $f1 = fL$, and draw 1D, to the point of ſight D, cutting *f*S in *l*, the image of L. Make $g2 = gP$, and draw 2D cutting *g*S in *p*, for the image of P. Make $hB = hM$, and draw BD, cutting *h*S in *m*, for the image of M. Make $k3 = kN$, and draw 3E, cutting *k*S in *n*, the image of N. Make $i4 = iO$, and draw 4E, cutting *i*S in *o*, the image of O. Then draw the lines *op*, *pl*, *lm*, *mn*, *no*, which makes the image of the pentagon, required.

Ex. 3.

Suppoſe the two ſquares O, P, ſeen corner wiſe, 35.
are to be drawn, joining upon the ground line AB. S is the point of ſight, D one point of diſtance. From all the angles of the ſquares let fall perpendiculars upon the ground line AB; from the points of interſection draw lines to the point of ſight S. Make $At = tt$, and from the points A, *t*, *t* draw lines to the point of diſtance D, to interſect the former; the firſt in *i*, *k*; the ſecond in *l*, *m*, the third in *n*. Then *iklt*, and *lmnt* are the images of O and P.

Shorter thus. Since the ſides of the ſquares make half a right angle with the ground line; their repreſentations will tend to the points of diſtance, on each ſide. Therefore from the angular points *t*, *t*, draw lines to the point of ſight D, and alſo to the point of ſight on the other ſide; theſe lines by their interſections will give the angles of the ſquares at *i*, *k*, *l*, *m*, *n*.

Or thus. Draw *t*S, *r*S, *t*S, and draw *t*D to interſect theſe in *l*, *m*. Thro' *l*, *m*, draw parallels, *nli*, *mk*, to the ground line AB; and making $li = ln$; this gives the angles *i*, *k*, *l*, *m*, *n*.

The

Fig. The reaſon of theſe operations, is evident; for 35. *t*M = *tt*, and *t*K = *t*A, and *r*L = *rt*, &c. Alſo ſince the lines ILN, and KM, are parallel to AB; *iln*, and *km* will alſo be parallel to AB.

Ex. 4.

To draw a floor of ſquares ſeen fore right.

36. This may be done either with or without a geometrical plan. Let ABGH be part of the plan, where the ground line AB is divided into the equal parts A*f*, *fg*, *gb*, &c. equal to the ſides of the ſmall ſquares. To the point of ſight S, draw AS, *f*S, *g*S, *b*S, &c. to BS. Then from A draw to the oppoſite point of diſtance, the line AE, which will cut the lines *f*S, *g*S, *b*S, &c. in ſeveral points, 1, 2, 3, thro' which, as many parallels muſt be drawn to the ground line AB; the laſt whereof is CF; in which all the diviſions are equal between C and F, and the ſame equal diviſions may be continued from C to K, and from F to I. And from theſe points of diviſion other lines may be drawn from S, which upon the line AE (going thro' the point of diſtance E) give other points, thro' which more parallels to the ground line are to paſs, and thus the whole ſpace ABIK may be filled with ſquares. And if more be deſired, draw from the point K to the oppoſite point of diſtance E, the line KE, which will cut all the lines drawn from S, in as many points, thro' which new parallels muſt be drawn to the ground line AB, which laſt is LN, paſſing thro' the point L, at the interſection of SB, KE.

For ſince *f*A = *fi*, the point 1 repreſents *i*, and *g*A being equal to *g*K, 2 repreſents *k*; and as *b*A = *bl*, 3 repreſents *l*, and ſo on. And the ſides of the ſquares being parallel to AB, their repreſentations in the picture, paſſing thro' 1, 2, 3, &c. will be parallel to AB, by Cor. 1. Prop. III.

If a line be drawn from *f* or *g* or *h*, &c. to E; Fig. that line will paſs thro' all the angles of the ſquares, 36. as well as the line AE, which paſſes thro' the angles 1, 2, 3, &c. For ſuch a line cuts the lines drawn from S to the ſeveral diviſions of AB, in the angular points. As AE paſſes thro' all the angular points; therefore 1E or 2E or 3E (which are parts of the ſame line) alſo paſſes thro' all the angular points. Therefore a line paſſing thro' any diviſion in the firſt parallel, or thro' any diviſion in the ſecond, or in the third, &c. will alſo paſs thro' all the angular points. And therefore a line drawn thro' F and E, or thro' C and E, or thro' K and E, will alſo paſs thro' all the angular points of the ſquares.

Ex. 5.

To draw a floor of ſquares ſeen cornerwiſe.

This alſo may be done either with or without a 37. geometrical plane. Let ABGH be a part of a ſquare ſeen fore right, which contains the little ſquares. Here AB is divided into equal parts by the diagonals of the ſmall ſquares. Let AB be the ground line. DE the horizontal line, D and E the points of diſtance, S the point of ſight. Draw AS and BS to the point of ſight, and from all the points of diviſion in AB, draw lines to D and E the two points of diſtance, of which the extreme rays AE, BD, will cut the former in C and F; draw CF. Then ABFC is the image of a ſquare which contains all the little ſquares; and the other lines drawn to D and E, will by their interſections form the images of the ſmall ſquares, ſeen angle wiſe. And to fill up the whole, divide CF into the ſame number of equal parts as AB is divided into; and draw lines from D and E thro' all the points of diviſion, which will form the remaining ſquares.

Fig. 37. For ſince the ſides of the ſmall ſquares, lying on the geometrical plane make half a right angle with the ground line, their appearances in the picture will tend to the points of diſtance, as their accidental points, by Cor 3. Prop. IV. And ſince AB is divided into equal parts by lines drawn from D or E; therefore all lines parallel to AB, and paſſing thro' the angular points, of the ſquares, will be divided into equal parts by the ſame lines; and therefore EF is divided equally, by the lines drawn from D and E; to the equal diviſions in AB continued.

Ex. 6.

To repreſent a floor of equilateral triangles.

38. Let AB be the ground line, DE the horizontal line, S the point of ſight; D, E, the points of diſtance. Upon the horizontal line DE, make an equilateral triangle PIQ; ſo that SP and SQ may each of them be half the ſide of the triangle. To the point of ſight S, draw AS, BS. And from A and B draw to the points of diſtance E and D, the lines AE and BD, interſecting the former in C and F. Then ACFB is a ſquare in which the equilateral triangles are to be repreſented. Let ABGH be part of the ſquare upon the geometrical plane. AB being divided into equal parts, each equal to the ſides of the triangles; from all the points of diviſion, draw lines to the point P, and from the ſame points of diviſion, draw lines to the point Q, and thro' the points of meeting draw lines parallel to AB. All theſe lines by their interſections with the former, will make the repreſentations of the triangles in the picture, as required. And to fill up the ſquare, divide CF into the ſame number of equal parts, as AB is divided into; then drawing lines from P and from Q thro' all the points of diviſion; the ſquare ABFC will be filled up with equilateral triangles.

For

a 46.

the figures in the geometrical ſquare ABGH, Fig. rminated by continued right lines which in- 38. t the ground line in an angle of 60 degrees. IP and IQ alſo making an angle of 60 de- with the horizontal line DE; therefore P and vill be the accidental points of the lines AH, and all others parallel to them; conſequently Prop. IV.) the appearances of all theſe lines d to the points P and Q. And ſuch lines as in the geometrical plane, being parallel to the und line AB, their repreſentations in the picre as CF will be parallel to AB (by Cor. 1. Prop. I). Alſo (by Cor. 2. Prop. V.) the diviſions of F will be equal; and therefore lines drawn from and Q, thro' theſe equal diviſions, will terminate the remaining triangles, and fill up the ſquare ABFC. And if CF was continued and divided nto the ſame equal parts, and lines drawn from P and Q thro' the ſeveral diviſions, theſe lines would form more triangles, and ſo as much ſpace as you pleaſe may be filled with theſe triangles.

Ex. 7.

To repreſent in perſpective a floor of regular hexagons.

Divide the ground line AB into as many equal 39. parts as you will, and deſcribe a floor of equilateral triangles, as in the laſt example, all in obſcure lines. Then join every 6 triangles together, meeting in the center with all the angular points, and draw lines round them, which will form a hexagon. ABGH is part of the floor drawn upon the geometrical plane. ABCF is the perſpective ſquare, formed by drawing AS, BS to the point of ſight S; and then drawing AE and BD, to the points of diſtance D and E, to cut AS, BS in C and F. All the ſides of the hexagons tend either to the point P or Q; or elſe are parallel to the ground line AB;

Fig. 39. AB; for each of them is the ſide of ſome equilateral triangle, all which is evident by the figure. When the hexagons are all finiſhed in the picture, the ſuperfluous lines may be rubbed out.

PROB. IX.

To draw any given curve line in perſpective, which is ſituated in the geometrical plane.

Find the images of a ſufficient number of points thereof, (by Prob. VI.) and then join them neatly by hand; that is, by drawing a curve line regularly thro' them, making no angles; and the more points are aſſumed, the more exact the work will be.

Ex. 1.

40. Let CGK be any curve, DE the horizontal line, S the point of ſight, D, E the points of diſtance, AB the ground line. Take the points C, F, G, H, I, Z, K of the curve, and let fall the perpendiculars CL, FM, GN, HO, KIP, ZQ. Make L1 = LC, and draw 1D and LS to interſect in *c* the image of C. Make MP = MF, and draw PD and MS to interſect in *f*, the image of F. Make NP = NG, and draw NS and PD to interſect in *g*, the image of G. In like manner make OP = OH, PB = PK, PQ = PI, and QB = QZ. And draw PD, OS interſecting in *h*; and draw BD, QD interſecting PS in *k* and *i*, and laſtly draw QS interſecting BD in *z*. Then *cfghizk* is the repreſentation of the given curve.

Ex. 2.

41. Let FNO be a circle, AB the ground line, DC the horizontal line, S the point of ſight. Draw SE perpendicular to DC, and equal to the principal ray. Draw any number of parallels as FNG thro' the

v.48.

the circle to cut the ground line, as at G, and from Fig. 41. E draw EC parallel thereto, cutting the horizontal line in C. Then to find the image of any point F, draw EF, and CG interſecting one another in *f*, which is the image of F. In like manner EN being drawn will interſect CG in *n*, the image of N. And thus EO will interſect CI in O, which is the image of O. And ſo of the reſt.

For ſince EC is parallel to FG, C is the accidental point of FG, and all its parallels. Therefore (Prop. IV.) the images of all theſe parallels tend to C; therefore (by Prob. VI. Method 5); the image of F will be found at *f*, at the interſection of EF and CG. And the like for the reſt of the points.

Otherwiſe for a Circle.

Let DS be the horizontal line, S the point of 42. ſight, D one point of diſtance, AB the ground line. FKH a circle, C the center. Draw the diameter FH perpendicular to AB, and as many lines as you will parallel to FH, as GL, &c. Then find the image of the diameter FH, which is *fh*; and of all the points G, L, K, &c. as *g*, *l*, *k*, as uſual; and thro' *l*, *k*, *g*, &c. draw lines parallel to AB, and continued to 1, 2, 3, ſo that the diameter *hf*, may biſſect the lines *l*1, *k*2, *g*3, &c. Then will 1, 2, 3, &c. be in the curve, which muſt be drawn by hand thro' all theſe points.

SCHOLIUM.

The appearance of a circle will ſometimes be a circle, when it is ſeen foreright; and to find when this will happen, make the height of the eye a mean proportional, between the two parts of the line of ſtation, taken upon the geometrical plane from the eye's perpendicular, to the neareſt and furtheſt points of the original circle. For let AB be the geometrical plane, LB the diameter of the

Fig. 43. circle ſeen foreright, E the eye, EA the eye's perpendicular, ES the principal ray, SH the picture, and FG the image of the circle LB; and that this may be a circle, the cone ELB muſt be cut in ſubcontrary poſition, by the planes AB and SH; that is, the angle ABE muſt be equal to the angle EGF. Draw CL parallel to SH. Then the triangles ABE and AEL are ſimilar; for the angle ABE = EGF = AEL, and the angle at A is common; therefore AB : AE :: AE : AL, and AE is a mean proportional between AB and AL.

PROB. X.

To draw any figure in perſpective, by help of a floor of ſquares; called the Method of Reticulation.

44. Let AB be the ground line, DE the horizontal line, D and E the points of diſtance, S the point of ſight. Make the ſquare ABGH upon the ground line AB, in the geometrical plane, and divide the ſide AB into as many equal parts as you pleaſe, and ſet off the ſame on BG; thro' theſe diviſions draw lines parallel to the ſides of the great ſquare, which will divide it into little ſquares or cells. Then deſcribe the given figure or figures in their true poſition, in the great ſquare ABGH. This done, draw the ſquare ABGH in perſpective, with all the ſmall ſquares within it, by Ex. 4. Prob. VIII. Then deſcribe each part of the object in ſuch a ſquare or cell of the picture, as you ſee it deſcribed in the correſpondent ſmall ſquare in the geometrical plane; and draw the lines in a like ſituation in one as you find them in the other. And thus by going thro' all the parts ſucceſſively, the whole object or objects will be deſcribed. This method by network is very uſeful, for putting any irregular plane figures in perſpective, conſiſting of many parts;

which would require a deal of time, and the drawing of a great many lines, the common way. Fig. 44.

Examp.

Let IKLMN be a piece of fortification. The part I is ſituated between the ſecond and third ſquare from B, in the firſt croſs row, therefore its image *i* muſt be ſituated between the ſecond and third ſquare of the picture in the firſt croſs row. The part K lies on the ſide of the third ſquare from AB, and of the ſecond from AH; and *k* its image likewiſe lies on the ſide of the third ſquare from AB, and of the ſecond from AC. L lies in the third cell from AH; and ſecond from HG And its correſpondent *l* lies in the third cell from AC, and the ſecond from CF. M runs into the corner ſquare at G; and *m* runs into the corner ſquare at F. N joins upon the ſide BG, in the third ſquare from B; and *n* joins upon the ſide BF, in the third ſquare from B. And the like for all other parts of it, as the center, &c.

PROB. XI.

To find the image of a right line perpendicular to the geometrical plane, at a given point.

1. Let AB be the ground line, DE the horizontal line, S the point of ſight, D and E the points of diſtance. And let P be the image of the given point. Thro' the point P, draw any line KI, cutting the horizontal line in K, and the ground line in I; make IH = the length of the perpendicular given, and draw HK. Draw PQ parallel to the ground line AB, cutting KH in Q; draw PG perpendicular to AB and equal to PQ; then PG is the repreſentation of the given perpendicular. 45.

For (by Cor. 2. Prop. III.) the image GP will be perpendicular to AB. Thro' P draw ML per-

Fig. 45. pendicular to AB, then by Prop. XV. as length of the perpendicular HI : to length of its image : : height of the eye ML : diſtance MP : : (ſimilar triangles) KI : KP : : HI : QP; therefore QP, or its equal PG, is the image of that perpendicular.

2. *Or thus.*

Let RV be the diſtance of the perpendicular from the picture, make VO = VR, and VN = given length of the perpendicular; and draw VS, NS to the point of ſight, alſo draw ND to the oppoſite point of diſtance, to cut SV in 1; and drawing 12 parallel to AB cutting NS in 2, then 12 ſet from P in the perpendicular PM to G; gives PG, for the image of the given perpendicular.

For it is evident by the proceſs, that 1 is the image of R, or of the foot of the perpendicular in the geometrical plane. And therefore 1 and P are equidiſtant from AB. And ſince VN = HI, therefore 12 = QP = PG.

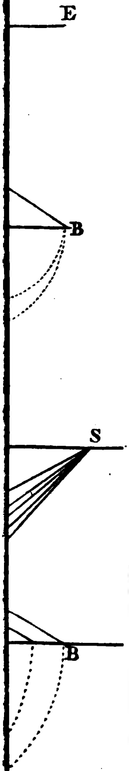
3. *Or thus.*

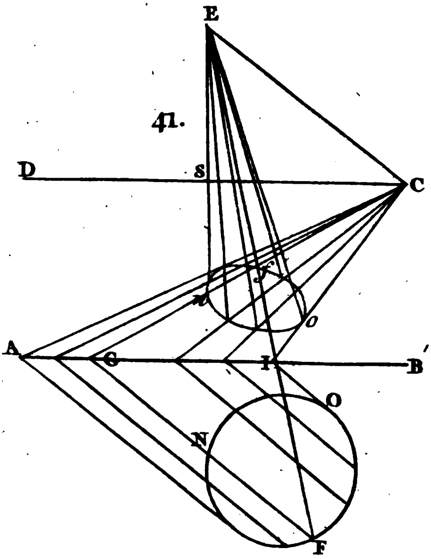
Draw EB perpendicular to AB, from any point E; in which ſet off BT equal to the given length of the perpendicular. And from any point C in the horizontal line DE, draw CB, CT. From P the image of the foot of the perpendicular, draw PX parallel to AB, and XZ parallel to BT; and laſtly ZG parallel to AB; and PG, in the perpendicular PM, is the image of the given perpendicular.

For HI : QP : : KI : KP : : CB : CX : : BT or HI : XZ. Therefore XZ = EP as required.

Cor. *Hence the image of a right line ſtanding obliquely upon the geometrical plain may be found; by letting fall a perpendicular from the top, and finding*

its





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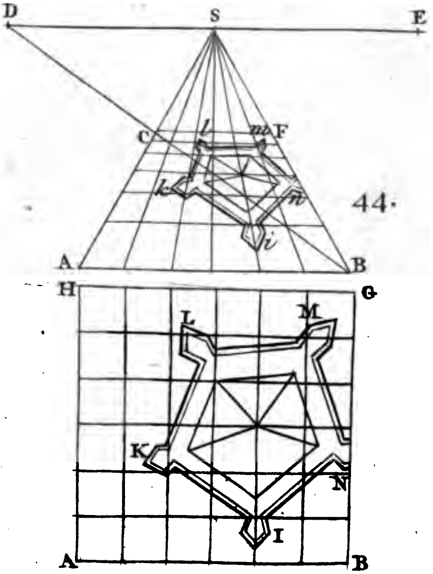


44.

Pl. VI. *pa. 52.*

its appearance, and alſo the appearance of the foot of Fig. *the line itſelf.* 45.

PROB. XII.

To make a ſcale to ſet off the repreſentations of all equal altitudes.

Let AB be the ground line, DE the horizontal line. Draw BE perpendicular to AB, and ſet off BT equal to the original height on the geometrical plane; and from any point V, in the horizontal line, draw VB, VT. And from any point F required, draw FG parallel to BT, and FG is the apparent altitude. And thus all lines parallel to BT as FG, *fg*, &c. will repreſent equal altitudes at the points F, *f*, &c. 46.

Or if the altitude be greater than the height of the eye; in the perpendicular AD, ſet off AH equal to that altitude; and draw lines from A and H to any point V in the horizontal line DE, as AV, HV. Then any line FG drawn parallel to AH, and comprehended between the lines AV, HV, will repreſent the altitude AH at the point F of the picture.

And if the perpendicular object be on this ſide of the picture, produce the lines VA, VH; and at any point *f* draw *fg* parallel to AH, and *fg* will be the repreſentation of AH at the point *f*.

The truth of this appears by the 3d method in the laſt Prob. This is called a *Flying ſcale.*

SCHOLIUM.

It may be proper ſometimes in finding an altitude, to do it in a ſeparate paper, and afterwards to put it into its proper place in the draught. And this is done where there is a multitude of lines, which will be apt to confound one another.

Fig.

PROB. XIII.

To draw a ſolid body in Perſpective.

From all the angles of the body, let fall perpendiculars upon the geometrical plane, and find the appearance in the picture of all the points where the perpendiculars fall (by Prob. VI); upon all theſe points raiſe perpendiculars (by Prob. XI. or XII.) of due lengths, as taken from the ſolid; the tops of theſe being joined with right lines, as they appear in the body, will give the perſpective draught thereof. But ſuch lines muſt be left out as are hid from the eye behind the body. And this being properly ſhadowed, makes the draught complete, and thoſe planes muſt be ſhaded the deepeſt that are moſt oblique to the eye.

If there be any curve lines in the body, perpendiculars muſt be let fall from ſeveral of their points, on the geometrical plane, and the points of interſection put in perſpective, and perpendiculars erected of a due length; then all the reſt is to be done by hand.

Before you begin, you muſt make choice of ſuch a poſition of the body, that the work may be the eaſieſt done; and that the moſt parts of it may be ſeen, and that it may appear to the beſt advantage. If a parallelopipedon be propoſed, it may be placed with one ſide parallel to the picture, ſo that two faces beſide the top may be ſeen. If a ſolid has a triangle for its baſe, it may be ſet to advantage, by placing one ſide parallel to the picture. And if they be ſet to touch the table it will ſhorten the work.

In drawing all bodies, you muſt have its plan drawn out upon the geometrical plane. And if the body be much compounded, you muſt have the front and profile, or the ſections thereof made by Fig. perpendicular planes, which is to direct you in the altitudes of the ſeveral parts, and their places and poſitions. You muſt work by the plan for making the baſe of the body; and by the front and profile for getting the altitudes, to be erected upon the ſeveral parts of that baſe. The profile ſhews theſe heights, and the plan ſhews where they are to ſtand.

Example 1.

To draw a cube in perſpective. This is ſo ſimple, that it needs neither profile nor plan. Let AB be the ground line, S the point of ſight, D the point of diſtance. Let one ſide of the cube be in the ground line at EF. To the point of ſight S, draw ES, FS. Alſo draw DF to the point of diſtance, to cut ES in H. Draw HG parallel to EF, to cut SF in G. Then EFGH is the baſe, upon which at the points E, F, G, H, are to be raiſed perpendiculars equal to EF. Therefore make EI and FK = to EF. Alſo make GL and HM = HG, then I, K, L, M, are the tops of theſe perpendiculars; then draw IK, KL, LM, MI, and they compleat the cube. 47.

For ſince the ſides of the cube are all equal, and EF, FK, KI, IE are all in the picture, their repreſentations are equal to theſe lines. And (by Prob. XI.) HG is the image of EF; and HG, GL, LM, MH being parallel and equidiſtant from the picture, are all equal in appearance. Therefore EGI is the cube drawn in perſpective.

Ex. 2.

To draw a triangular priſm, whoſe baſe is an equilateral triangle. Let one ſide be in the ground line AB, at NO. Find the appearance of the other angle of the baſe at P, by Prob. VI. Then NOP is the baſe. Make NQ equal to the height of the priſm, 47.

Fig. priſm, and draw QS, and thro' P draw RV parallel to NQ, then ſet off Pp equal to RV, and 47. perpendicular to BA; alſo make N*n*, O*o* equal to NQ, and perpendicular to AB. Then draw *no*, *op*, *pn*; and NOP*pno* is the priſm required.

Ex. 3.

48. To draw a parallelopidedon, whoſe baſe is a rectangle. Here P is the plan and Q the profile; AB the ground line, DS the horizontal line, S the point of ſight, D the point of diſtance. By Prob. VI, find the appearance in the picture of all the points in the plan, as D, D, D, D, I, I, I, I, make FO equal to the perpendicular height of the body, draw OS, and thro' D, and I, draw the lines KD, LI parallel to AB: at D, D, erect two perpendiculars equal to KD, as D*d*, D*d*. And at I, I, raiſe two perpendiculars equal to LI, as I*i*, I*i*. Then drawing the ſlant lines as in the profile, and alſo the lines *di*, *ii*, *id*, *dd*. You have the parallelopipedon required.

Ex. 4.

49. To draw a cylinder in perſpective. Let AB be the ground line, SD the horizontal line, S the point of Sight, D the point of diſtance, OGHK the baſe of the cylinder on the geometrical plane. Let the diameter OH be perpendicular to AB, and the diameter GK parallel to it; from all the points O, G, H, K, and as many more intermediate points as you will, let fell perpendiculars to the ground line AB; from which points draw lines to the point of ſight S. Make FC, FA = to FO, FH; draw AD, CD to the point of diſtance D, cutting SF in *h* and *o*, ſo will *ho* be the image of HO. In like manner *g* is the image of G, and *gk* drawn parallel to AB, to cut DC in *k* gives *gk* for the image of GK. And as many intermediate points may thus be found as you deſire. Then joining all theſe points by a curve line, gives the baſe of

the cylinder *gbko*. Make CB the altitude of the cylinder, and draw BD; and from *o*, *k*, *b*, draw parallels to AB to cut BD; and the lengths of theſe parallels ſet perpendicular at the points *o*, *k*, *b*, *g*, gives the altitude of the cylinder in theſe ſeveral points; and as many more intermediate altitudes may be ſet up as you pleaſe, by the ſcale CDB. Then the curve 1234, drawn thro' theſe points, determines the top of the cylinder. If the cylinder be an oblique one, inſtead of raiſing the perpendiculars from *o*, *k*, *b*, *g*, they muſt be raiſed from the points where the perpendiculars from the top cut the plan, and at laſt the oblique lines *o*1, *k*4, *b*3, *g*2 muſt be drawn, and the curve thro' the points 1, 2, 3, 4, as before. Fig. 49.

Ex. 5.

Let a pyramid be put in perſpective with a hexagonal baſe. Let CEFGHI be the plan of the baſe. From all the angles of the baſe let fall perpendiculars upon the ground line AB, and from the points of interſection, draw lines to the point of ſight S. Then ſetting off the diſtances of theſe angles from AB, from theſe points, towards B, draw lines to interſect the former. Thus H, I, C are projected into *b*, *i*, *c*. Then drawing lines from *b*, *i*, *c*, parallel to AB, will by their interſections with the correſpondent lines, find the points *g*, *f*, *e*, which are the projections of E, F, G. Then if P be the point where the perpendicular from the vertex cuts the baſe, find its projection *p*, as you did the reſt. Set off the altitude of the pyramid from 1 (in the perpendicular P1) to 2, and draw 2S; and thro' *p*, draw 3*p* parallel to AB, which ſet from *p* to V, perp. to AB. Then draw lines from the vertex V to all the angles of the baſe, *cefgbi*; as V*e*, V*f*, V*g*, V*b*, V*i*, V*c*; or rather leaving 50.

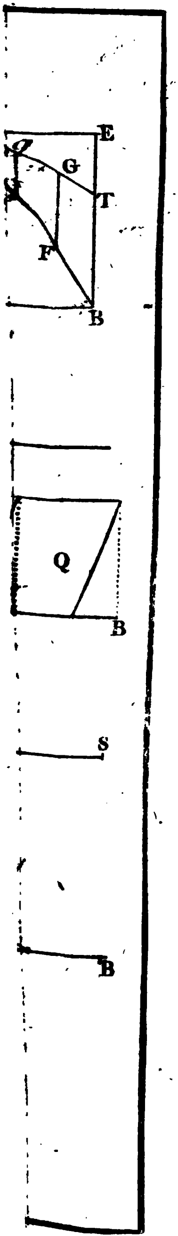
Fig. 50. ing out V*c* and V*e*, which are inviſible, draw the reſt of the lines, and the pyramid is finiſhed.

Ex. 6.

51. To draw the fruſtum of a ſquare pyramid in perſpective. Let P be the plan of the baſe, Q the profile. From all the angles of the plan, let fall perpendiculars on the ground line AB, from the points of interſection draw lines to the point of ſight S. Then take ſucceſſively the diſtances of theſe angles from AB, and lay them from their correſpondent perpendiculars towards A; and from the points where they fall, draw lines to the point of diſtance D, to interſect the correſpondent lines drawn towards S; the points of interſection joined by right lines, gives the perſpective of the baſe LMNO, and likewiſe of the interior ſquare 1234. Then make BC the perpendicular altitude of the fruſtum, according to the profile, and draw lines from any point S, as SB, SC. From 1234 draw parallels to AB, to interſect SB, in certain points, from which other parallels muſt be drawn to BC, reaching to SC; and theſe laſt parallels muſt be ſet off from the correſponding points 1, 2, 3, 4, perpendicularly; theſe give the angles at the top at *l*, *m*, *n*, *o*. And the lines *lm*, *mn*, *no* *ol*, being drawn, give the top end of the fruſtum; and laſtly, the lines *l*L, *m*M, and *o*O drawn, and the ſide L*o*, and top *mo*, ſhaded; the fruſtum is finiſhed.

Ex. 7.

52. To repreſent a hollow cube in perſpective. Let P be the baſe, from all the points of it, let fall perpendiculars to the ground line AB, and ſet off their diſtances along BA, as before. Draw lines from the firſt points to S, and from the laſt to D, to interſect in the points of the perſpective baſe, as at L, M, N, O, and 1, 2, 3, 4. Make BC the



pa.58.

height, equal the ſide of the cube. And make a ſcale SCB as before for ſetting off the altitudes, by drawing from L, 1, 4, O parallels to AB, and from the points of their interſecting SB, draw parallels to CB, for the reſpective altitudes at L, 1, 4, O, and thoſe oppoſite on the other ſide. Several lines are left undrawn to avoid confuſing ſo ſmall a figure; but the way of proceeding is evident. Fig. 52.

Ex. 8.

To draw ſeveral rows of ſquare pillars, at equal diſtances and of equal heights. 53.

Firſt deſcribe the baſes or ſituations of all theſe pillars, which is beſt done by deſcribing a floor of ſquares, as in Ex. 4. Prob. VIII. and upon each row of ſquares taken at equal diſtances, erecting perpendiculars upon the ſeveral baſes, equal to the given height. Here the ſquares are all equal to the deſigned baſes of the pillars; the pillars being all finiſhed, ſuch ſquares as are hid behind the pillars muſt be rubbed out, and the parts that are viſible may be drawn in full.

If inſtead of ſquare pillars you would have cylinders, then the ſquares muſt be made equal to the diameters of the cylinders, and deſcribe the repreſentation of the circles of the baſes within the ſquares, upon which perpendiculars muſt be erected as before. Or if they be cones, perpendiculars muſt be erected from the middle of every correſpondent ſquare, from the top of which two right lines muſt be drawn to the ſides. And after this manner any other priſmatic bodies may be drawn, having any poligonal baſes, deſcribing firſt the baſes of the polygons in their proper ſquares, and erecting perpendiculars at every angle, of the height given; and compleating the ſolids.

As to the ſquare pillars, it is evident that two parallel ſides are parallel to the ground line AB, and

Fig. 53. and the other two ſides, which are alſo parallel, range to the point of ſight S.

Here the foremoſt pillars ſtand in the ground line AB; but if they had been at ſome diſtance, the images of the firſt would have been diminiſhed, which requires ſo much labour, more than if they had ſtood in the ground line.

Ex. 9.

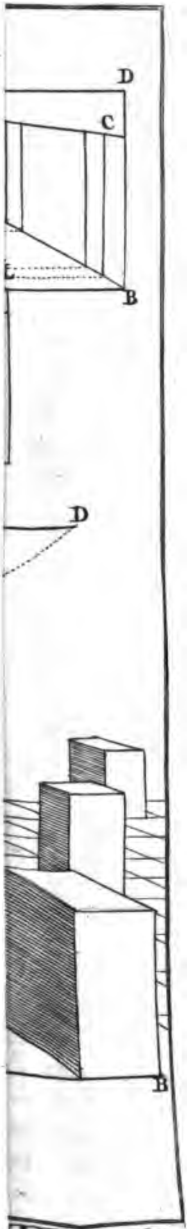
54. To draw an equilateral croſs, ſtanding upright upon the geometrical plane. Draw the plan PP and the profile QQ. Let the plan PP be drawn in perſpective, which will be *cccc*; and from the points *c*, *c*, *d*, *d*, &c. raiſe perpendiculars to the ground line AB, upon which ſet off the heights of the croſs in theſe particular places, according to the profile, but diminiſhed, becauſe it does not ſtand upon the horizontal line. Then all the lines being drawn as in the figure, will give the perſpective appearance of the croſs.

If the croſs had been double, the plan would be the ſame as the profile PQ, and the fore part would come to the ground line AB, and perpendiculars muſt be erected at R, *r*, &c. of a proper height, and lines drawn to terminate the other ends of the croſs.

Ex. 10.

55. To repreſent a hollow priſm ſtanding upon an edge. Let P be the plan, Q the profile when cut thro' the middle. Produce the lines of the plan till they cut the ground line AB, and from thence draw lines to the point of ſight S. Then ſetting off the diſtances in the plan, 3, 1, 4, &c. along the ground line towards A, draw lines to the point of diſtance D as uſual, which will find the points 3, 1, 4, &c. on the lines drawn to the point of ſight, correſponding to the points 3, 1, 4 in the plan.

Then



III. pa. 60.

Fig. 55. hen from the points 2, 2 in the picture, raiſe per- endiculars by the plan, which will give the exter- al angles 2, 2 of the priſm, the points 1 and 1 are he angles of it below, and perpendiculars erected t 1, 1, by the plan, will give the angles 1, 1 at he top. Then the lines 12, 21, 11, 12, 21, 11, being drawn, give the external circumference of the oreſide of the priſm. Then to find the inner cir- umference bounding the cavity, from 3, 4, &c. n the line S2, raiſe perpendiculars according to the rofile, which will be *r*5, *r*5; theſe will give the oints 5, 5, 5, 5, being the inner angles of the nternal poligon. And from 3, 3 raiſe other per- endiculars, which are equal to *r*2 by the profile, nd theſe will give the points 3, 3. Then theſe points connected by right lines will give the inner polygon, 535535. In drawing theſe perpendiculars, they muſt be diminiſhed according to their ſituation by Prop. XV. and ought to be drawn in a ſeparate paper, to avoid ſo great a confuſion of lines; then for the back ſide of the priſm, or that from the eye; ſince the poſition of the priſm is ſuch, that its axis is parallel to the ground line, the interſections of all the faces with one another, will be parallel to the picture; and therefore to find the angles, draw from the points 1, 2, &c. lines parallel to AB, whoſe length muſt be 2B, but diminiſhed according to their ſituations; theſe give the angles 6, 6, 6, which joined by right lines give the appearance of the back ſide of the priſm 666, as far as it is viſible. Then for the inner part, from 3, 5, &c. draw lines parallel to AB as before, but diminiſhed by Prop. XV; which will give the points 7, 7; and lines being drawn from one to another, will give part of the inner poligon on the back ſide, ſo far as it is viſible thro' the hole. And the planes that bound it, being properly ſhaded, we have the repreſentation of the hollow priſm as required.

Fig. required. But it is impoſſible to repreſent all the operations in ſo ſmall a draught.

Ex. 11.

56. *To draw a trough in perſpective, being a hollow parallelopipedon.*

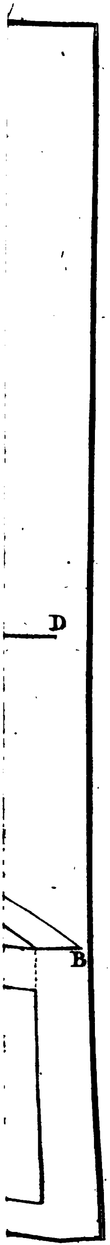
Here P is the plan, and Q the profile or perpendicular ſection. From all the angles in the plan, draw lines perpendicular to the ground line AB, and draw lines from thence to the point of ſight S. Set their diſtances toward A, and draw lines from thence to the point of diſtance D, to cut the former, which will give the points *n*, *n*, &c. in the picture. From the ſeveral points *n*, raiſe the perpendiculars *no*, in the picture, being the repreſentation of *no*, *no*, &c. in the profile. Likewiſe erect the perpendiculars *nr*, *nr*, &c. to repreſent *nr*, *nr* in the profile, which will give the bottom of the trough. Having got the points *n*, *n*, on one ſide, you need only draw *nn*, *nn*, &c. parallel to AB, to cut the lines proceeding from S, which will give the points *n*, *n*, on the oppoſite ſide. In like manner, having got the points *o*, *o*, &c. on one ſide, thoſe on the other ſide are had by drawing parallels to AB, to cut the perpendiculars on the oppoſite ſide, which gives the points *o*, *o*. Then the lines *nn*, *nn*, &c. being drawn give the baſe; and *oo*, *oo*, &c. likewiſe drawn, give the edges at top, on the outer and inner ſide. Likewiſe *rr*, *rr*, &c. drawn makes the bottom. And *no*, *no*, &c. drawn give the corners, which finiſhes the trough.

Ex. 12.

To draw a parallelopipedon leaning againſt a wall.

57. Let P be the plan, Q the profile, or front view. From all the points 1, 2, 1, 1, where the plan cuts the ground line AB, draw lines to the point of ſight

S;



a.62.

S; then ſetting the diſtances of 1, 3, &c. towards B, draw lines to the point of diſtance D, to interſect the former in points of the baſe, where perpendiculars are to be erected; which being done, by the profile, diminiſhing thoſe that are on the further ſide; the points 4, 4, 4, and 5, 5, 5, are found, which are the corners at the top. Then thro' theſe points, drawing lines as in the ſolid itſelf, we have 1444 for the firſt face, and 555 for the further face of the ſolid. Draw 45 and 54, and the ſolid is finiſhed. Fig. 57.

PROB. XIV.

To find the image of the Sun in the picture.

Let DR be the horizontal line, AB the ground line, S the point of ſight, D the point of diſtance. Draw QS perpendicular to DR, and make SQ = SD, and make the angle SQN equal to the ſun's declination from the vertical plane, on the left or right, according as the ray drawn thro' the eye and the ſun interſects the picture. Then draw NL perpendicular to DS, and take NR = NQ, and make the angle NRL equal to the ſun's altitude, above or below the horizontal line DS, according as the ſun's ray drawn thro' the eye interſects the picture; then L is the image of the ſun in the picture. Alſo draw LI parallel to DS, to cut QS in I, and N is the point of *declination*, and I the point of *inclination* in the picture. 58.

For if the triangle NQS be raiſed up till it ſtand perpendicular over NS; then ſince the line SQ is equal to SD the diſtance of the eye from the picture; and the angle SQN, is the declination of the ſun from the picture; therefore the ſun's image will be ſomewhere in the line NL. Again, ſince NR is equal to NQ, the diſtance of the eye from N, and the angle NRL is the ſun's altitude, therefore

Fig. fore he is in the point where RL interſects NL,
58. and that is at L.

Cor. *If the ſun is in the plane of the picture, his place* L *will be at an infinite diſtance, in the line* RL.

For then his declination from the vertical plane, is 90 degrees.

PROB. XV.

59. *To find the appearance of the ſhadow of a right line, perpendicular to the geometrical plane, to the picture, or to the vertical plane; caſt upon that plane, or their parallels, by the ſun's light.*

1. Find the appearance of the ſun in the picture at L by the laſt Prob. and alſo the points N and I, the points of declination, and inclination, And let GH be a line perpendicular to the geometrical plane. Thro' the point of declination N and the foot of the line H, draw the line NH, which muſt be terminated by drawing the line LG from the place of the ſun thro' the top of the line G to interſect NH in M, then HM is the image of the ſhadow of GH. Alſo the ſhadow of GH upon the picture or vertical plane will be parallel thereto. And therefore the image will be perpendicular to AB.

For (by Prop. XXI) the ſhadow MH tends to N, the point of declination; and M is the ſhadow of G. Therefore MH is the ſhadow of GH. The reſt follows from Cor. 2. Prop. III.

2. Let FK be a line perpendicular to the picture. Let fall FC perpendicular to the geometrical plane, and thro' N and C draw N*cf*; thro' L the ſun's place and F, draw LF*f*, cutting NC*f* in *f*; which is the ſhadow of F. Draw *fk* toward S, likewiſe draw LK cutting *fk* in *k*, then *k* is the ſhadow of K, and conſequently *fk* is the image of

the ſhadow of FK. And by Prop. XXI, *fk* muſt tend towards the point of ſight S. And if the ſhadow of FK falls upon the vertical plane, its ſhadow will be parallel to FK, and its image will tend to S. Fig. 59.

Let PQ be alſo perpendicular to the picture, draw QR parallel to SL, and QR will be the direction of the ſhadow in the picture; ſuppoſing the ſun on your back, and it is terminated by drawing LP from the place of the ſun, to the end P of the line, to cut QR, ſomewhere in the plane ARQ; for the part of the ſhadow that falls without the plane is loſt. For if it cut it at R, then R is the ſhadow of P, and QR the ſhadow of QP in the picture; and if a line perpendicular to the picture was erected at S, it is evident its ſhadow would tend to L. But PQ being parallel to the line at S, its ſhadow by the ſun's light, will be parallel to the ſhadow of the other, or to the line SL, as is evident by Prop. XXI.

3. Let VZ be a line perpendicular to the vertical plane. Thro' the point of inclination I, and the foot of the line Z, draw the line IZ*v*, which is the indefinite ſhadow of VZ, and muſt be terminated by drawing the line LV thro' the place of the ſun, and the end of the line VZ, to interſect the ſhadow Z*v* in *v*; then Z*v* is the ſhadow of VZ in the picture, or its appearance.

For (by Prop. XXI.) the image of the ſhadow *v*Z tends to I the point of inclination, and *v* is the ſhadow of V; therefore Z*v* is the ſhadow of ZV, upon the vertical plane Z*v*1, ſuppoſing the ſun in your face; otherwiſe the ſhadow muſt be directed the contrary way.

Again the ſhadow of VZ upon the geometrical plane will be parallel thereto (by Cor. 1. Prop. XVIII), and therefore parallel to the ground line AB; and this ſhadow being taken as an object, its

Fig. 59. appearance in the picture will alſo be parallel to AB, by Prop. III. Likewiſe the ſhadow of VZ upon the picture will be parallel to VZ, and that ſhadow coincides with its image.

Cor. 1. *Hence if a line be perpendicular to the geometrical plane, the image of its ſhadow upon the vertical plane or the picture, is perpendicular to the ground line. If a line be perpendicular to the picture, the image of its ſhadow upon the geometrical plane and vertical plane, tends to the point of ſight. And if a line be perpendicular to the vertical plane, the image of the ſhadow upon the geometrical plane, or the picture, will be parallel to the ground line. And the ſame of their parallel planes.*

Cor. 2. *When the ſun is in the plane of the picture; the image which a right line perpendicular to the geometrical plane, caſts upon that plane or its parallels, is parallel to the ground line; upon the vertical plane or its parallels, is perpendicular to the ground line.*

For the point N in this caſe is removed to an infinite diſtance along the line SD, and therefore NHM will be parallel to AB, which is the appearance of the ſhadow of GH.

Cor. 3 *When the ſun is in the plane of the picture; the image which a line perpendicular to the picture, caſts upon the picture or its parallels, is parallel to* LS. *Upon the geometrical plane or its parallels, tends to the point of ſight; upon the vertical plane or its parallels, tends to the point of ſight.*

For if a line be erected at L, perpendicular to the picture, LS will be the ſhadow thereof; and all lines parallel to that line, will have their ſhadows parallel to one another and to LS. When the ſhadow falls upon the geometrical or vertical plane; then the real ſhadow will be perpendicular to the picture; and conſequently its image tends to the point of ſight. Fig. 59.

Cor. 4. *When the ſun is in the plane of the picture; the image of a line perpendicular to the vertical plane, falling on it or its parallels; will be perpendicular to the ground line. On the geometrical plane or its parallels, is parallel to the ground line. Upon the picture or its parallels, is an infinite ſurface.*

For the point I being removed to an infinite diſtance, in the line SI, the line IZv will be parallel to IS, or perpendicular to AB. And on the geometrical plane the real ſhadow is parallel to the ground line, and therefore its image is alſo parallel to the ground line.

PROB. XVI.

To find the image of the ſhadow of a right line falling upon an inclined plane, by the ſun's light.

60. Let CE be a right line or any upright object upon the geometrical plane; PFQ an oblique plane cutting the geometrical plane in the line RV; let OKB be a plane parallel to the vertical plane, and therefore perpendicular to the geometrical plane, and cutting the oblique plane RQ in the line PQ, and the geometrical plane in KB. Let N be the point of declination, and I the point of inclination, L being the ſun's place. Thro' E the foot of the object, and N the point of declination, draw NEG to the foot of the plane OKB. From G raiſe the perpendicular GH, to cut the top of the oblique plane PQ in H, and from F, where the line NG cuts the baſe of the oblique plane, draw FH. Laſtly from the place of the ſun L, draw the top of the object C, the line LCZ cutting FH in Z, where the ſhadow is terminated. Then EFZ is the ſhadow of the object CE.

Fig. 60. For (by Prob. XV.) EFG is the direction of the ſhadow of the object CE, and GH is a part of the ſhadow when the oblique plane is taken away. Therefore FH is the ſhadow upon the oblique plane. And this is terminated at Z by the line LC produced to cut FH in Z; where Z is the ſhadow of C.

Cor. 1. *If the oblique plane* RQ *was taken away, the ſhadow would be* EGH; *the part* EG *upon the horizontal plane, and a part of* GH *upon the perpendicular plane.*

Cor. 2. *If the object* CE *had been higher, a part of the ſhadow would have fallen upon the line* HT; *and then the ſhadow of* CE *would have fallen upon the three planes,* NB, RQ, *and* BO.

Cor. 3. *After this manner it will be eaſy to trace the ſhadow of any perpendicular object thro' any number of planes however ſituated.*

Cor. 4. *If the object is not perpendicular to the geometrical plane, yet its ſhadow may be found, by letting fall a perpendicular from the top as* CE, *and finding the point* Z *as before; and drawing thro' the foot, which ſuppoſe to be at e, the line* NeV, *and then drawing* VZ.

SCHOLIUM.

This ſuppoſes the ſun to be beyond the picture; but if it is behind your back, you muſt raiſe the plane OB perpendicular to the geometrical plane as before, but parallel to the picture. And the reſt of the work will be the ſame. Here if the line NE does not cut the baſe RV of the oblique plane, or if LC does not meet FH; the ſhadow will not fall upon the inclined plane. If a line is drawn from L thro' any point 1 of the line CE, to cut the line NE produced in 2; then will 2FZ repreſent

preſent the ſhadow of the part of the line C1; and E2 the ſhadow of the part E1 upon the geometrical plane. The ſhadow of the point 1 will be repreſented by 2; and ſo the repreſentation of any point of the line CE may be found by drawing a line thro' L and thro' that point; and ſo it will be known whether that ſhadow falls upon the geometrical plane, or upon the oblique plane, or upon the perpendicular one. When the ſun is in the plane of the picture, the points N and I vaniſh, being at an infinite diſtance, and then the line EF muſt be drawn parallel to the ground line AB, till it meet the oblique plane in F, at the common interſection of it with the vertical plane; and proceed with the reſt as before. Fig. 60.

PROB. XVII.

To find the image of the ſhadow of a body upon the geometrical plane, when the ſun is in the plane of the picture.

Let fall perpendiculars from all the angles of the 61. body upon the geometrical plane; from all the points of interſection, draw lines parallel to the ground line, continued on the oppoſite ſide of the light. Then from all the angles of the body, draw lines parallel to the rays of the ſun, to cut the former lines drawn thro' the feet of the perpendiculars, each with each; the points of interſection will determine the ends of the ſhadow.

Examp.

Let DEFG*o* be an inverted pyramid, whoſe vertex is at *o*, in the geometrical plane; from the points D, E, F, G, let fall the perpendiculars D*d*, E*e*, F*f*, G*g*, upon the geometrical plane. From *e*, *f*, *g*, draw the lines *e*1, *f*2, *g*3, parallel to the ground

Fig. line AB. Then thro' E, F, and G, draw the lines 61. E1, F2, G3, parallel to the ſun's ray LS; TSL being the ſun's altitude. The firſt interſects *e*1 at 1, the ſhadow of E; the ſecond interſects *f*2 at 2, the ſhadow of F; and the third interſects *g*3 at 3, which is the ſhadow G. The point D is omitted, being on the other ſide of the body, can give no ſhadow. Then drawing *o*1, 12, 23, *o*3; theſe lines terminate the ſhadow of the pyramid. For 1*o* is the ſhadow of the ſide E*o*, and 12 of the line EF; 23 of the line FG, and 3*o* the ſhadow of the line G*o*. And therefore *o*123*o* is the ſhadow of the pyramid *o*DEFG.

PROB. XVIII.

To find the image of the ſhadow of a body upon the geometrical plane; when the ſun is out of the plane of the picture.

Find the ſun's place in the picture, as alſo the point of declination. Then from all the angles of the body let fall perpendiculars upon the horizontal plane. Thro' the places where they fall, draw lines from the point of the ſun's declination, continued beyond the body. Then draw lines from the ſun's place thro' all the angles of the body, to cut the former; the points of interſection will terminate the ſhadow of theſe perpendiculars; and the points of interſection being joined by right lines, terminate the ſhadow of the body, as repreſented in the picture. Note, ſuch lines as lie next the light and caſt no ſhadow may be left out.

Examp.

62. Let CD3 be the fruſtum of a pyramid. Find L the place of the ſun, N the point of declination, in the horizontal line NS; AB is the ground line. From

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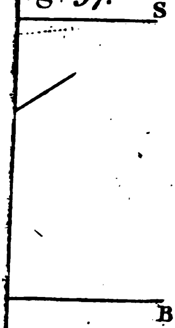

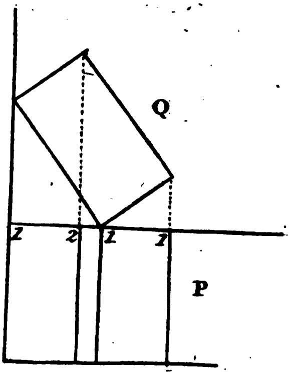

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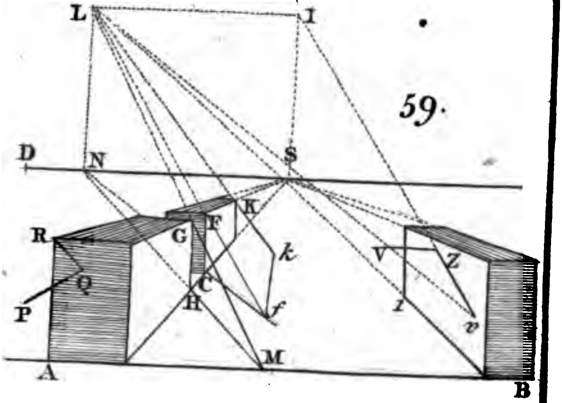

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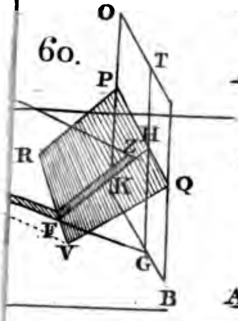

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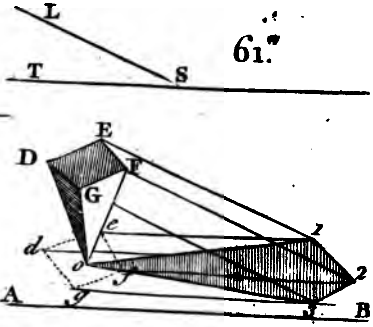

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Fig. 62.

From the points C, D, E, which lye from the ſun, let fall the perpendiculars C*c*, D*d*, E*e*, upon the horizontal plane. Then thro' *c*, *d*, *e*, draw from N, the lines N*c*, N*d*, N*e*, continued beyond the fruſtum, and theſe are the indefinite ſhadows of the perpendiculars C*c*, D*d*, E*e*. Then to determine the ſhadows of the points C, D, E, draw from L the place of the ſun, thro' theſe points, the lines LC4, LD5, LE6; cutting the former lines drawn from N, in the points, 4, 5, 6, which are the ſhadows of C, D, E. Then drawing the lines 14, 45, 56, 63, they give the ſhadow of the fruſtum required.

SCHOLIUM.

If part of the ſhadow fall upon an inclined plane, it may be found thereon, by finding the ſhadows of the ſeveral perpendiculars, after the manner of Prob. XVI; and then drawing lines from the extreme points which will terminate the ſhadow. And by Prob. XV. the appearance of the ſhadow of a line falling perpendicular to the picture or vertical plane may be found, or of any number of lines, and conſequently that of a ſurface or ſolid may be found on any of theſe planes.

In drawing any ſorts of ſhadows, they muſt be taken for objects; and therefore their real poſitions, upon the planes they fall on, muſt be known.

PROB. XIX.

To find upon the geometrical plane, the image of the ſhadow of a point, a line, a ſurface or ſolid, expoſed to a ſmall light.

From the point given let fall a perpendicular to the geometrical plane; likewiſe from the light, let fall a perpendicular to the ſame plane, to have the foot of the light. Then from the foot of the light

Fig. draw a line thro' the foot of the perpendicular, which is the direction of the ſhadow. Then draw from the light thro' the given point, a right line cutting the former, then the point of interſection is the ſhadow of the given point in the picture.

And the line contained between the foot of the perpendicular, and this point, is the ſhadow of the perpendicular.

After the ſame manner if another point is given, the image of its ſhadow may be found, and the line drawn between them is the image of the ſhadow of a line drawn between the two given points.

And thus the repreſentations of the ſhadows of all the lines and points in a ſolid may be found, and conſequently the image of the ſhadow of the whole body.

Exam. 1.

63. Let EHG be a cube ſtanding upon the geometrical plane; L a ſmall light, LP a perpendicular upon that plane. Then to find the ſhadow of the point F, draw from the foot of the light P, thro' the foot of the perpendicular G, the line PG3, for the direction of the ſhadow. Then draw thro' the light L and the given point F, the line LF continued to interſect the former in 3; then 3 is the ſhadow of the point F; and G3 is the ſhadow of the line FG.

Again for the point E; from the foot of the light P draw PI thro' the foot of the perpendicular EI, continued; then thro' the light L and the point E, draw LE continued to interſect the other in 2; then 2 is the ſhadow of the point E; and I2 the ſhadow of IE. And if 23 be drawn it is the ſhadow of EF. And G32I is the ſhadow of the plane GFEI. Again, for the point C and line CK; thro' P and K draw the line PK continued; and thro' L and C draw LC to cut the former in

1;

1; then 1 is the ſhadow of C, and K1 of KC. Fig. Therefore drawing 12, 23, 3G; we ſhall have 63. 123G for the image of the ſhadow of the cube CGE.

Ex. 2.

Let DI be a cylinder, CE a perpendicular beam, 64. L a ſmall light, P its foot, GF a perpendicular wall. To find the ſhadow of the beam CE. Thro' the foot of the light P, and the foot of the beam E, draw PEF, which is the direction of the ſhadow upon the geometrical plane; where it touches the foot of the wall at F, raiſe FG perpendicular to the ground line. Then from the light L, and thro' the top of the beam C draw LCG, cutting FG in G, which is the height of the ſhadow. At the place I where the ſhadow meets the cylinder, ſet off the diameter of the cylinder, above the line EF, which will find the higheſt point of the cylinder, thro' which the ſhadow is to paſs, and thro' that deſcribe a ſemicircle for the form of the ſhadow. From L to touch the top of the ſemicircle at I, draw the line LH, which will interſect PF in H, the point to which the ſhadow of the cylinder reaches, at the point I. So that between H and the cylinder, we have the ſhadow of the cylinder; and beyond that is the ſhadow of the beam. As it paſſes over the cylinder, it is in form of a ſemicircle; and ſtreight, from E to the cylinder. The truth of all this appears from Prop. XXII.

Ex. 3.

Let CE be an upright poſt, L a ſmall light, P 65. the foot of it, FG a flight of ſtairs; to find the ſhadow of the poſt CE, upon the ſteps. Thro' P and E, the foot of the light, and the foot of the poſt, draw the line PED, interſecting the horizontal line SD in D; and where it meets with the bottom of the firſt ſtep at 1, there is a plane per-

Fig. 65. pendicular to the geometrical plane, draw the line 12 upon it, perpendicular to the ground line AB; till meeting with another plane at 2, parallel to the geometrical plane; draw 23 towards D upon that plane, till you come to 3, where is another perpendicular plane; upon that draw 34 perpendicular to AB, till you come at 4, where you meet another horizontal plane, thro' which draw 45, ſtill directed to D; and at five where you meet with another perpendicular plane draw 56, ſtill perpendicular to AB; and then upon the next horizontal plane a line from 6 tending to D, which runs off the ſteps, and ſo there is an end of the ſhadow. If from L the point of light, and C the top of the poſt, the line LC be drawn, it cuts none of the former lines, and therefore the remainder of the ſhadow at top is loſt, and no where to be found.

For the plane of the ſhadow of CE, being perpendicular to the horizontal plane, will cut the planes of the ſteps which are alſo perpendicular to the horizon, in the lines 12, 34, 56, which are perpendicular to the ground line AB, which being parallel to one another, and to the picture, their appearances will alſo be perpendicular to the ground line, by Cor. 2. Prop. III. Alſo ſince the plane of the ſhadow interſects the horizontal ſteps in lines parallel to one another and to the line PD, their images 23, 45, all tend to their accidental point (by Prop. IV.), but (Prop. VIII.) their accidental point is in the horizontal line SD, and therefore at D, where PE interſects it.

SCHOLIUM.

By a like method, the repreſentation of the ſhadow of any point, line, &c. may be found upon any other plane, by letting fall a perpendicular upon that plane, from the point of the light, and alſo from the point given; as may eaſily be gathered from what is delivered at Prop. XXII. and the foregoing Problems. For the ſhadow of any point in the perpendicular, will be ſomewhere in the direction of the ſhadow, and the direction of the ſhadow is the line drawn upon that plane thro' the foot of the light, and the foot of the line. And the ſhadow of a point in it is in the line drawn thro' the light and that point, and is therefore where it interſects the direction of the ſhadow. And if the line of ſhadow meet any oblique plane, it may be traced upon that plane, by help of another plane drawn perpendicular to the firſt plane, as has been explained in Prob. XVI. in regard to the ſun. Fig.

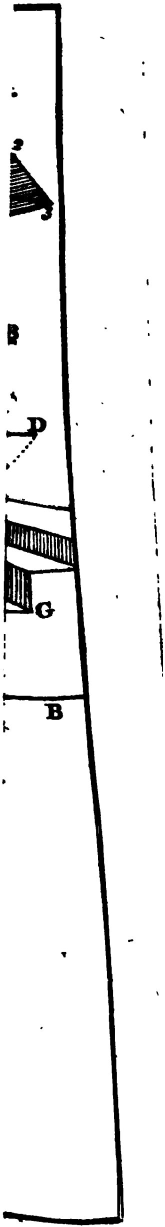
PROB. XX.

To repreſent ſuch objects as appear by reflexion in the water.

Let AB be the ground line, SD the horizontal line, ABEF a canal or pool of water. GHN a tower with a ſpire upon it. Let LK be the level of the water under the tower HI, make K*b* = KH in the perpendicular H*b*, then the reflexion of the point H by the water, will be at *b*. Alſo let NI be the level of the water juſt under the point G at the vane, make N*g* = NG in the perpendicular G*g*, and *g* will be the reflexion of the point G by the water. And in like manner meaſuring the diſtance from N to the top of the vane at G, and ſetting it down in the line N*g*, gives the reflexion of the top of the vane at 1. If you have a mind to know how much of the tower HN appears in the water, meaſure the diſtance from N or I to the water-ſide at O, and ſet it for I to P; then the part from P to G appears by reflexion, the reſt below being incapable of being ſeen by reflexion, by reaſon of the interpoſed earth between L and O. 66.

Again

Fig. 66. Again, if T be a tree near the water-ſide, and the level of the water under the top T be at V; then if V2 be made equal to VT, in the perpendicular T2, then 2 will be the appearance of T by reflexion. And thus meaſuring from any part of the tree, perpendicularly to the ſurface of the water or its level, and ſetting that length downwards towards AB, in a perpendicular line, from that ſurface, it will there give the ſame part of the tree as it appears by reflexion. And it will do the ſame for any other object, which is high enough, or near enough the water. For all objects ſeen by reflexion appear to be turned upſide down, the higheſt part appearing the loweſt in the water; and the loweſt to be the higheſt, where they touch at the ſurface. And all this is evident from the known laws of reflexion, which is this, that any object appears by reflexion, to be as far beyond the reflecting plane, as it really is on this ſide; that is, the object and its image are equidiſtant from the reflecting ſurface, and on contrary ſides. Whence if Nl be a part of the reflecting ſurface, then will NG be equal to N*g*. And if LK be the reflecting ſurface, then will KH = K*h*. For the ſame reaſon will TV = V2. As ſome objects may be ſeen directly which cannot be ſeen by reflexion, as the part PK of the tower. So ſome others may be ſeen by reflexion which cannot be ſeen in a direct line. Thus you may ſee the hollow arch of a bridge both by reflexion and a direct view, if your eye is lower than the arch; but if your eye is raiſed higher than the bridge, you may ſee the top of the bridge directly, but not the arch, and yet you may ſee the arch by reflexion; for nothing hinders the rays from the under ſide, from falling on the water, and being reflected to the eye. And thus part of the tree T, which is covered by the other branches,



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branches, cannot be ſeen directly; but may appear by reflexion in the water. Fig. 66.

PROB. XXI.

To take the perſpective draught of any place, as a Country, City, a Harbour with Ships, &c.

All the problems that have been hitherto delivered, are preparatory to the ſolution of this. For in a general view, all ſorts of objects offer themſelves, of which ſome lye upon the ground, ſome ſtand erect, and ſome ſloaping, ſome are ſtreight, and ſome crooked, bodies ſtanding ſome in one poſition and ſome in another; of various ſorts and forms, ſome near hand, and ſome far off; ſome with ſhadows, and ſome without. So that a perſon before he ſets himſelf to drawing in a general way, muſt make himſelf acquainted with all the foregoing problems, and have them ready in practice, to be applied as there is occaſion. To proceed then,

Take a piece of paper big enough for the purpoſe, as half a ſheet or a ſheet; and near the bottom draw a line for the ground line; and above that, at a diſtance equal to the deſigned height of the eye, draw another line parallel to it, for the horizontal line. The height of the eye, or the diſtance of theſe lines ſhould not be above half the height of the picture, and may be about a third part of it. When the objects are high in the air, the horizontal line muſt be drawn nearer the ground line, to make room for the appearance of the objects above. Then take the point of ſight in the middle of the horizontal line, or in ſome caſes near one end of it; and from it, on one or both ſides, ſet off the points of diſtance, equal to the principal ray, or the diſtance of the eye; which muſt at leaſt be equal to the breadth of the picture; but it ought to be more, ſo that the picture may ſubtend an angle at the eye, of 45 or 50 degrees. The principal ray, or diſtance from the picture, muſt be more than the height of the eye, it ſhould be half as much more, or ſometimes even twice or thrice as much; and eſpecially when the point of ſight is near one end; and it ſhould never be leſs than eight inches, but may be as much more as you will.

This being done, the diſtances of all the objects, and their heights, muſt be meaſured (or at leaſt eſtimated) by ſome given meaſure, as miles, furlongs, yards, &c. And their ſituations, and poſitions, truly taken; all by the ſame ſort of meaſure. Then they are all to be laid down or planned upon paper, according to their ſeveral places, ſituations and diſtances; to be taken off ſome ſcale of equal parts of a convenient bigneſs, as a ſcale of inches, &c. keeping to the ſame ſcale thro' the whole work. And as the plan or ſituation of all theſe objects is commonly made, by drawing them all below the ground line in an inverted ſituation; yet this will be better done by drawing the plan in a ſeparate paper, of the ſame breadth as the picture, and placing all the objects in their true ſituations. The reaſon of drawing them on the contrary ſide of the ground line, in an inverted ſituation, is to hinder them from interfering with their images in the picture; and to preſerve the true diſtances of all and every part, from the ground line. But this is better done by drawing all the objects above the ground line in their true poſitions in a ſeparate paper; and for that reaſon it is more natural, than inverting all the objects, and ſo changing their places.

The plan of the work being laid down, it muſt be drawn in perſpective in the picture, according to the former rules, one part after another till the whole is inſerted. And if any of the parts be much compounded, and requires a great many lines; it will be better to do ſuch part in a ſeparate paper, and then transfer it to the picture. It will be proper to do this as often as there are any difficult parts to be laid down. Fig.

When any buildings, or bodies elevated in the air, happen to be in the view, as few are without; it will often be neceſſary to take the front view and the profile thereof by the ſame meaſure as the reſt; and then lay them down upon paper by the ſame ſcale. And from this draught, they muſt be drawn in perſpective by the rules of drawing ſolids. And here alſo they may be laid down on ſeparate papers when they require many lines; and at laſt transferred to the picture; and thus the picture will be kept clean and neat.

In the practice of all this, one muſt remember that all ſtreight lines are repreſented by ſtreight lines in the picture; and ſuch as are parallel to the ground line, on the earth, will be parallel to the ground line; and thoſe perpendicular to the earth, will be perpendicular to the ground line; and all lines perpendicular to the picture, tend to the point of Sight. Likewiſe all lines parallel to the baſe or the ground, have their accidental points in the horizontal line. And lines parallel to the vertical plane, have their accidental point in the vertical line.

Such objects as ſtand on your right hand, muſt be placed on the right hand of the vertical line; and thoſe on the left, muſt be on the left hand; and thoſe before, in the middle of the picture. And thoſe far off, high up in the picture. And objects that are higher than the eye, muſt be raiſed above the horizontal line.

In deſcribing objects at a great diſtance, their altitudes muſt be diminiſhed in proportion to the diſtance. And therefore a ſcale muſt be made (by Prob. XIII.) for every altitude concerned, which will

Fig. ſhew the altitude diminiſhed in every place of the picture. And here the profile muſt be made uſe of, when different heights are concerned. And theſe heights taken from the ſcale, muſt be raiſed upon their proper baſes in the picture, the tops of which joined by right lines, give the boundaries of the bodies, and then theſe bodies may be finiſhed, and duly ſhadowed. When this is done, in reſpect to all the objects that are to appear in the draught, then your view is finiſhed, as far as depends on the rules of geometry and perſpective. When all is done, the ground line, horizontal line, and other lines not belonging to the proſpect muſt be rubbed out; and to make the draught complete, it ought to be coloured, every object with its proper colour.

When objects are ſeen by reflexion in water, they muſt be drawn according to the rules laid down for that purpoſe; and then coloured of the ſame colour as the objects themſelves, but a great deal fainter, by reaſon of the loſs of light, by reflection at the ſurface of the water.

Likewiſe the colours of all objects ought to be ſo much fainter as the diſtance is greater; till at laſt they vaniſh in a blueiſh miſt.

Every one that would be a practitioner in the art of drawing, muſt furniſh himſelf with a drawing board and a Tee. This board or table muſt be right-angled at all the corners, and as big as to contain a ſheet of paper, and made ſmooth and flat. The tee (called ſo from its ſhape) is made after the manner of the carpenters ſquare, with the head thicker than the tong, and muſt be made exactly ſquare. Then a paper being paſted to the board at the four corners, it is ready to draw upon; and when you want to draw parallel or perpendicular lines, apply the head of the tee to one or other of the edges of the board, and then draw by the ſide of the tongue; and thus you may ſoon draw as many parallels or perpendiculars as you pleaſe, without any trouble. Fig.

Example 1.

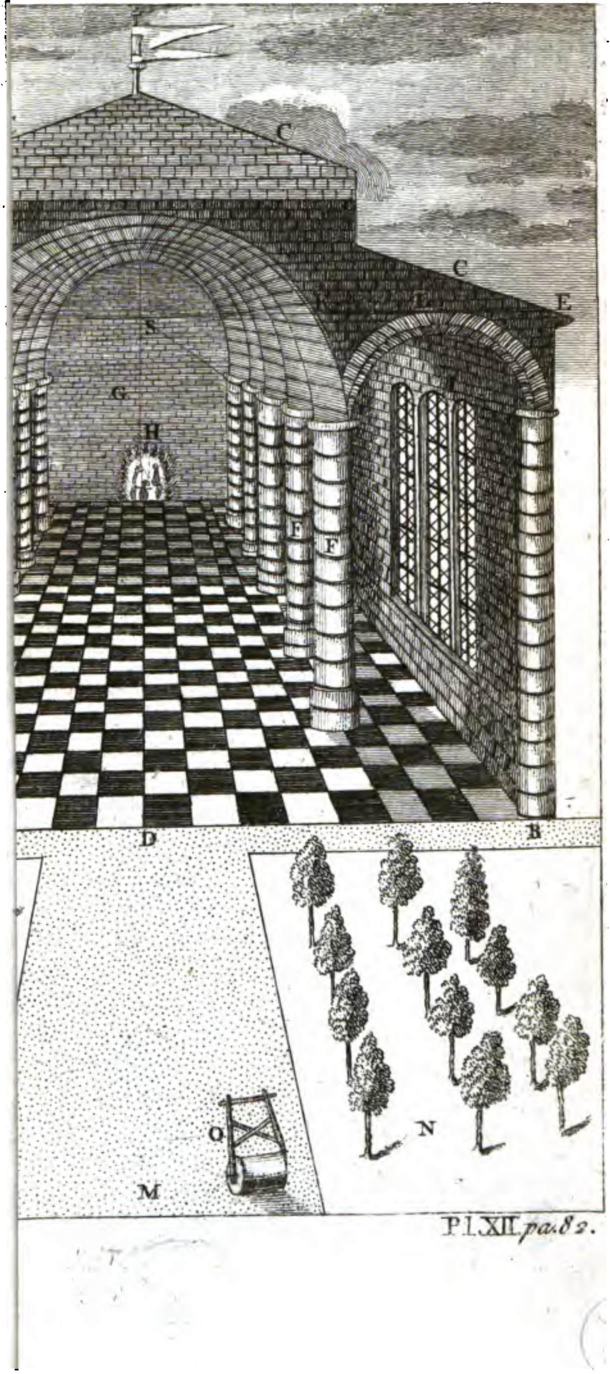
67. ACEB is a building, whoſe roof is ſupported by pillars; AB is the ground line, SE the horizontal line, S the point of ſight, and the point of diſtance is far out of the picture. Here the breadth of the houſe is AB, and the height of the eye is SD or EB. The floor here is laid with ſquares, alternately black and white, they lye in rows that run towards the point of ſight, and the other ſides are parallel to the ground line AB. F, F, &c. are two rows of pillars, one on each ſide, theſe are placed at equal diſtances, and range directly towards the point of ſight. G is the further end of the building, being a perpendicular wall of free ſtone; at that end of the houſe are four lamps or tapers H, placed to give light over the room. Between the pillars F, F, and the ſides of the building, are two walks laid with the ſame ſort of ſquares. In the two ſides of the room AC, BC, are two large windows I, I; theſe have each of them three lights, with iron ſtainſhals down the middle, and iron bars acroſs, to keep the jaums faſt. K, K, are arches carried over the building from the top of one pillar to that of another, all manner of ways; upon theſe the roof is built. C, C, &c. is the roof, the two wings on each ſide lye upon the arches L, L, and on others beyond them. The higher part of the roof C, C, ſtands upon the main arches. On the very top at the end of the roof, is placed a vane, to ſhew the wind. Before the entrance is a gravel walk DM, and a roller O to roll it with. On each ſide of it are three rows of trees, which tend to the point of ſight. Along AB is alſo another gravel walk.

Fig. 68.

Example 2.

Let SD be the horizontal line, S the point of ſight, D the point of diſtance, AB the ground line; C is a houſe on the left hand, whoſe profile runs up to the point of ſight S; in this profile you may ſee the windows which are all perpendicular to the ground line; from the end of it runs a wall E; which is perpendicular to the picture, and therefore it alſo tends to the point of ſight. At the end of the wall is another houſe, which ſtands in the ſame poſition. Beyond at F upon an aſcent, is a houſe ſtanding in a wood among a deal of trees; and at ſome diſtance from this is another houſe G upon a hill, with a garden walled in, before it; and near it is the ſummer houſe H, and beyond theſe is the country, riſing high, and covered with buſhes at I, I. On the ground line at K is a garden incloſed with a wall, in this are ſeveral beds with flowers, and ſome with fruit trees; being all perpendicular to the picture, tend to the point of ſight S. Adjoining to the garden is a church L with a tower indented at the top; this has its foreſide parallel to the ground line; and the ends being perpendicular to the picture, run towards the point of ſight. Behind the church is a houſe M, with its front parallel to the picture; this joins upon a garden N, with trees, herbs and flowers. On the other hand is the houſe O, ſtanding croſs the former, or with its face perpendicular to the picture, and therefore, it tends to the point of ſight S. Behind theſe houſes is another large church P, with a tower and a ſpire upon it, on the top of which is a weather cock, to ſhew the wind; this ſtands fronting the picture. Behind this church is another houſe or two Q, ſtanding in the ſame poſition, and having a chimney ſmoaking, as likewiſe have ſeveral of the reſt. R is an artificial fountain, where the aſcending water plays,

and



Pl. XII. *pa.* 82.

and then falls into the baſon below. T ſeveral incloſures, and ſmall fields. V ſeveral rows of trees. W an obeliſk in a walk between ſome graſs plats. X ſeveral fowls flying in the air. Y a windmill on the top of the hill. Z the clouds hanging in the atmoſphere. Fig.

SCHOLIUM.

From what has been laid down before, concerning the nature of perſpective; the following obſervations may be drawn.

All horizontal planes ſeated above the eye, ſeem to ſink downwards, more and more, the further they are produced; till being infinitely produced, they at laſt become level with the eye. Likewiſe ſuch planes as are below the eye, ſeem continually to riſe upward, the further they go. Alſo thoſe on the right hand approach to the left; and thoſe on the left, towards the right. Therefore floors and pavements of buildings, when continued to a great length, ſeem to a ſpectator to riſe upwards; and the roofs and cielings of buildings, appear gradually to deſcend, and therefore the longer ſuch roofs are, the higher they ſhould be made; to prevent their appearing too near the ground at a great diſtance.

All rows of columns, or of trees; and thoſe of walls or the ſides of buildings, ſeem to the eye of the ſpectator to contract themſelves, the further they run; and continually grow narrower. And therefore the breadth thereof ſhould be greater, where the length is greater; that their appearance to the eye may be the more delightful. And thus the capitals of pillars ſeem to incline downwards, and their pedeſtals to riſe upwards. The horizon ſeems continually to riſe from the eye, and at laſt to appear higher than it really is. But if a row of columns, perpendicular to the horizon, and of equal height, be lower than the eye; the tops of them

Fig. them ſeem continually to riſe as well as the bottoms; and thoſe furtheſt off would appear the higheſt. But the contrary happens if they are above the eye, for then the furtheſt would appear the loweſt. Thus the ſurface of the ſea ſeems continually to riſe, and at laſt to be higher than the eye. Hence alſo a plane ſurface continued to a great diſtance will appear hollow. Statues and images raiſed on high, will appear only in their true proportion at a certain height; and at other heights will appear more or leſs deformed or out of proportion. And therefore the optical appearance of any object, in any place, muſt be conſidered.

From the tedioufneſs of drawing a perſpective view throughout, by geometrical rules, as has been now taught; no body ever puts it in practice. But inſtead of that, they make uſe of a mechanical method of doing it, which is this; they take a ſquare box, with a hole in the top facing the object, in which is placed a convex lens; and behind that, within the box is a reflector. This box being placed ſo as to face the country whoſe draught is to be taken, the rays of light flowing from all the objects, enter the convex glaſs, and are reflected downwards by the reflector, which is placed obliquely; and ſo they paint upon a white paper laid at the bottom of the box, the images of all theſe external objects, very diſtinctly if the convex lens be at its true diſtance; if not, the lens muſt be moved in or out, till the images are diſtinct. Then the draughts man, putting his head within the box, has nothing to do, but run over all the out-lines of the ſeveral objects with a pencil; all which being done, they may be finiſhed afterwards. Such a box for taking a perſpective view mechanically you have deſcribed to you in Prob. 24. Book IV. of the Optics. For it would be an endleſs taſk to go through all the particulars of ſuch a draught, by the common rules of Perſpective;

ſpective; not only for the number of lines it would require, but for the great length of time requiſite for ſuch a work. And yet theſe rules are the eaſieſt that can be contrived, where the picture is ſuppoſed perpendicular to the horizon. For if the picture was oblique, the labour would be far more, to draw the objects in ſuch a poſition; and new rules are neceſſary to work by, and the objects when drawn make no better appearance than in the common way; and therefore no body would ever follow the practice upon oblique tables, and conſequently they are of no uſe, but merely for curioſity; and to ſhew what can be done by art. Therefore the common method is the moſt uſeful, and its principal uſe, is that of drawing ſingle objects, which require to be done very exactly; and to initiate the drawing by hand, which is by far the moſt uſeful and expeditious method. Hence no landſkips or views of countries are ever drawn by the method of perſpective, but by ſuch mechanical contrivances, as we have ſhewn; and that for the ſake of expedition. And this method is very exact too; for the images are diſtinctly repreſented on the paper, and in their true proportions, as they appear; and the drawing them truly, depends on the ſteady hand of the drawer. Fig.

I muſt obſerve, that when any view or landſkip is to be drawn in a publick place, as in a garden, on the ſcenes in a playhouſe, or the walls of a large room; the point of ſight is not to be taken in the air, where people cannot come. But it ought to be taken where moſt people reſort, and ſtand or ſit to look at it. For then it will appear to the beſt advantage and moſt natural. But if the point of ſight is taken in a different place, the proſpect will appear unnatural and deformed; as may be gathered from Cor. 2. Prop. XII.

Fig.

PROB. XXII.

By the inverſe method of perſpective, having a point given in the picture; to find its original in the geometrical plane.

In what goes before, we have treated only about the direct method of perſpective, or the finding the place of any object in the picture, having its ſituation on the geometrical plane given. We ſhall here add a few propoſitions about the inverſe method, which is finding the original, from having its appearance in the picture. As the place in the picture is found from having that in the geometrical plane, ſo by working backward, its place in the geometrical plane is eaſily found from having that in the picture.

69. Let N be the point given in the picture. From the given point N and the point of ſight, draw the line SE to cut the ground line CB in E. Alſo from the point of diſtance D, thro' the ſame point N, draw the line DC, cutting the ground line in C; then CE is the diſtance of the point, in the geometrical plane, from the ground line. And therefore if E*n* be made perpendicular to AB, and equal to EC, *n* will be the point ſought, but on the contrary ſide of the ground line.

For if *n* was given, its repreſentation at N is found by drawing lines from S to E, and from D to C, to interſect in N, EC being made equal to E*n*. Therefore by the reverſe operation, *n* is truely found.

PROB. XXIII.

69. *Having an object given in the picture, to find its real height, on the geometrical plane.*

Let OP be the object given; find the ſituation F, of the foot O, by the laſt Prob. And thro' F draw

Fig. 69. draw FI perpendicular to AB, and from the point of ſight S draw SP thro' the top of it, cutting the perpendicular FI in I. Then FI is the real height of the object upon the geometrical plane. For if FI be the real height, OP will be found to be the apparent height in the picture, by Prob. XI.

Cor. *In like manner, if* OR *be a line in the geometrical plane, parallel to the ground line; then if* SO, SR *be drawn to cut the ground line in* F *and* G; *then* FG *is the real length of* OR.

PROB. XXIV.

Any oblique line in the picture being given; to find its original in the geometrical plane.

70. Let OP be the given line in the picture. Find the ſituation of the point P at F, and then its place *p* upon the geometrical plane, by Prob. XXII. Then produce PO to cut the ground line in E, and draw *p*E, which will be the original of PE.

Likewiſe find the original of the point O, which will be at *o*; then *po* will be the original of the line PO, but inverted.

This is evident from the contrary proceſs of finding the repreſentation PO, from having the original *po*.

Cor. 1. *Hence the original of any plane figure given in the picture, may be found. For it is but finding the originals of all the lines, that incloſe the figure; and then you'll have the original figure.*

Cor. 2. *And thus a ſolid being given in the picture, its original may be found, but with more labour: For all the baſe lines, and all the perpendiculars will be found as before; and thence all the ſurfaces that incloſe it, and conſequently the whole ſolid.*

Fig. 70. Cor. 3. *Hence the original of any angle may be found, by finding the original lines that form the angle.*

PROB. XXV.

Two or more lines being given in the picture, whoſe originals are perpendicular to it; and having the real length of any ſuch baſe line; to find the points of ſight and diſtance.

71. Let AB be the ground line of the picture, AEG a houſe, and BHM a wall; both the originals being perpendicular to the ground line AB. Produce any two ſides, as AE, IK, BM, HL, which are perpendicular to the ground line in the geometrical plane; till they interſect, as AE and BM, which interſect in S, then S is the point of ſight.

Again draw SD parallel to AB, and ſet the real length of the baſe line BF (which tends to the point of ſight) from its ſituation B to C in the ground line; and thro' C and F draw the line CF cutting SD in D; then is D the point of diſtance; and SD the horizontal line; and the length of SD is the principal ray, or the diſtance of the eye from the picture.

For (by Cor. 2. Prop. IV.) the lines AE and BM tend to the point of ſight at S. And therefore the point of diſtance is ſomewhere in the line SD drawn parallel to AB, which is the horizontal line. But if BC be the diſtance of a point from the ground line, and D the point of diſtance, and DB and DC be drawn to interſect in F, then F will be the image of that object. Therefore on the contrary, when CF is drawn thro' the image at F, it will go thro' the point of diſtance D.

Cor. 1. *Hence, in any perſpective draught, having the horizontal line given; its center, or the point of ſight may be found.*

For

D

pa. 88

For produce any line whoſe original is perpendicular to the picture, till it cuts the horizontal line, and there is the point of ſight. Or if there is no horizontal line, produce two ſuch lines till they cut one another. Fig. 71.

Cor. 2. *But the diſtance of the picture, or the principal ray, cannot be found, unleſs the real diſtance of ſome point or object (from the ground line) be known.*

For we only know, that the eye is ſomewhere in the perpendicular, raiſed upon the point of ſight. But if a real diſtance be known, the diſtance of the eye will then be known, as laid down before.

Cor. 3. *If in any draught, it be known that ſeveral original lines are parallel, in the geometrical plane. Then their accidental point, and the horizontal line may be found.*

Produce two of theſe lines till they interſect, and the point of interſection is their accidental point; and that point is in the horizontal line. Therefore thro' that point, draw a parallel to the ground line, and that is the horizontal line.

PROB. XXVI.

Having the angle which any original object ſubtends at the eye, and alſo its image; to find the point of diſtance, or the point of view in the air.

Make the angle CEF upon the horizontal line, equal to the angle which the original object ſubtends at the eye. Let OP be the image of it in the picture. Make CF = OP, and place CF in the ſame poſition to EC, as the image has to the viſual ray paſſing thro' O. Then CE is the diſtance of the eye from the point O in the picture. When that diſtance is had, erect a perpendicular at S the point of ſight (found before), which muſt 72.

ſtand

Fig. 72. ſtand upright upon the plane of the picture; then the diſtance EC ſet from O, will interſect that perpendicular in the point of view. Then the diſtance between the point of view and point of ſight, or the principal ray, ſet from S to D, gives the point of diſtance D.

Having ſaid ſomething before of inclined pictures, I ſhall add a little more here; more out of curioſity, than any real uſe they are of.

PROB. XXVII.

To draw objects in perſpective upon an inclined picture.

I have ſhewn in Prop. XXIII. that the rules for drawing upon inclined pictures are the ſame as thoſe for upright ones; taking here a line parallel to the vertical line, inſtead of the height of the eye, in the upright picture; and taking lines ſtanding on the geometrical plane, parallel to the vertical line, inſtead of perpendiculars. I ſay lines in this poſition, and lines parallel to the ground line, will be drawn by the very ſame rules. Therefore in theſe caſes, you muſt work by Prob. VI. VII. VIII. and Prob. XI. for altitudes, And when perpendiculars are given upon the geometrical plane, you muſt firſt draw lines from their tops parallel to the vertical line, to cut the geometrical plane, and there you have their baſes; and then finding the images of one of theſe baſes, you muſt draw an oblique line obſcure. Then find the image of the foot of the perpendicular itſelf, and joining this laſt image, and the top of the former obſcure line, by a right line, you'll have the image of that perpendicular upon the inclined plane; and ſo of all the reſt of the perpendiculars. By this means planes may be drawn, and ſolids perpendicular upon on the horizon or geometrical plane. And any Fig. other ſort of oblique lines may be drawn, by a like method, firſt drawing a ſlant line parallel to the vertical line; then you have two lines meeting at the top, or whoſe top is common to both; find the images of both the baſes or feet, and the altitude of the ſaid ſlant line, then join the top and the proper baſe belonging to that line, ſee Cor. Prop. XXI.

But the beſt way is, firſt to draw the profile of the picture, and the principal lines belonging to it; and find the accidental point of lines perpendicular to the horizon; or of any other lines, which are to be drawn in perſpective. Thus,

Let SH be the picture, E the place of the eye, 73. or the point of view. Draw ES and AH parallel to the horizon, draw EC perpendicular to AH, to cut SH produced in F, then (by Def. 17.) F is the accidental point of all lines parallel to EC, or perpendicular to the horizon. Alſo draw EA parallel to SH; and then EA is to be taken for the height of the eye, ES the principal ray, AH the line of ſtation, and SH the picture. Then in order to draw upon ſuch a picture, (in Fig. 74.) draw SD for the 74. horizontal line, and make SD equal to SE, and D is the point of diſtance. Draw SH perpendicular to SD, and equal to SH (fig. 73.) for the vertical line; and draw AHB parallel to SD for the ground line; or this may be drawn firſt, if you will; alſo make HF equal to HF (fig. 73), and F will be the accidental point of all lines perpendicular to the horizon. Thus all the requiſites are tranſferred from fig. 73. to fig. 74. which done, the operations are to be performed as before-mentioned. Examples will explain the thing.

Ex. 1.

To find the image of a given point P. Let 74. fall PI perpendicular to AB, and ſet PI from I to

A.

Fig. 74. A. Then draw SI from the point of ſight S, and DA from the point of diſtance D; and where they interſect at *p*, is the image of P. This is juſt the ſame proceſs as in the upright picture.

Ex. 2.

To find the image of a right line, parallel to the ground line.

Let PQ be the line given; from P and Q let fall perpendiculars upon the ground line AB, to interſect it at I and K. From I and K draw lines to the point of ſight, IS, KS. Set IP from I to A, and draw AD to the point of diſtance D, to interſect IS in *p*. Draw *pq* parallel to AB, cutting KS in *q*; then *pq* is the image of PQ. Or find the point *q* the repreſentation of Q, as *p* was found, and draw *pq*.

Ex. 3.

To find the image of any line in the geometrical plane.

Let PL be the line given. Find *p* the image of P, one end of the line as in the firſt example. And likewiſe *l* the image of L the other end of the line, the ſame way; then draw *pl*, which will be the repreſentation of PL, as required.

Ex. 4.

To find the image of a plane figure given in the geometrical plane, on an inclined picture. Let PQLR be a ſquare, whoſe two oppoſite ſides are parallel to the ground line AB. Find the repreſentations of all the points P, Q, L, R; at *p*, *q*, *l*, *r*; and draw the lines *pq*, *ql*, *lr*, *rp*; then *pqlr* is the image of the ſquare PQLR.

Or thus, let fall perpendiculars from all the angles upon the ground line AB, to interſect it at I and K; draw IS, KS to the point of ſight S. Set off IR from I towards A, from which point draw a line to the point of diſtance D, interſecting IS in *r*. Alſo ſet off IP from I toward A, and from that point draw a line likewiſe to the point of diſtance D, to interſect IS in *p*. From *r* and *p* draw two lines parallel to AB, as *rl*, *pq*, interſecting KS in *l* and *q*; then *pqlr* is the repreſentation of the ſquare PQLR, whoſe two ſides are parallel to the ground line AB. Fig. 74.

Ex. 5.

To draw the image of a line parallel to the vertical line of the inclined picture. Let SD be the horizontal line, AB the ground line, S the point of ſight, D the point of diſtance, as before; and let P be the foot of the line in the geometrical plane. Draw PI perpendicular to AB, make IL = PI. Then draw SI, LD to interſect in *p* the image of P, the foot of the line. Make IA equal to the length of the line, which ſtands not perpendicular but ſloping; draw AS, and from *p*, draw *p*G parallel to AI, cutting AS in G. Draw *pg* parallel to SH, and equal to *p*G, then *pg* is the image of the line given. 75.

If the line had ſtood in the ground line at I, you had no more to do, but to draw a line from I parallel to SH, and equal to IA, the length of the line propoſed.

Ex. 6.

To find the repreſentation of a line perpendicular to the horizon. Let Q be the foot of the perpendicular upon the geometrical plane, and CN its height. Find the image of Q, by Ex. 1. which will be at *q*. The picture being in the poſition SH, fig. 73. From the top of the line C, draw CM parallel to the vertical line, then MCN will be the profile of the lines CN, CM. Therefore make QO = NM, and O will be the ſituation of

Fig. 75. M upon the geometrical plane. Then find the image of O, which will be at *o*. Alſo find the image of MC whoſe foot is at *o*, by Ex. 5. this will be *oc*, which is drawn parallel to SH, and equal in repreſentation to MC. Laſtly, draw *cq*, and that will be the repreſentation of the perpendicular CN, ſtanding at Q, whoſe image is *q*.

For ſince CM is drawn parallel to the vertical line, the line NM is perpendicular to the ground line or baſe of the picture; therefore the triangle CMN is the profile of the ſlant line, the perpendicular line, and the baſe. And ſince N is at Q, M will be nearer the picture as at O; therefore *q* is the repreſentation of Q or N; and *o* is the repreſentation of O or M; and *oc* being the repreſentation of MC, therefore the triangle *ocq* is the repreſentation of the triangle MCN; and *cq* is the repreſentation of CN.

Otherwiſe.

Find the accidental point F of lines perpendicular to the geometrical plane, as directed before. Alſo find *q* the repreſentation of Q, the foot of the perpendicular as before. Thro' F draw *qc* for the indefinite perpendicular ſtanding at *q*, which muſt be terminated thus; draw the ſlant line MC as before, and ſet its length from K to T in the perpendicular KV; then draw ST which will cut *qc* in *c*, the top of the perpendicular line required.

For the images of all lines perpendicular to the horizon tend to the accidental point F; and the image of N or Q is at *q*. Draw *oc* perpendicular to AB, to cut SK in *o*. Then if KT be the appearance of CM at K in the ground line; *oc* will be its appearance at *o*, by Prob. XI. therefore *c* is the top of the image, and therefore *qc* is the image of the perpendicular required.

Ex.

Fig. 76.

Ex. 7.

To draw the repreſentation of any ſloaping line. Let the foot of the given line be at P in the geometrical plane; ſuppoſe a line drawn from the top of this line parallel to the vertical line, to cut the geometrical plane in the point Q, and let the length of that be CM. Find the repreſentations of the points P, and Q, at *p* and *q*, by Ex. 1. Set off MC from K to B, in the horizontal line AB, and draw SB from the point of ſight. Draw *ql* parallel to AB, to cut SB in *l*; and draw *qc* parallel to SH and equal to *ql*, then *c* is the image of the top of the line. Therefore draw *cp*, and it will be the image of the given ſloaping line drawn to P. For it is evident by the proceſs in Ex. 5, that *cq* is the repreſentation of the line CM, drawn to Q. Therefore *cp* is the image of the given line.

By the help of this example any ſolid may be drawn upon an inclined picture; by drawing all the out lines thereof, and joining their tops by right lines, as thoſe in the ſolid are drawn. But it is beſt to draw all the parts in ſeparate papers.

Ex. 8.

To draw an oblique parallelopipedon, whoſe axis is parallel to the vertical line, and one face parallel to the picture, and the baſe a ſquare. 74.

Let AB be the ground line, S the point of ſight, D the point of diſtance. Let the baſe of the ſolid be deſcribed in the geometrical plane, according to the given ſituation, as at PQLR, which is a ſquare baſe. Draw this in perſpective, as ſhewn in Ex. 4. which let be *pqlr*. This done, we muſt proceed to raiſe the heights, at the places *p*, *q*, *l*, *r*, as directed in Ex. 5. thus, ſet the length of the axis or any ſide of the parallelopipedon, from B to C in the perpendicular BC; draw the lines SB, SC; then

SBC is a flying ſcale to take off the altitudes. Draw $rl1$ and $pq2$ parallel to AB, to interſect SB in 1 and 2. Then draw 13 and 24 parallel to BC, to cut SC in 3 and 4. At p, q, r, l raiſe perpendiculars to the ground line AB; and from 3 and 4 draw lines parallel to AB, to interſect theſe perpendiculars, which will determine the top of the ſolid. Thus the parallel $3wx$ will cut the perpendiculars at l and r in the points w and x; and the parallel $4t$ will cut the perpendiculars from p and q in t and v. Therefore joining the points t, v, w, x with right lines, you have the upper face of the ſolid. Here the lines pq, ql, and qv, which are hid from the eye, are not drawn. In this example all the lines terminating the ſeveral planes, are either parallel to the ground line AB, or tend to the point of ſight S.

Thus all bodies which lean the ſame way as the picture, and incline as much, are as eaſily drawn in perſpective, upon an inclined picture, as bodies perpendicular to the horizon, are upon an upright picture, and by the very ſame rules. But when they ſtand in any other poſition, there is required more labour to draw them; and ſo there is in an upright table, when the bodies are not perpendicular to the horizon; but the labour in one caſe is no greater than that in the other; and they are both done by ſimilar methods.

Ex. 9.

To draw a cube upon an inclined table, ſeen angle ways. Let AB be the ground line, SD the horizontal line, S the point of ſight, and D the point of diſtance. PQLR is the baſe of the cube laid down on the geometrical plane, in the ſituation required. Let this ſquare PQLR be drawn in perſpective, by the method in Ex. 4. and let its image be pqlR; p being the image of P, q of Q, and

C
B

and *l* that of L; the point R in the ground line being common. Then at the points *p*, *q*, *l*, R, we muſt raiſe the ſides of the cube, which are perpendicular to the baſe in the ſolid, but muſt paſs thro' their accidental point in the picture. Therefore find the accidental point F, of lines perpendicular to the horizon as before. And from F draw lines thro' *p*, *q*, *l*, R. Then to terminate them at the top of the cube, proceed by examp. 6; thus. Let CN be the perpendicular height of the cube, draw MN perpendicular to it, and draw CM, making the angle CMN equal to the angle which the picture makes with the horizon. Then take CM in your compaſſes, and ſet it perpendicular at the points G, R, I, where S*p*, S*q*R, and S*l*, interſect the ground line. Let theſe perpendiculars reach to *g*, *r*, and *i*. Then thro' S and theſe points draw lines to interſect the lines drawn from F; that is, draw S*g* to interſect F*p* in the point 1, the top of the ſide *p*1. And draw S*r* to interſect F*q* and FR in 2 and 4; then 2 is the top of the ſide *q*2; and 4 is the top of the ſide R4. Alſo draw S*i* to interſect F*l* in 3, the top of the ſide *l*3. Then draw the lines 12, 23, 34, 41, for the upper face of the cube. Fig. 77.

One may obſerve, that ſince the ſides PR, RL, &c. make half a right angle with the ground line, the point of diſtance D, will be the accidental point of the ſides *pq*, R*l*, and alſo of their parallels 12, 43. And for the ſame reaſon the other point of diſtance will be the accidental point of R*p*, *lq*, 41, 32. And if the cube was in any other poſition, the accidental points might be eaſily found in the horizontal line SD; becauſe SD is the vaniſhing line of the baſe, and top of the cube, being both in or parallel to the geometrical plane.

In all theſe examples, I have made uſe of the ſame picture, in the ſame poſition, the ſame point

Fig. 77. of ſight and point of diſtance; and in general of the ſame profile of the picture as you have in fig. 73. And in all theſe examples the picture is ſuppoſed to lean backward; but all theſe things may as eaſily be done, if it leans towards you. In that caſe, the accidental point of lines perpendicular to the horizon will be above the point of ſight; but all operations will be the ſame. But for variety, I ſhall give one example where the picture leans forwards.

Ex. 10.

78. To draw a hexagonal right priſm, upon an inclined picture. We ſhall here ſuppoſe the picture to lean as much forward, as it before leaned backward, and that the height of the eye and the principal ray, remain the ſame. Let PQRTVZ be the hexagonal baſe deſcribed upon the geometrical plane, AB the ground line, SD the horizontal line, S the point of ſight, D the point of diſtance. Draw P1, Z2, V3 perpendicular to AB, the parallel ſides PQ, TV being ſo. From the point of ſight draw S1, S2, S3; by means of which, draw the baſe PQRTVZ in perſpective, which let be *pqrtvz*. Then to raiſe the images of the perpendiculars at the points *p*, *q*, *r*, *t*, *v*, *z*, proceed thus, at the points 1, 2, 3, raiſe the perpendiculars 16, 27, 38, each equal to the ſlant line CM, in the right-angled triangle MCN; where the angle CMN is the inclination of the picture, and CN the height of the priſm. Draw the line S8 from the point of ſight S; then from *v* and *t* draw lines towards F, (the accidental point of lines perpendicular to the horizon,) to cut S8 in the points 4 and 5, then *v*4, *t*5, repreſent two of the perpendicular ſides of the priſm. Again, draw the obſcure line S7; and from *z* and *r*, draw lines to F, to cut it in *o* and 9; then *zo*, *r*9 will be two more ſides of the priſm. Draw alſo the obſcure line S6; and from *p and q*, draw lines to

F, to cut S6 in the points *n* and *c*; then *pn*, *qc* are the two other ſides of the priſm; then joining all theſe points by right lines, you have the top of the priſm *nc*9540. And the planes being ſhaded, you have the true repreſentation of the priſm *vzpqn*5, upon the inclined picture. Fig. 78.

I ſuppoſe theſe examples are ſufficient to ſhew the method of drawing upon inclined pictures.

SCHOLIUM.

It appears from the practice, in all theſe examples, that the image of any original figure in the geometrical plane, will fall exactly in the ſame place of the picture, whether the picture lean forward or backward, or be upright; provided the principal ray, the length of the picture, and diſtance of the object from the ground line remains the ſame. But in objects elevated above the geometrical plane, the caſe is otherwiſe; and they will have a different appearance according to the different poſition of the picture. The principal uſe of drawing upon inclined pictures, is for painting any images upon the cieling of a room, or on any vaulted roof, or on the cupola of a church, &c. where the ſurface to be painted on, is in an oblique poſition.

As to the ſhadows of bodies; the rules are the ſame as in upright pictures; only inſtead of lines perpendicular to the horizon we muſt uſe lines parallel to the vertical line; and for lines perpendicular to the picture, make uſe of lines parallel to the line of ſtation or to the principal ray. In difficult caſes, the progreſs of the ſhadow may be traced out as it falls upon any planes, or upon any bodies, that are in its way; which is eaſily done from the ſituation of the light. And then theſe ſhadows, or parts of ſhadows, muſt be taken for

Fig. real objects, and ſo drawn in perſpective by the common rules.

PROB. XXVIII.

To draw the picture of any object mechanically.

This may conveniently be done by help of a candle or lamp. Let the object be placed near a wall, and a light, ſuch as a candle or lamp, placed at a good diſtance from it, or the ſun's light may be made uſe of; let this light project the ſhadow of the given object upon the wall behind it in lines nearly perpendicular to the wall. Then the ſhadow of the body would ſhew the appearance of the body to an eye placed where the light is, if the body was removed, and the ſhadow remained there. Therefore we muſt trace out all the out-lines of the ſhadow, with a pencil, upon the wall; which is very eaſy to do where they appear; but where the ſhadow of any part is intercepted by ſome other part; there will be ſome trouble. To find the ſhadow of any point, ſo placed as that its ſhadow cannot fall upon the wall; take a ſlender rod, and put one end of it to touch that point; and faſten the rod there by ſome means or other; then removing the body, whilſt the rod remains fixed; the ſhadow of the end of the rod will ſhew the place where the ſhadow of that point of the body would fall. Then replacing the body, you may proceed thus with any other point of the body, or as many points as you will. If any line in the object is hindered from caſting its ſhadow upon the wall; the place of the ſhadow will be found in like manner; lay a ſlender rod cloſe to the line, and mark the length with chalk; then remove the body, and where the ſhadow of that part of the rod, which you marked, falls upon the wall; there is the ſhadow of that line in the object. And thus any lines in it may be drawn. Fig.

Or the ſhadows of theſe points and lines may be found thus, take two ſtrings and extend them croſs one another; ſo that the ſhadow of their interſection may fall on the given point of the object. Then if the body be removed, the ſhadow of the interſection of the ſtrings, will fall upon that point of the wall where the ſhadow of the given point of the body would fall. In like manner extend a ſingle ſtring, ſo that a knot in it may fall upon the given point of the body, and fixing it there, remove the body, and the knot will caſt its ſhadow, where the ſhadow of the given point of the object would fall upon the wall. Alſo extend a ſtring, ſo that its ſhadow may fall upon any line in the body which we want to deſcribe; and mark the length on the ſtring, by tying two knots, or any other way. Then removing the body, that part of the ſtring will caſt a ſhadow upon the wall, where the ſhadow of the line in the object would fall. Other methods may be contrived, to get the ſhadows of ſuch points, as are hindered from falling on the wall. And ſome ſuch method may be contrived without removing the body, and being at the trouble to place it again. It may be thus, if your croſs ſtrings are long enough to caſt their ſhadows upon the wall, on every ſide of the body, whilſt their interſection caſts its ſhadow upon the given point of the object; continue theſe ſhadows behind the body, upon the wall, till they interſect; and it is evident, the point of interſection will be the place where the ſhadow of the given point of the body will fall. And thus you may find the ſhadows of as many points as you will, and conſequently of as many lines as you will, without ſtirring the body.

Inſtead of a light, the eye may be uſed, keeping it fixed in the place where the light ſhould be.

Fig. For the eye will project any point of the object upon the oppoſite wall, which may be marked there by help of an aſſiſtant; and thus one part after another, and ſo the whole body, may be projected upon the wall. Alſo if one end of a ſtring be fixed where the light ſhould be, and taking hold of the other end, and drawing it ſtreight; if the ſtring be made to paſs along the out-lines of the object, the end of the ſtring, by help of a pencil, may be made to deſcribe the figure thereof upon the wall. And thus by means of the ſtring, the image of the body will be projected upon the wall, the ſame as by the ſhadow.

Otherwiſe.

79. To delineate any object; make a ſquare frame DE; divide the ſpace into ſmall ſquares, by threads fixed to the ſides, parallel to one another and to the ſides of the frame, and at equal diſtances. Set up a ſtand FH, upon the baſe FG; this muſt have a ſmall hole at H to look thro; it is fixed in the baſe, parallel to the frame DE. Alſo let the paper on which the draught is to be made, be divided into the like ſmall ſquares. Then placing the object beyond the frame in a proper ſituation; look at all the parts thereof ſucceſſively thro' the hole; and obſerve in what ſquares the ſeveral parts appear, and put them into the correſpondent ſquares in your paper; ſo at laſt you will have the picture of the whole object on your paper. Theſe ſquares may be drawn in black lead, and rubbed out afterwards.

There are alſo inſtruments contrived, by the help of which one may draw the appearance of any object; but the conſtruction of them is difficult, and therefore I ſhall ſay no more about them.

What is here ſaid of the projecting the image of a body upon a wall or any plane ſurface, is eaſily applicable to any curve ſurface. For wherever the ſhadow of any point falls by the light, there will be the image of that point. And likewiſe where the ſhadow of any line falls upon the curve ſurface, there will be the image of that line; for it will be where the plane paſſing thro' the luminous point and that line, cuts the curve ſurface. And the caſe is the ſame, if the figure is projected upon that curve ſurface, by rays drawn from the eye, placed where the light was. Fig.

PROB. XXIX.

To draw a deformed or monſtrous picture upon a plane, which ſhall appear regular, from a certain point.

Make a ſquare ABCD, and divide the ſides in- 80.
to as many equal parts as you will; and draw lines 81.
thro' all the points of diviſion, parallel to the ſides; theſe lines will divide the great ſquare into a number of little ſquares. In this ſquare draw the figure true which you want to appear deformed.

Again draw any line *ab*, which divide into as many equal parts as AB is divided into. From the middle point *e*, draw the line *e*V perpendicular to *ab*, and VS perpendicular to *e*V. Alſo draw the lines *a*V, *g*V, *h*V, *b*V, thro' the equal diviſions *g*, *h*, &c. Alſo draw *b*S, and where it cuts the former lines drawn to V, draw lines parallel to *ab*; then *abcd* is a deformed ſquare, and all the ſmall ſpaces repreſent the correſpondent little ſquares in the great one ABCD. Therefore in all the cells or little ſquares of *abcd*, draw the ſame part of the figure as you find in the correſpondent cells of ABCD; ſo you have the deformed image required. Then ſuppoſe VS to ſtand perpendicular to the plane *abcd*; if the figure be viewed by the eye

Fig. 80. 81. at S, it will appear in its true ſhape, as you have it in ABCD. But from any other point it will appear more or leſs deformed. And if the ſide *dc* was equal to the ſide DC, the figure would appear equally large, as that in ABCD would do, ſeen at the diſtance S*d*.

For if a ſquare whoſe ſide is *dc* be erected upon *dc*, and as many parallel lines drawn in it; and if SV be erected perpendicular to the plane *a*V*b*; then the firſt parallel line in the ſquare will be projected into the line 11, by the eye at S; and the ſecond parallel line into 22, the third into 33, and the laſt or top of the ſquare into *ab*; and the diagonal of the ſquare would be projected into *db*. Conſequently all the little ſquares would be projected into the correſpondent cells in the ſpace *abcd*. On the contrary, if the deformed figure *abcd* be given to be drawn in perſpective, its image will be the ſaid ſquare ſtanding upon *dc*, and all the little ſquares therein will be images of the reſpective cells in the figure *abcd*. And whatever is deſcribed in theſe cells, will appear in the little ſquares; and therefore the deformed picture drawn in the ſpace *abcd* will be a regular picture in the ſquare; that is, it will appear a regular head, ſeen from the point S in the air; but from any other point, the parts will appear to have different proportions, and therefore will be in ſome meaſure deformed.

This problem may be ſolved mechanically, by holding a candle at S in the air, and tracing out the ſhadow of the object (as the head here deſcribed) upon the plane ſurface given *abcd*, which may as eaſily be done upon a curve ſurface. But it is better if the candle ſhine thro' a ſmall hole, made in a plate ſet at S. The ſun's light may alſo be made uſe of for drawing either the original, or the deformed picture; which will be more or leſs ſo,

according to the obliquity of the plane, it is to be Fig.
drawn on. 80.

And by a like method, a deformed figure may 81.
be drawn upon the ſurface of a cone, or any ſolid; which will appear regular from a certain point of the axis.

Cor. 1. *Hence, the trapezium abcd is the ſhadow of the perpendicular ſquare whoſe ſide is dc. And every particular cell is the ſhadow of the correſpondent ſmall ſquare in the upright one.*

For the ſhadow of any point of the ſquare ſtanding on *dc*, is where the line drawn from the luminous point S, cuts the plane V*ab*. And the ſhadow of any line in the ſquare, will be where the plane paſſing thro' that line and the luminous point, cuts the ſaid plane. Therefore the ſhadow of the top of the ſquare, is the line *ab*; the ſhadow of one ſide is *ad*, and that of the other ſide of the ſquare is *bc*; and the baſe *dc* coincides with the ſhadow. And thus the ſhadows of the lines or ſides of any ſmall ſquare, are the lines that circumſcribe the correſponding trapezium or cell. And conſequently the ſpace contained in each cell, is the ſhadow of the correſponding ſmall ſquare.

Cor. 2. *Hence, a picture that appears rude and irregular, when viewed directly, will appear regular and uniform, when viewed from a proper point.*

For the object *abcd* looked at directly, appears only a long trapezium; but to the eye of a ſpectator at S, it appears a regular ſquare. For the ſhape of the image and all the parts of it in the picture, to the eye; is the very ſame as if the rays had come at firſt from that picture ſtanding perpendicular upon *dc*; becauſe they come in the ſame lines whether they come from the picture, or from the original. And if the colours of the ſeveral parts, and the ſhadows be the ſame in both, a perſon

Fig. a perſon will rather judge it to be regular than de-
80, formed; by being more accuſtomed to that, and
81. ſeeming more natural.

Cor. 3. *The practice of making deformed objects from regular ones, is no more than the inverſe method of perſpective.*

For in the direct method of perſpective, the trapezium *abcd*, lying on the geometrical plane is projected into a ſquare upon the line *dc*, to the eye at S, in the air. So on the contrary, a ſquare in the picture is projected into the deformed image *abcd*, on the ground plane. In the firſt caſe the picture is between the eye and the object; but in the latter, they lie contrary.

SCHOLIUM.

Since ſhadows are no more than the deformations of objects falling upon ſome plane or other: Therefore the drawing of the ſhadows of bodies depends on the ſame rules, as finding their deformations. The rays of light paſſing by the outlines of a body project their ſhadows upon theſe planes; and the places where they fall will be found, by having the poſition of the light, and the ſhape of the body. Therefore the practice of theſe things depends upon ſome or other of the rules before laid down.

PROB. XXX.

To ſhew the uſes of perſpective in drawing.

The rules of perſpective before laid down, are employed either in repreſenting figures that lye in the geometrical plane, or objects that are elevated above that plane; and all objects whatever are either one or the other of theſe; and therefore the foregoing rules are ſufficient for deſcribing any objects, and ought to be followed when we want any objects to be drawn perfectly exact, and mathematically true. And this is the beſt method when there are only right lined objects concerned, and but a few of them. But there are numerous objects that cannot be drawn this way, ſuch as mountains, trees, ſhips, &c. In ſuch caſes all we can do is to get a few of the principal points of ſuch objects, truly laid down; for to get all the points, is impoſſible, eſpecially where curve lines are concerned, and the figures are irregular; but the reſt of the work is to be done by hand. For the art of perſpective only lays down geometrical rules for finding the images of certain points, but as the number of points in any object is infinite, the work would be infinite; except in their ſingle caſe, where the points in the object happen to be connected by right lines; for then they will be right lines in the drawing, but in no other caſe; and therefore our only refuge is drawing by hand.

Then to gain a dexterity and exactneſs at drawing by hand; the beſt way is, firſt to draw ſome plain and ſimple objects in perſpective, and then try to imitate them by hand; and where you find any fault, rub it out, or elſe throw them away, and do them over again; ſtill endeavouring, to mend the former faults, and to make them more exact; which is as eaſy to do, as learning to write. When you have ſucceeded pretty well with one object, take a new one, and proceed the ſame way with that; and the like for as many as you pleaſe, till you improve in the art; and then you may make choice of more compounded objects which are harder to draw.

In objects that are more difficult to draw, let ſome of the principal lines be drawn by the rules of perſpective, and finiſh the reſt by hand; repeat this

Fig. this ſeveral times till you get them to your mind. Afterwards draw fewer of the lines in perſpective, and finiſh the reſt by hand. And laſt of all draw none of it in perſpective; but do the whole by hand; and that ſeveral times over, till it be tolerably exact. And thus by proceeding from eaſy objects to thoſe that are harder, you'll at laſt acquire a habit of drawing truly and readily. But it is practice alone that muſt compleat any perſon in this art of drawing, a multitude of rules ſignifying little or nothing.

When a complicated figure is to be drawn, it would be a difficult thing to do every ſmall part of it by the rules of perſpective; but by incloſing it in a ſquare divided into little ſquares, it may be done then by a perſon ſkilled in drawing. For where curve ſurfaces are concerned, the practical rules can only ſerve to find a few points thereof, or ſome right lines, that may incloſe it; by which means the drawer may then deſcribe them ſufficiently true.

From what has been ſaid, any one may ſee that it is in vain to endeavour, by the practical rules of perſpective, to deſcribe the hollows or riſings of objects, their lights and ſhadows, or any irregular turnings and windings thereof. Thus the capitals and ornaments of pillars, cannot be deſcribed that way, from the multitude of lines required to do it, and would never be exact. Nor can ſuch objects as have not any determinate ſhape, be deſcribed this way; ſuch as clouds, hills, trees, valleys, rivers, &c. but are far better done by hand. For in ſuch as theſe, there is more latitude allowed, and one can hardly miſtake ſo far as to make them appear monſtrous or unnatural. The turnings and windings of rivers, the ſhapes of trees, their branches and leaves; the limbs of animals, the features of men and women, the folds of their garments, could never be performed by the ſtrict rules of perſpective. Fig.

We muſt be content therefore, to apply the Art of Perſpective to what it is capable of doing to perfection; and that is, laying down a general draught, which is to ſerve for a ground-work for our future operations; in which the ſeveral places and ſituations of objects are to be aſſigned. But then the finiſhing and beautifying the piece muſt be left to the ſkilful hand of the drawer. Indeed it is moſt uſeful where it is moſt wanted, and that is in deſcribing ſuch objects as are terminated by right lines, eſpecially perpendicular ones; ſuch as buildings, pieces of architecture, regular figures, and ſuch like; where a deviation from their true ſhape or poſition would be very ſenſible. In other caſes, all we can do by the mathematical rules, is to find a few points which ſerve for a guide to draw the reſt by. Thus in deſcribing a circle, we find as many points in it by rule and compaſs, as we think ſufficient, and then carry a curve thro' them by hand; ſo that ſtrictly ſpeaking no part of this image is found by geometrical rules, but theſe few points; all the reſt owes its being and exactneſs to the judgment and hand of the artiſt.

The principal uſe then of Perſpective, is for informing the judgment; for by being accuſtomed to draw objects in perſpective, one may better know which way ſuch and ſuch lines ſhould run, and where they ſhould end, and to what points they ſhould converge; and from thence judge better how an object ſhould appear, by ſhewing the relation between the object and its image; and by that means directs the hand to draw them, and the eye to diſcover any defect in the drawing, which otherwiſe would not be ſeen. Likewiſe when the general plan, and the principal parts are laid down exactly by rule; the minuter parts will naturally come

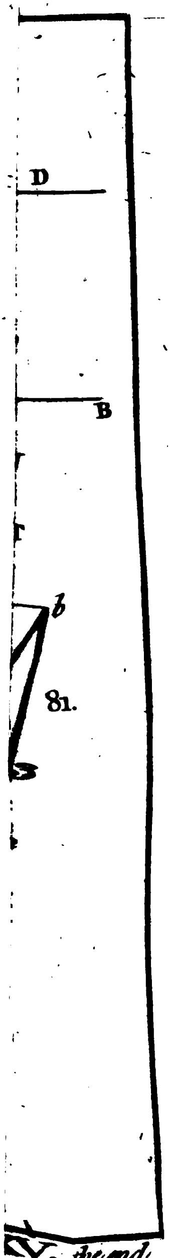
Fig. into their proper places, and all errors will then be eaſily avoided, and eaſily rectified. So that makeing uſe of the rules for drawing the principal parts, where they can conveniently be uſed; will give the picture ſuch a form as will guide the eye of the drawer, in deſcribing the other parts; and direct him to draw the reſt, and to finiſh the draught, and make it perfect and complete.

When objects are at a great diſtance, the magnitude muſt be diminiſhed in proportion; becauſe they appear under a leſſer angle than when nearer. And in ſhading them, the ſhadows muſt fall all one way, that is from the ſun. And in colouring them which ought to be done at laſt; the colours muſt be ſtronger near hand, and fainter further off; till at a great diſtance they loſe themſelves in a bluiſh miſt. And in all things you muſt perform the ſame thing in the picture, as you obſerve with your eye.

The buſineſs of *deſigning*, is the work of a maſter; this is forming a general idea of ſome large work, conſiſting of many parts, whoſe relation and connection, is entirely the invention of the artiſt. This may be ſome hiſtory piece, or contrived to ſhew ſome great tranſaction or event.

At firſt the artiſt muſt make only a rough draught of his deſign, and in a ſmall compaſs: this he is to correct and mend, as he ſees it neceſſary; and the outlines need only be drawn in black lead, to be rubbed out occaſionally. When he has fixed on the manner he would have it done, he may draw it larger and more compleat. And when he has got all the parts to his ſatisfaction, he muſt at laſt draw it, and all the parts of it, according to his deſign, and then proceed to the colouring it, if it is to be painted. Here drawing according to a deſign, is a different thing from drawing the pictures of objects, we have before us. For this picture or deſign has no object but what exiſts in the ideas and mind of the artiſt. And therefore to make an elegant piece, the artiſt ſhould have a good invention, and be well acquainted with the nature of things, that he may proportion things truly, in his draught; and obſerve that the connection of the ſeveral parts be according to nature; and that they be pleaſing and inſtructive. To do this requires a good genius, and a great deal of practice. His deſign is formed entirely in his own head, which is to be executed by the rules beforementioned. And this branch of the art is gained by degrees, firſt by drawing pictures by ſight; and afterwards copying from good original draughts; obſerving to draw the outlines firſt, and then the inner parts, and then to ſhadow them. And laſt of all, they that would become complete, muſt make themſelves maſters of Perſpective.

FINIS.



the end.

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